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A Novel Fuzzy Time Series Forecasting Method Based on Probabilistic Fuzzy Set and CPBD Approach

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Abstract

Probabilistic fuzzy set is used to model the non-probabilistic and probabilistic uncertainties simultaneously in the system. This study proposes a cumulative probability-based discretization and probabilistic fuzzy set based novel fuzzy time series forecasting method. It also proposes a novel discretization approach based on cumulative probability to tackle the probabilistic uncertainty in partitioning of datasets. Gaussian probability distribution function has been used to construct probabilistic fuzzy set. The advantage of the proposed work is that it addresses the uncertainties due to randomness and fuzziness simultaneously and also improves accuracy rate in time series forecasting. A proposed forecasting method is applied on two time series data set of enrollments of University of Alabama and Taiwan Exchange (TAIEX). A reduction of the amount of average forecasting error rate (AFER) and root mean square error (RMSE) shows the proposed method outperforms over other existing forecasting methods.

Keywords: Uncertainties; Forecasting; Probabilistic fuzzy set; Probabilistic fuzzy logical relation

MSC 2010 No.: 62-07, 65C50, 65C60

1. Introduction

Time series forecasting plays a challenging role in various research areas. Statistical methods, e.g., Moving average, Weighted moving average, Autoregressive moving average, and Autoregressive integrated moving average are generally used for time series (TS) forecasting. However, these models deal with probabilistic uncertainty in TS forecasting but are unable to handle uncertainty due to linguistic representation, vagueness and imprecision of TS datasets. Fuzzy set (Zadeh (1965)) based TS forecasting resolves these kinds of limitations of forecasting methods.

Song and Chissom (1993a,b, 1994) established time variant and time invariant TS forecasting methods using fuzzy set and implemented to forecast enrollments of University of Alabama. Chen (1996) proposed arithmetic operators based fuzzy time series (FTS) forecasting method to avoid the complicated computations of max-min composition operators but later on he was observed that more accuracy in forecast using max-min composition operators rather than arithmetic operators. Later on Chen (2002), Song (2003), Chen and Chung (2006), Lee et al. (2007), Huarng and Yu (2006), Aladag et al. (2009, 2010), Huang et al. (2011), Wang et al. (2013), Aladag (2013), Kuo et al. (2009, 2010), Egrioglu et al. (2014, 2019, 2021) developed various FTS forecasting models with novelty in fuzzification or in partitioning the length of intervals to increase accuracy rate in forecasting. Jain et al. (2018), Gangwar and Kumar (2015, 2016), Joshi and Kumar (2012), Singh (2007) proposed computational method for constructing intervals and high order FTS forecasting to deal with large datasets.

Fuzzy set may not be handling with non-determinacy in the system using non-membership and membership grades. Atanassov (1986, 1999) extended fuzzy set to intuitionistic fuzzy set (IFS) to model with non-determinacy in the system using separate function for non-membership and membership grades. Later on Wang et al. (2016), Kumar and Gangwar (2016), Gautamand Singh (2018), Bisht and Kumar (2019), Bas (2020), and Gupta and Kumar (2020) proposed intuitionistic FTS forecasting method based on intuitionistic fuzzy reasoning, IFS and probabilistic fuzzy set (PFS) to deal with non-determinacy and enhance the accuracy in forecasting. Simultaneous occurrence of non-probabilistic and probabilistic uncertainties in system is the challenge of researchers. To overcome these issues, Meghdadi and Akbarzadeh (2001) introduced the concept of PFS to model non-probabilistic and probabilistic uncertainties simultaneously. Fialho et al. (2016), Hinojosa et al. (2011), Almeida and Kaymak (2009), and Liu and Li (2005) have implemented it in risk analysis, mortality rate prediction and control problem. Recently, Pattanayak et al. (2021, 2022a,b), Gupta and Kumar (2019a,b,c, 2022) and Kocak (2017) proposed many forecasting methods based on fuzzy theory in probabilistic environment.

Motivated with incorporation of fuzzy and probability theory in the form of PFS and its application in TS forecasting, we proposed cumulative probability-based discretization (CPBD) and PFS based TS forecasting method. Advantage of CPBD is that it may be handle probabilistic uncertainty during partitioning of TS data and PFS simultaneously deal with probabilistic and non-probabilistic uncertainties. In the proposed method, Gaussian probabilistic distribution function used to assign the probabilities to membership grades. Verification of the suitability of the proposed method is applied on TS data of enrollments of University of Alabama and TAIEX and to compare with various existing forecasting methods.

2. Cumulative Probability-Based Discretization (CPBD) Approach

The proposed method uses a new discretization approach for partitioning TS data. Cumulative probability-based discretization (CPBD) approach uses cumulative probability to find length of intervals and it is explained in following steps.

1. Define $X=[X_1,X_2]$ as universe of discourse. Here, $X_1=X_{\min}-\sigma$ and $X_2=X_{\max}+\sigma$. σ and $X_{\min}$, $X_{\max}$ are standard deviation and minimum, maximum of dataset.
2. Choose k intervals (u_i; $i=1,2,...,k$) using round of $\sqrt{n}$. Here, n is the size of data set. Intervals of datasets may be defined by the following expression:
$$u_i=[X_1+(i-1)h, X_1+ih],\ i=1,2,...,k;\ \ h=\frac{X_2-X_1}{k}.$$
3. Assign the data to their respective intervals and noted their probabilities by favorable number of elements to the interval divided by total number of elements in dataset. Compute the cumulative probabilities of the intervals and take cumulative probability (C_{Pmax}) corresponding to maximum probability of the interval.
4. Compute deciding factor (D_F) for the intervals as:
$$D_F=\begin{cases} k/2; & \text{if } k \text{ is even,} \\ (k+1)/2; & \text{if } k \text{ is odd.} \end{cases}$$
5. Determine new interval (NI) length using C_{Pmax} and h as follows:
$$NI=\frac{C_{P_{\max}}}{D_F}\times h;\ \text{here, } h \text{ is the previous interval length.}$$
6. Now, X is divided into m intervals (u_i; $i=1,2,...,m$) based on NI. Each interval of time series data set defined as follows:
$$u_i=[X_{1_i},X_{2_i}],\quad i=1,2,...,\quad 1\le X_{2_i}<X_2.$$
Here, $X_{1_i}=X_{1_i}+(i-1)NI,\ X_{2_i}=X_1+iNI$.

3. Proposed method with experimental study

In this section, we have to explain methodology of CPBD and PFS based FTS forecasting method and forecasting results of implementation to forecast TS data of enrolments of University of Alabama. Every step of CPBD approach is applied in proposed forecasting method.

Step 1: Apply CPBD approach to construct the intervals $u_i=[X_{1_i}+(i-1)NI,\ X_1+iNI]$, $(i=1,2,...m)$ for TS dataset of University of Alabama enrollments (Table 1). Construct fuzzy sets $(A_i, i=1,2,...,m)$ as follows:

$$A_i = \begin{cases} [X_{1_i} + (i-1)NI, X_{1_i} + iNI, X_{1_i} + 2iNI], & i = 1,2,...,m-1, \\ [X_{1_i} + (i-1)NI, X_{1_i} + iNI, X_{1_i} + iNI], & i = m. \end{cases}$$

Table 1. Dataset of University of Alabama enrollments

Year	Enrollments	Year	Enrollments	Year	Enrollments
1971	13055	1979	16807	1987	16859
1972	13563	1980	16919	1988	18150
1973	13867	1981	16388	1989	18970
1974	14696	1982	15433	1990	19328
1975	15460	1983	15497	1991	19337
1976	15311	1984	15145	1992	18876
1977	15603	1985	15163		
1978	15861	1986	15984		

Construct nineteen fuzzy sets $A_1, A_2, A_3, ..., A_{19}$ in accordance with $u_i (i = 1, 2, 3, \ldots, 19)$ using CPBD approach of data of University of Alabama enrollments. The triangular fuzzy sets $A_i (i = 1, 2, 3, \ldots, 19)$ are described as follows:

$A_1 = [11280, 11786.54, 12293.09]$, $A_2 = [11786.54, 12293.09, 12799.63]$, $A_3 = [12293.09, 12799.63, 13306.18]$,
$A_4 = [12799.63, 13306.18, 13812.72]$, $A_5 = [13306.18, 13812.72, 14319.27]$, $A_6 = [13812.72, 14319.27, 14825.81]$,
$A_7 = [14319.27, 14825.81, 15332.36]$, $A_8 = [14825.81, 15332.36, 15838.9]$, $A_9 = [15332.36, 15838.9, 16345.45]$,
$A_{10} = [15838.9, 16345.45, 16851.99]$, $A_{11} = [16345.45, 16851.99, 17358.54]$, $A_{12} = [16851.99, 17358.54, 17865.08]$,
$A_{13} = [17358.54, 17865.08, 18371.62]$, $A_{14} = [17865.08, 18371.62, 18878.17]$, $A_{15} = [18371.62, 18878.17, 19384.71]$,
$A_{16} = [18878.17, 19384.71, 19891.26]$, $A_{17} = [19384.71, 19891.26, 20397.8]$, $A_{18} = [19891.26, 20397.8, 20904.35]$,
$A_{19} = [20397.8, 20904.35, 20904.35]$.

Step 2. Fuzzify TS data using A_i and Gaussian probability distribution function is used to associate the probabilities to corresponding membership grades,

$$p(\mu_i) = \begin{cases} \dfrac{l}{\sqrt{2\pi}\zeta_j}\left(e^{-\frac{\left(x_i - (\mu_i - 1)l_j - m_j\right)^2}{2\zeta_j^2}} + e^{-\frac{\left(x_i - (1-\mu_i)l_j - m_j\right)^2}{2\zeta_j^2}} \right), & \mu_i \in [0,1], \quad \left(p(\mu_i) \in [0,1], \quad i = 1,2,...,n\right) \\ 0, & \textit{otherwise.} \end{cases}$$

Here, m_j, l_j, ζ_j and μ_i are standard deviation, length of interval, mean and membership grade of fuzzy set. PFS with maximum probability of membership grade is assigned to TS data by the following algorithm.

for i= 1 to n (last data point)
    for j= 1 to m (last intervals)

choose

$p(\mu_{ki}) = max\left[p(\mu_1), p(\mu_2), \ldots, p(\mu_k), \ldots, p(\mu_m)\right],\ 1 \le k \le m$

if F_k is PFS corresponds to $p(\mu_{ki})$ *then assign F_k to x_i.*

end if

end for

end for

Step 3. Proposed method use data of year 1 to *n* for framing rules to apply on probabilistic fuzzy logical relation (PFLR) $F_i \to F_j$; here, F_i is current state and F_j is the next state. To forecast next data, take union of PFLRs which are assigned to the previous observations. PFLRs are constructed and probabilistic fuzzy logical relationship groups (PFLRGs) are constructed (Table 2).

Table 2. PFLRs and PFLRGs for enrollments

PFLRs		PFLRGs
$F_4 \to F_5$	$F_8 \to F_8$	$F_4 \to F_5$
$F_5 \to F_5$	$F_8 \to F_8$	$F_5 \to F_5, F_7$
$F_5 \to F_7$	$F_8 \to F_8$	$F_7 \to F_8$
$F_7 \to F_8$	$F_8 \to F_9$	$F_8 \to F_8, F_9$
$F_8 \to F_8$	$F_9 \to F_{11}$	$F_9 \to F_9, F_{11}$
$F_8 \to F_9$	$F_{11} \to F_{14}$	$F_{10} \to F_8$
$F_9 \to F_9$	$F_{14} \to F_{15}$	$F_{11} \to F_{10}, F_{11}, F_{14}$
$F_9 \to F_{11}$	$F_{15} \to F_{16}$	$F_{14} \to F_{15}$
$F_{11} \to F_{11}$	$F_{16} \to F_{16}$	$F_{15} \to F_{16}$
$F_{11} \to F_{10}$	$F_{16} \to F_{15}$	$F_{16} \to F_{15}, F_{16}$
$F_{10} \to F_8$		

Establish fuzzy row vector (FRV) by applying max-min composition operations on PFLR.

Step 4. Use aggregation operator to aggregate elements of probabilistic FRV $(\mu_i \mid p_i; i = 1, 2, \ldots, n)$ and obtain a FRV,

$$f_i = \left(1 - (1 - \mu_i)^{p_i}\right)^{1/p_i}\ ; f_i \in [0,1].$$

Finally, apply following centroid defuzzification formula to defuzzify FRV to have numerical forecast.

$$\text{Numerical forecast} = \frac{\sum_{i=1}^{n} f_i c_i}{\sum_{i=1}^{n} f_i};\ \text{here, } c_i \text{ are the centroids for the intervals.}$$

Proposed FTS forecasting method is implemented on Taiwan Exchange (TAIEX) dataset from 01-11-2004 to 31-12-2004 (Table 3). Twenty three intervals (u_i; i = 1,2, …, 23) are constructed for TAIEX dataset using CPBD approach and triangular fuzzy sets (A_i; i = 1,2, …, 23) are constructed in their accordance. PFSs are constructed using triangular membership function of fuzzy sets (A_i) and also PFLRs and PFLRGs are constructed for TAIEX time series data. Table 5 shows the outperformance of forecasted TAIEX dataset in terms of RMSE using proposed method.

Table 3. Actual TAIEX data

Date	TAIEX data	Date	TAIEX data	Date	TAIEX data	Date	TAIEX data
01-11	5656.17	17-11	6028.68	03-12	5893.27	21-12	5987.85
02-11	5759.61	18-11	6049.49	06-12	5919.17	22-12	6001.52
03-11	5862.85	19-11	6026.55	07-12	5925.28	23-12	5997.67
04-11	5860.73	22-11	5838.42	08-12	5892.51	24-12	6019.42
05-11	5931.31	23-11	5851.1	09-12	5913.97	27-12	5985.94
08-11	5937.46	24-11	5911.31	10-12	5911.63	28-12	6000.57
09-11	5945.2	25-11	5855.24	13-12	5878.89	29-12	6088.49
10-11	5948.49	26-11	5778.65	14-12	5909.65	30-12	6100.86
11-11	5874.52	29-11	5785.26	15-12	6002.58	31-12	6139.69
12-11	5917.16	30-11	5844.76	16-12	6019.23		
15-11	5906.69	01-12	5798.62	17-12	6009.32		
16-11	5910.85	02-12	5867.95	20-12	5985.94		

4. Performance analysis

Verify the superiority of proposed method based on CPBD approach by RMSE and AFER. Following mathematical expression define RMSE and AFER,

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(F_i - O_i\right)^2}{n}},$$

$$AFER\ (in\ \%) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|F_i - O_i\right|}{O_i}\times 100.$$

The proposed method is compared with previously existing methods proposed by Pant and Kumar (2021), Pattanayak et al. (2020), Gupta and Kumar (2019c), Egrioglu (2012), Kumar and Gangwar (2016), Joshi and Kumar(2012), and Bisht and Kumar (2016) in terms of RMSE and AFER and is shown in Table 4. Reduce amount of RMSE and AFER of proposed method over previous existing methods (Pant and Kumar (2021), Pattanayak et al. (2020), Gupta and Kumar (2019c), Bisht and Kumar (2016), Kumar and Gangwar (2016), Joshi and Kumar (2012) and Egrioglu (2012)) and verify superiority of CPBD approach and PFS based proposed FTS forecasting method.

Table 4. Forecasted outputs of enrolments and compare with other existing models

Actual Enrolment	Pant and Kumar (2021)	Pattanayak et al. (2020)	Gupta and Kumar (2019c)	Bisht and Kumar (2016)	Kumar and Gangwar (2016)	Joshi and Kumar (2012)	Egrioglu (2012)	Proposed method
13055	-	13055	-	-	-	-	-	-
13563	13682	13637	13559.71	13595.67	13693	14250	13480.63	13573.83
13867	13682	14120	13559.71	13814.75	13693	14246	13480.63	14130.11
14696	14722	14408	14781.24	14929.79	14867	14246	14242	14130.11
15460	15427	15195	15333.51	15541.27	15287	15491	15710	15311.4
15311	15544	15712	15677.52	15540.62	15376	15491	15484.55	15563.48
15603	15544	15635	15677.52	15540.62	15376	15491	15935.65	15563.48
15861	15544	15786	15677.52	15540.62	15376	16345	15935.65	16250.29
16807	16665	15918	15965.56	16254.5	16523	16345	16837.86	16250.29
16919	15994	16406	17001.21	17040.41	16606	15850	17499.69	16972.75
16388	17230	16466	17001.21	17040.41	17519	15850	17499.69	16972.75
15433	15994	16190	15965.56	16254.5	16606	15850	16737	15315.84
15497	15544	15698	15677.52	15540.62	15376	15450	15484.55	15563.48
15145	15544	15731	15677.52	15540.62	15376	15450	15484.55	15563.48
15163	15516	15550	15677.52	15541.27	15287	15491	15710	15563.48
15984	15516	15559	15677.52	15541.27	15287	15491	15710	15563.48
16859	16665	15982	15965.56	16254.5	16523	16345	16837.86	16250.29
18150	17230	16433	17001.21	17040.41	17519	17950	17499.69	16972.75
18970	18820	17366	19159.7	18902.3	19500	18961	19144.4	18900.54
19328	19311	17967	19132.11	19357.3	19000	18961	19144.4	19381.67
19337	19311	18230	19132.11	19168.56	19500	18961	19144.4	19131.44
18876	19311	18236	19132.11	19168.56	19500	18961	19144.4	19131.44
RMSE	**420.3**	**754.21**	**470.08**	**428.63**	**493.56**	**433.76**	**484.61**	**418.92**
AFER (%)	**1.87**	**3.46**	**2.21**	**1.94**	**2.33**	**2.24**	**2.21**	**1.94**

The proposed forecasting method is compared and found superior than existing methods based on fuzzy set, IFS and PFS developed by Chen et al. (2012), Joshi and Kumar (2012), Cai et al. (2015), Chen and Chen (2015), Cheng et al. (2016), Wang et al. (2016), Kocak (2017), Gupta and Kumar (2020) in terms of RMSE. Even though RMSE in forecasted TAIEX is slightly higher than that of Wang et al. (2016), but the capability of the proposed method to handle both types of uncertainties make it better than the method of Wang et al. (2016).

Table 5. Comparison of forecasted TAIEX data in terms of RMSE with existing methods.

Methods	RMSE	Methods	RMSE
Chen et al. (2012)	52.27	Wang et al. (2016)	43.23
Joshi and Kumar (2012)	52.63	Kocak (2017)	50.12
Cai et al. (2015)	50.33	Gupta and Kumar (2020)	49.03
Chen and Chen (2015)	53.63	**Proposed method**	**45.36**
Cheng et al. (2016)	54.25		

5. Conclusion

Simultaneous occurrence of non-probabilistic and probabilistic uncertainties is the main constraints of statistical method and fuzzy logic-based TS forecasting methods. In this study, we have proposed a CPBD approach and PFS based FTS forecasting method that may be handle both non-probabilistic and probabilistic uncertainties simultaneously and also improves the accuracy in TS forecasting. Outperformance of proposed method is shown in forecasting of two time series data of enrollments of University of Alabama and TAIEX. Reduced amount of AFER and RMSE in TS data confirms outperformance and validity of proposed FTS forecasting method based on CPBD approach and PFS.

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