8-1966

The Use of the Function Concept in Teaching Arithmetic

Elece Tucker
Prairie View A&M College

THE USE OF THE FUNCTION CONCEPT IN TEACHING ARITHMETIC

TUCKER

1966

THE USE OF THE FUNCTION CONCEPT IN TEACHING ARITHMETIC

by

Elece Tucker

A Research Paper

Presented to

the Faculty

of the Department of Mathematics

In Partial Fulfillment

of the

Requirement for the Degree

Master of Science

Prairie View Agricultural and Mechanical College

Prairie View, Texas

August, 1966

This Thesis for the Degree Master

of Science Has Been Approved for the

Department of Mathematics

by

[redacted]

Adviser

[redacted]

Head of Department

7/15/66

Date

ACKNOWLEDGEMENT

The writer wishes to express her utmost appreciation to Dr. Stewart and Dr. Whitmore for their guidance, helpful suggestions and constructive criticism in the preparation of this research paper.

Secondly, she wishes to pay tribute to her husband, Mr. K. S. Tucker and daughters, Alice Faye, Sandra Louise, Patricia Ann, and Dimple Jean whose understanding and encouragement assisted in the final completion of this project.

E. T.

TABLE OF CONTENTS

eager, creative mind would surely rebel. However, if we broaden our thinking and look everywhere for patterns and relationships, the subject stimulates the mind to inquire and search out and discover.

Throughout discovery in mathematics, the main recurrent themes are structure and discovery. It must be remembered that the facts and mainpulation of arithmetic are not neglected. In fact one should find in the context of structure that pupils become more aware of patterns that relate these experiences, while in the context of discovery pupils search for relationship and become more actual partners in the learning process which helps them see where particular experiences fit into the general shape of things. The real concern is finding a solution for two interrelated problems: how can the children be helped. (a) to learn more mathematics, and (b) develop a deeper appreciation for important phases of mathematics.

The mathematics the children have been learning, arithmetic is completely valid--they simply aren't learning enough of it at an early age. Most of the children will drop mathematics as soon as they can, losing contact with a tool that is essential in coping with the problems of the time.

CHAPTER I

INTRODUCTION

In the present critical period the mathematical problem in our schools takes on added significance, causing educators to re-examine their objectives in order to ascertain ways in which the school can help develop more mathematicians. No other single problem is such a challenge and causes so much teacher frustration as the problem of the use of function concepts in teaching arithmetic.

An extended investigation of a single situation provide opportunities to nurture important mathematical ideas. As a record is kept, patterns begin to emerge. Without noticing those patterns, one would have no way to be quite sure he has exhausted the possibilities. Students should be encouraged to proceed systematically in investigation of this type. The problem stated has no solution yet, the children have all the tools they need to find the solution.

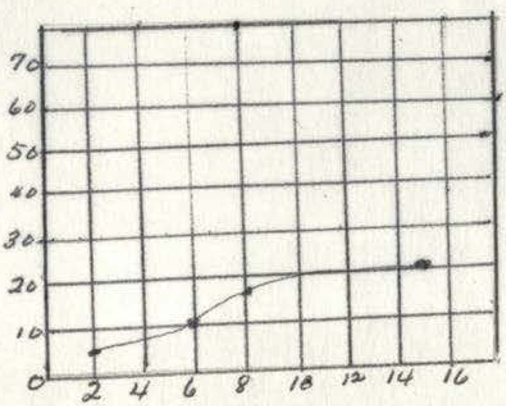
Mathematics is fundamentally a study of patterns and relationships and if arithmetic is limited to memory of number combination and rules for manipulation, then it is a dead, dull dreary task against which an active, There is much hue and cry to bring new and unusual content into the elementary school curriculum so the use of the function concepts should be introduced in teaching arithmetic.

Our daily activities continually furnish us with examples of things that are related to one another, of quantities which depend on certain other quantities, which change when certain other quantities change. Thus, a man's health is related to the food he eats, the exercise he takes and many other things.

The price paid for a certain quantity of sugar depends on the number of pounds bought and the price per pound. In all such cases, where some quantity depends on some other quantity or quantities, it can be stated that the former is related to the latter.

Speed in terms of time is shown and the curve is an idea of a function.

Number of Seconds After Start



Speed in Miles per hr.

Number of seconds after start

5	10	15	20

Speed in

2	6	8	15

Miles per hour

The Problem

Problems not only exist in mathematics but in every class room. These problems cannot or should not be ignored or avoided. They must be dealt with in one way or the other.

The major role of the elementary teacher is to help a pupil improve his adjustment and develop skills for dealing more successfully with the problems he encounters.

In arithmetic, there are a number of problems pupils are faced with. Certain verbal problems have been drawn from real situations, others have been invented to resemble these situations without reproducing them.

There is no reason for supposing that this shifting of mathematics has reached its end, a considerable change may be anticipated as the result of the demand of modern science that the student be prepared to make constant use of the tools and concepts of mathematics.

The problem content of texts in mathematics is dynamic, not static, adapting itself to human needs and to human interest and constantly changing to suit new circumstances.

Statement of the Problem

To facilitate the providing of learning experiences for children in a classroom a teacher must be able to cope with the numerous mathematical changes that are present and existing.

In this study the writer purposes to (1) Review studies that deal with most types of mathematical problems. (2) Determine the causes of mathematical problems. (3) Discover some effective techniques for minimizing these problems among children in classrooms.

Significance of the Study

The teaching of functional concepts should start in the early grades for learning of arithmetic is a gradual growth process that should be guided and directed at all stages by a systematical planned program. Learning should be goal centered and the learning activity should be purposeful. Learning is reacting. There can be no learning without pupil response and activity.

A teacher definitely should plan to explore and develop the interest and aptitudes of children who have unusual capacity and talent in the field of arithmetic. It is time that interest will ultimately determine in a general way, what the individual will do or become, provided he possesses the necessary abilities and opportunities.

Objectives

To further the development of the concepts of numbers.

To increase facility in using functional skills and develop new skills.

To develop, understand and appreciate the part mathematics has played in various fields of human activity.

To meet the needs of those who wish to acquire additional mathematics that is desirable for entering higher education.

To relate skills in one-to-one correspondence with the idea of a function.

Definitions, Symbols and Notations

Set - has the meaning of collection of elements, class or aggregate of elements enclosed in brackets - $\{ \}$

Binary operation - a rule where by to each pair of elements of the set there corresponds exactly one element.

raised dot - $\cdot$ multiplication

$\oplus$ Addition

R - relation - a set of ordered pairs $\{(1, 2), (3, 4)\}$

N - natural numbers, element of, belong to.

R_e - real numbers function

I - integers R^{-1} inverse of relation

$<$ less than F^{-1} " of the function F

CHAPTER II

FUNCTIONS

Given a set of numbers and a rule which assigns to each number in this set exactly one number in R, the resulting assoication of numbers is called a function. The given set is called the domain of the definition of the function, and the set of assigned numbers in R is called the range of the function.

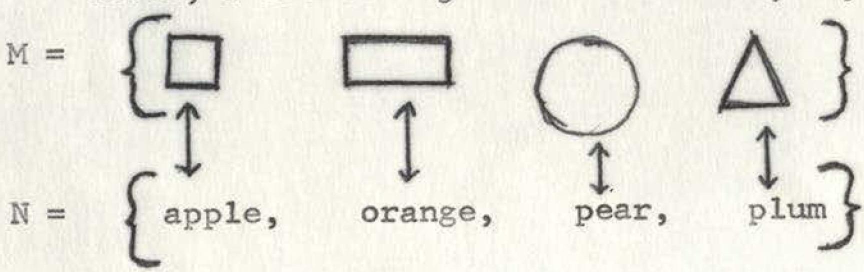
A function is usually designated by a letter, such as f, if f assign to each element in S exactly one element in R which can be indicated in the following ways.

$$f: x \longrightarrow y; \quad f(x) = y; \quad (x\ f(x)); \quad x \text{ in } s; \quad y \text{ in } R.$$

Note that f(x) is not f times x but rather f(x) is

Example:

Given, a set of objects or elements, say:



For each object or element in set M there is one and only one corresponding name or element in set N.

$>$ greater than

$\leq$ less than are equal to

$=$ equal to

Finite set - a set which contains a definite number of elements.

Sets are denoted by capital letters and written; for example as:

A = {1, 2, 3, 4} or A = {x 1 x is one of the 1st 4 counting nos.}

F (x) indicate that each x is paired off with the unique number assigned to x by the F- function.

formula - algebraic expression

domain - values on the x-axis

range - values on the y-axis

This is an idea of one-to-one correspondence and can be paired as a set of ordered pairs.

{(□ , apple) ; (▭ , orange) ; (○ , pear) ; (△, plum)}

A common misconception among students is that functions cannot be defined, in fact, do not exist unless there is a formula involved in the definition. He must be convinced that a function is a concept, an idea, and not a formula. There are many ways of representing a function. For example, a set of ordered pairs:

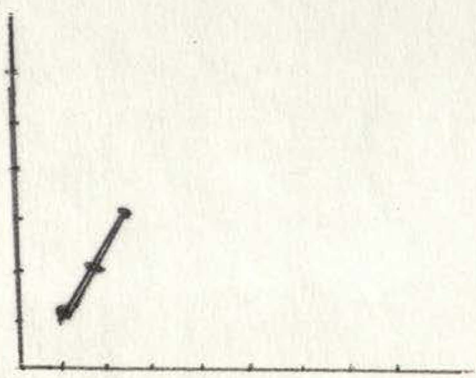
$$\{(1, 1), (2, 2), (4, 2) \ldots\}$$

A verbal statement: To each x in N assign one number $\sqrt{x}$ in R_e

An equation: $y = \sqrt{x}$, x in N, y in R_e

A formula: $f(x) = 2x + 1$

A graph



No one of the representations is the function but each describes the function. The point is that a function does not depend for its definition on its representation but only on its domain of definition and its rule of assignment. In general two functions are equal if their domain are the same set and their rule of assignment are the same, regardless of the manners in which they are represented.

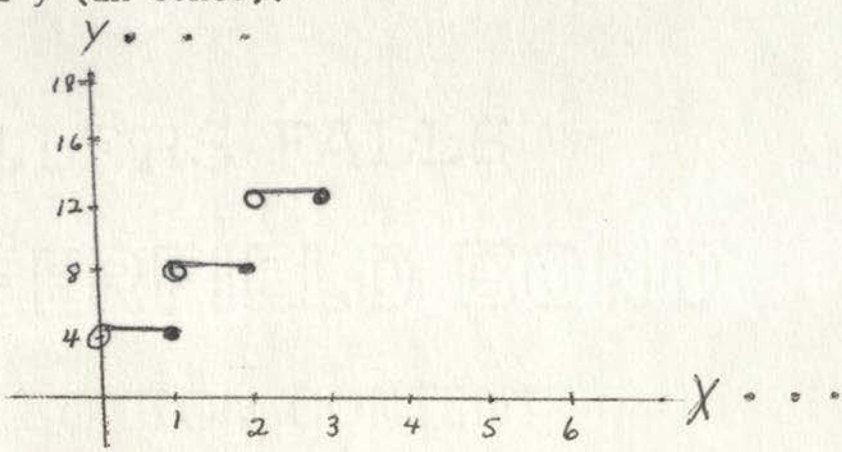
Example:

f: $a \longrightarrow 2a + 1$, a in I

g: $x \longrightarrow 2x + 1$, x in R_e

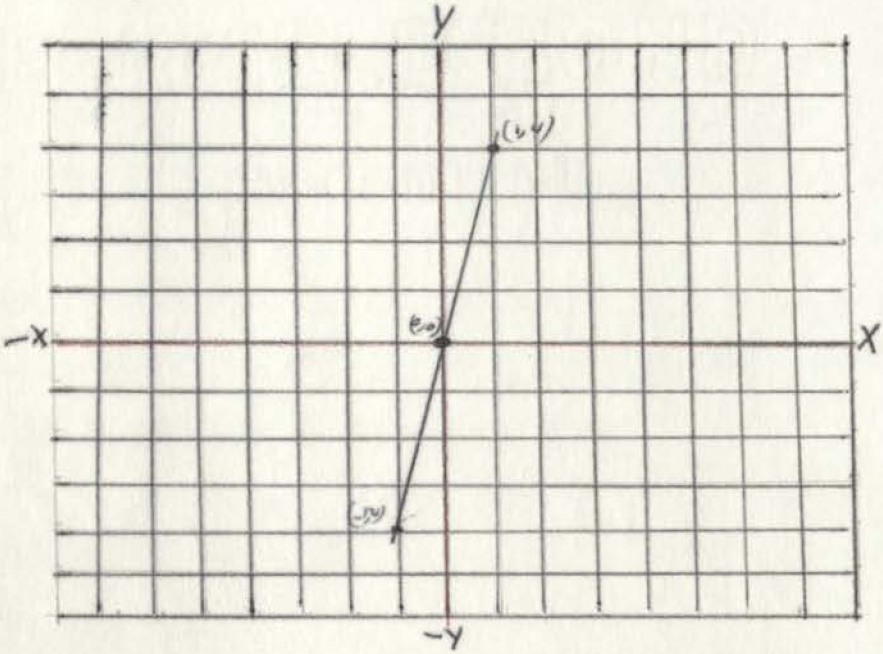
These are different functions because they have different domains, even though their rule of assignment are the same.

The graph below defines a function (a class postage regulation): to each real number x(in ounces) in the set $\{0 < x \leq 210\}$ there is assigned a natural number y (in cents).



To students the graphical representation of a function is probably more informative than any other.

Suppose we plot the set of ordered pairs $\{(0, 0), (1, 4) (-1, -4)\}$ on the squard chart below.



The straight line which connect the points of the relation represent the graph or function.

Given, a set of ordered pairs, say k = $\{(1, 2), (2, 4), (5, 7), (9, 10)\}$: each first element of the set is called the domain and each second element is called the range; such as domain = D $\{1, 2, 5, 9\}$; range = R $\{2, 4, 7, 10\}$

In reviewing the major ideas of algebra, note such statement as:

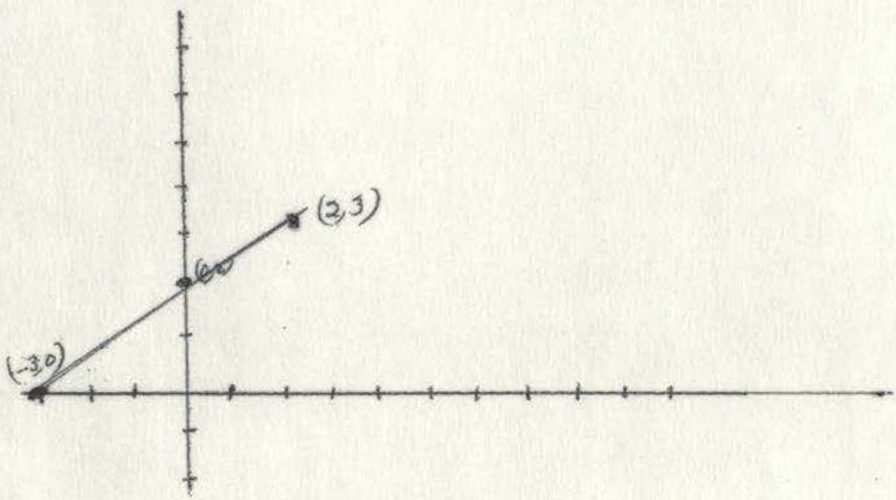
(1) Operation: For each pair of numbers a and b in R there is a unique number a + b in R. This operation assign to each pair of elements in R exactly one element in R. (2) Variables: Let X be the number of feet in the length of a rectangle. The variable X assign to each rectangle in the set of all rectangles exactly one number (of feet in its length) in R.

(3) Set of ordered pairs $\{(0, 2); (1, 2); (3, \pi); (4, \sqrt{2})\}$

This set of ordered pairs assigns to each element in $\{0, 1, 2, 3\}$ exactly one element in R.

(4) Graph of sentences:

This graph below assigns to each element x (abscissa) in R exactly one element y(ordinate) in R.



A common concept run through the above examples. In each some rule or operation or association or correspondence assigns to each element in a given set a unique element in R, resulting in a pairing off of elements from the two sets in such a way that <u>no two distinct elements of the second set are assigned to the same element of the first set</u>. This constitutes an idea of a function.

Laws or properties with respect to the Binary Operations $\oplus, \circ$, on R_e.

1. The commutative law for addition

   For all numbers x and y, $x + y = y + x$

2. Associative law for addition

   For all numbers x, y and z $(x + Y) + z = x + (y + z)$

3. The commutative law for multiplication

   For all numbers x, and y, $xy = yx$

4. The associative law for Multiplication

   For all numbers x, y, and z $(xy)z = x(yz)$

5. The distributive law for multiplication over addition

   For all numbers x, y, and z $x(y + z) = xy + xz$.

6. The law for adding 0

   For each number x, $x + 0 = x$

7. Closure

   A set of numbers have the closure property with respect to some operations if when we perform that operation on any numbers from the set we get a number that is in the set. Note both $\oplus, \circ$ are closed operations on R_e.

CHAPTER III

BINARY OPERATION OF ADDITION AS A FUNCTION

The binary operation of $\oplus$ is defined on the finite set - $\{0, 1, 2, 3\}$

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

If the set $\{0, 1, 2, 3\}$ is denoted or named by the letter T, then the example above may be denoted (T, $\oplus$).

In the table above, every entry is an element of T. If a, b are any elements of T then $a \oplus b$ is an element of T

If a, b are any elements of T, then $a \oplus b = b \oplus a$; ex.: say $a = 2$, $b = 3$; $2 + 3 = 3 + 2$

This state that the operation performed on the set $\{0, 1, 2, 3\}$ is commutative.

For any element a, b, c in T $a \oplus (b \oplus c) = (a \oplus b) + c$

ex. given $a = 2$, $b = 3$, $c = 4$; $2 \oplus (3 \oplus 4) = (2 \oplus 3) + (4)$.

This idea of regrouping of the operation performed on the set $\{0, 1, 2, 3\}$ is associative. Therefore, any elements a, b, c in T $a \oplus (b+c) = (a \oplus b) + c$ is associative.

Among the elements of T the element 0 is the identity element for $\oplus$ because $0 \oplus a = a \oplus 0 = a$ for any element a in T. That is, any element of T is left unchanged when operating with the identity.

In the table each row and each column contains the identity element, 0 exactly once. This follows that each element of T has an inverse under $\oplus$. That is to each element a in T there corresponds an element b in T such that $a \oplus b = b \oplus a = 0$.

On the set of numbers $\oplus$ is a binary operation for combining two numbers at a time you will get one and only one third number, that is if a and b are numbers there is just one number c, $c = a \oplus b$.

This type of operation can be considered as a function. ex. let $a = 2$, $b = 3$, be two numbers. Their sum $2 + 3 = 5 = c$, for the pair (2, 3) we get 5 as a result of our operation.

A set is said to be closed under a binary operation if when you perform the operation on two elements of the set and get a number which is in the set. ex. given, the set $A = \{1, 2, 3, 4\}$, under addition the set is not closed, because 1 ⊕ 4 = 5 and 5 is not an element of set A.

Consider the set of even numbers - $\{2, 4, 6, 8 \ldots\}$ Each time you add two even numbers the sum will be even. This set satisfies the closure property and is closed under addition.

Given a set of real numbers

Let F denote +

G denote ·

Then F(a, b) = F(b, a) = a + b = b + a commutative with respect to addition

G(a, b) = G(b, a) = ab = ba

Commutative with respect to multiplication

example: Let a = 3, b = 2

then F (3, 2) = 3 + 2 = 5

and F(2, 3) = 2 + 3 = 5 satisfies the commutative property G(3, 2) = 3 x 2 = 6

F(a, 0) = F(0, a) = a state the identity property

ex. F(3, 0) = F(0, 3) = 3.

If a, b and c are real numbers then F(F(a, b), c) = F(a, F(b, c) = (a + b) + c = a + (b + c) shows that the operation is associative.

For any a, b

F(a, -a) = F(-a, a) = 0 = the inverse

example: F(4, -4) = 4 + (-4) = 0

Other examples of inverse

Open the door .... Close the door
Turn on the light .... Turn off the light
Add 6 .... Subtract 6

CHAPTER IV

BINARY OPERATIONS OF MULTIPLICATION AS A FUNCTION

From the first set $\{0, 1, 2, 3\}$ the operation is defined

(A)

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Reverse Order (B)

•	3	2	1	0
3	1	2	3	0
2	2	0	2	0
1	3	2	1	0
0	0	0	0	0

From the table A above we can follow (2.2).3 = (3.2).2 If we check all possible cases we will find that the associative property hold.

If for any distinct element a of a set, the result of every binary operation of multiplication on the set involving a and any second element of the set is the same as the second element, then a is the identity. example, let b = to the second number then a.b = b; if a = 1, b = 3 also 1.3 = 3

The reverse order shows that the operation is commutative.

Again let G denote . and F denote + For any a, b G(a, b) = ab G(b, a) = ba then for G(3, 2) = 6; G(2, 3) = 6. This satisfies the commutative property for multiplication.

For any number a, b, c

G(a, G(b, c)) = G(G(a, b), c)

example let a = 2, b = 3, c = 4

then G(2, G(3, 4)) = 24 = G(G(2, 3), 4) = 24

which satisfies the associative property for multiplication.

For the distributive property over addition

Let a, b, c, be any number, then a(b+c) = ab+ac

example, G(a, F(b, c) - G(a, (b + c)) =

F(G(a, b), G(a, c) G(2, F(3, 4) = G(2, (3 + 4) =

2(3 + 4) = 6 + 8 = 14 = F(G(2, 3), G(3, 4)

Suppose that we have the function F and h where

F(x) = 2x and h(x) = x - 1. Consider F(h (4);

h(4) = 3 Then F(h (4)) = F(3). Since F(3) = 6, we

see that F(h (4) = 6

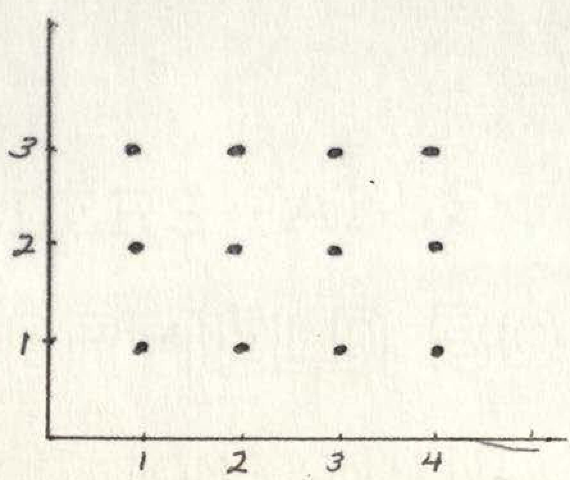
For the inverse operation

G(a, 1/a) = G(1/a, a) = 1 for a $\neq$ 0

and G(a, 1) = G(1, a) = a

Example - Given A = $\{1, 2, 3, 4\}$ c = $\{1, 2, 3\}$

Graph of A x C



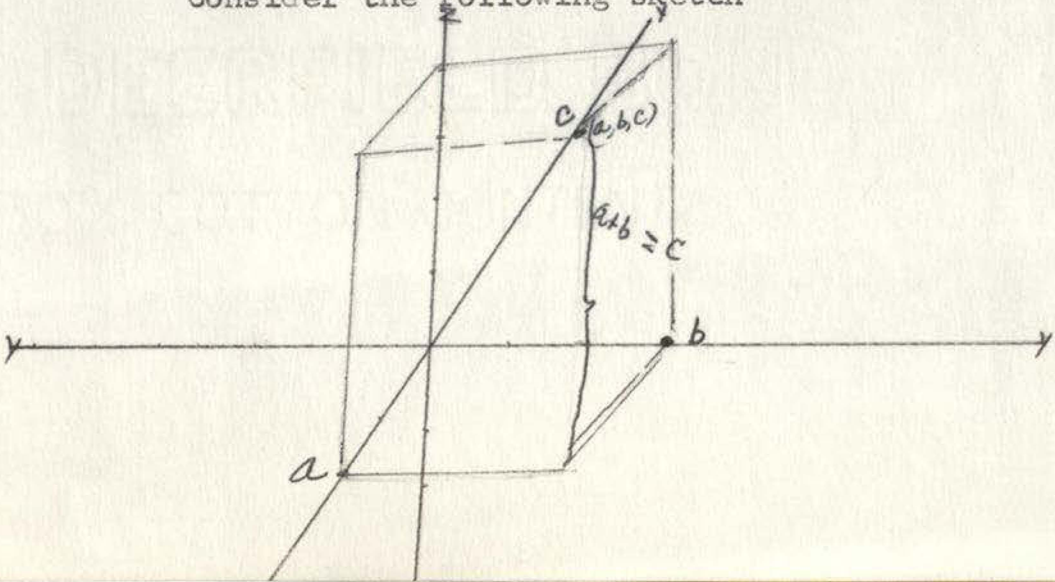
Then A x C = (1, 1); (1, 2); (1, 3);
(2, 1);(2, 2);(2, 3) (3, 1)
(3, 2) (3, 3), (4, 1),(4, 2)
(4, 3)

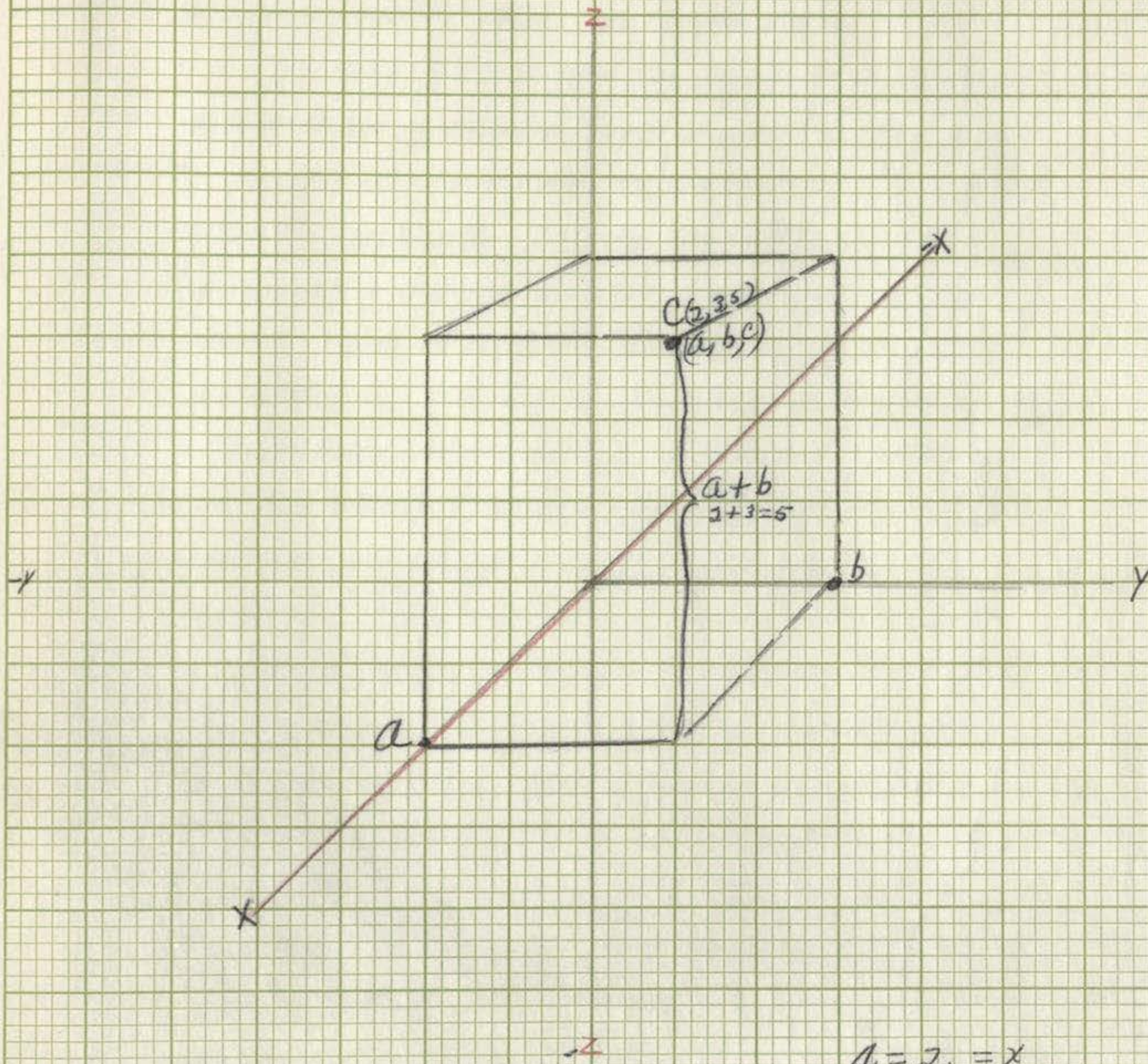
a + 1 correspondence can be established between the sets of points of a plane and the set of ordered pairs of real numbers by associating each point in the plane with an ordered pair of real numbers.

If the $R = \{(1, 2), (1, 3), (1, 4)\ (2, 3)\}$ and you interchange the set $\{(2, 1), (3, 1), (4, 1), (3, 2)\}$ you get R^{-1} called the inverse Sum Function $(F + G)x = F(x) + G(x)$ Let F be the function defined by $F(x) = 2x$ and G be the function defined by $G(x) = x^2 - 3$

Then $F = \{(x, F(x) \mid F(x) = 2x\} = \{(x, 2x) \mid F(x) = 2x\}$ and $G = \{(x, G(x) \mid G(x) = x^2 - 3\} = \{(x, x^2 - 3) \mid G(x) = x^2 - 3\}$ and $F + G = \{(x, F(x) + G(x) \mid F(x) + G(x) = x^2 + 2x - 3\}$

Consider the following sketch





$a = 2 = x$

$Y = 3$ or $b = 3$

$Z = 5$ or $c = 5$

Taken in pairs, the three coordinate axes determine three coordinate planes, called the xy-plane, the yz plane, and the xz plane.

For this illustration the domain of F is all the number plane or ordered pairs (a, b) and the range is the set of all numbers on the number line perpendicular to the plane. This is the combining of two numbers at a time to give one and only one third number. That is if a & b are numbers there is just one number c, $c = a + b$. This type of operation can be considered as a function in this way. Given a set E we can define a binary operation on E as a function from the product set (E x E) to the set E. Our function in this case is a set of ordered triples rather than pairs. Consider E to be the set of all numbers then E x E is the set of all ordered pairs in the plane representing points. If we add the two elements in our ordered pair we obtain another in E. For example: let $a = 2$, $b = 3$. The two numbers then their sum $2 + 3 = 5 = c$. For the pair (2, 3) we get 5 as a result of our operation. So we can consider the binary operation of adding as a function (F) from E x E to E that is F = (a, b, c) a and b are numbers and $a + b = c$

We can see that for each first pair (a, b) there is one and only one c therefore, F is a function in that it is a subset of (E x E) x E and no two of the triples have the same first pair.

If a, b are numbers there is just one number c, c = ab also. If we multiply the two elements in our ordered pair we obtain another in E. example: let a = 2, b = 3 then their product = 2(3) = 6 = c; for the pair (2, 3) we get 6 as a result of our operation. So we can consider the binary operation of multiplication as a function (F) from E x E to E that is F = (a, b, c) a and b are numbers

and a (b) = c

SUMMARY

Functions, in the arithmetic program, in its ultimate sense describes the relationship of objects and how they are related with like problems.

The arithmetic program must be related to the objectives of the High School programs if it is to make an appropriate contribution. Objectives change from time to time, reflecting the changing needs of society; present day mathematics is concerned with the development of talents of every individual to the maximum of his capabilities so that he may assume his proper role in the field of mathematics.

The arithmetic program should include those activities designed to assist the pupil in his adjustment and to assist toward the attainment of worthwhile goals and objectives. The mathematic program is only one phase of the total educational program; The elementary and high school mathematics teacher should work cooperatively with the curriculum in assisting students to get a sound background or basic fundamentals in mathematics which will enable them to succeed in higher mathematics.

It is essential and possible that we utilize a systematic approach to the evaluation of the arithmetic program.

RECOMMENDATION

On the basis of the research in this study, the following recommendations are made:

1. Give clear understanding of the operation for it is one of the major goals of all mathematics.

2. Graphs are used for motivation and to help the students see relationships.

3. Material should progress from concrete examples to abstract symbols.

From the research data collected here, teachers could use the above recommendations as a frame of reference from which to operate if they really want to get started with helping elementary children become well adjusted.

CONCLUSION

As a result of the study the following conclusions seem justifiable.

1. Encouragement of the widest possible use of manipulative material in the early years of children's experiences with arithmetic. We want them to understand clearly the relationship between the real world and arithmetic. That is arithmetic is a language we use to talk about events such as moving objects around. We are using every opportunity to dramatize

this fact. There is an attempt to present patterns or structure in arithmetic and new ideas. The notions of relations and functions are explored and used to unify the project.

2. Drill can be interesting, it can be effective and it can draw children deep into the excitement of mathematics. We must be basically concerned with the study of patterns and relationship in mathematics if we are to pass on to children the important aspects of this past of our culture.

All scientists, all observers of the physical world, most of us in every day life are continually playing game of "Whats my Rule?" with nature - and nature has some very tricky rules. Some might like to explore further by looking for clues in the statement of a rule. How to follow certain rules will lead to the plotting of graphs in arithmetic.

When we search for patterns or relationships, we must always be prepared for children to use normal and interesting approaches to a problem.

BIBLIOGRAPHY

Books

Baldwin, George and Gorch, Ralph, Mathematics for Elementary Teachers, (New York: John Wiley and Sons, Inc,. 1954).

Brumfield, Charles E. and Others, Fundamental Concepts of Elementary Mathematics, (New York: Addison-Wesley Publishing Co, Inc., 1962).

Clarkson, Donald R., Understanding Today's Mathematics, (New York: The Shoe String Press, Inc.)

Hamilton, Norman T. and Landin, Joseph, Set Theory The Structure of Arithmetic,.(Boston: Allyn and Bacon, Inc., 1961).

Morgan, Frank M., Elementary Mathematical Analysis, (New York: The MacMillan Company, 1918).

Selby, Samuel and Sweet, Leonard, Sets- Relations - Functions, (New York: McGraw-Hill Book Co., Inc, 1963).

Articles and Periodicals

Gray, Roland F., and Others, "The Arithmetic Teacher," Journal of The National Council of Teachers of Mathematics, Vol. 13, No. 5, (1966) pp. 183-185.

"The Sevolution in School Mathematics," National Council of Teachers of Mathematics, (Washington, D. C.: 1961), pp. 15, 22-26.

Writz-Botel Nunley, "Discovery In Elementary School Mathematics", Encyclopedia Britannica Press, (1963), pp. ii, iv.