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Aparajita Borah
National Institute of Technology, Sikkim, India

Gajendra Pratap Singh
Jawaharlal Nehru University

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Some Generalizations of Corona Product of Two Graphs

[1]Aparajita Borah and [2*]Gajendra Pratap Singh

[1]Department of Mathematics
National Institute of Technology
Sikkim, 737139, India
[2]School of Computational and Integrative Sciences
Jawaharlal Nehru University
New Delhi, 110067, India
[1]aparajita.juri@gmail.com; [2]gajendra@mail.jnu.ac.in

*Corresponding Author

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Abstract

In this paper we are seeking to conceptualize the notion of corona product of two graphs to contrive some special types of graphs. That is, here our attempt is to regenerate a familiar graph as a product graph. We are considering seven familiar graphs here to reconstruct them with the help of corona product of two graphs. Such types of families of the graphs and operations can be used to study biological pathways as well as to find the optimal order and size for the special types of graphs.

MSC 2020 No.: 05C30, 68R10

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1. Introduction

The concept of corona product is a neoteric addition in the mathematical dictionary. It is a novice concept to the mathematical world which is still very much in its developmental stage. In layman's terminology, Corona Product can be viewed as the concoction of two or more different/same structures to create a gigantic structure in no time. In mathematical parlance, it is the product of two or more difference/same graphs, i.e., having same/different degrees/nodes to create a larger network in a limited time. It may be noted that there are other methodologies that do exist for creating a larger network. The enigma may emanate in the minds of few regarding the utmost emphasis that the mathematical community is putting on this novice concept. One can easily create a large network from a given set of nodes and links by simply adding one after another. However, such a methodology demands lots of time and involves a bucket of costs.

The significance and importance of corona product (Barik et al. (2007); Indulal and Dragan (2015); Gopalapillai (2011); Laali et al. (2016); Liu and Zhou (2012); McLeman and McNicholas (2011); Rinurwati et al. (2017); Singh et al. (2020); Suprajitno (2016)) resides not only in creation of a larger network/graph but the time within which it is created and the cost incurred in its creation. In the modern era, where data and objects are moving in such a rapid speed, corona product will be certainly considered more advantageous in terms of speed and cost, and therefore, it outweighs its concomitant and contemporary algorithms/methodologies on that score. Further, the corona product can be utilized in enhancing the recent emergence of science such as Internet of Things, Virtual Reality and Augmented Reality, etc. Hence, corona product is of prime importance in our study (Frucht and Harary (1970); Jannesari and Omoomi (2012)). The formal mathematical definition of corona product of two graphs is given in Section 2.

2. Preliminaries

The corona product of two graphs, say G and H, was introduced by Frucht and Harary in 1970, and it is a graph constructed by taking n instances of H and each such H gets connected to each node of G, where n is the number of nodes of G. In this paper we will implement the definition of corona product to visualize some existing graphs.

For example, let $G = \bullet_1 \text{——} \bullet_2 \text{——} \bullet_3$ and $H = \bullet_a \text{——} \bullet_b$. Then GoH is given by

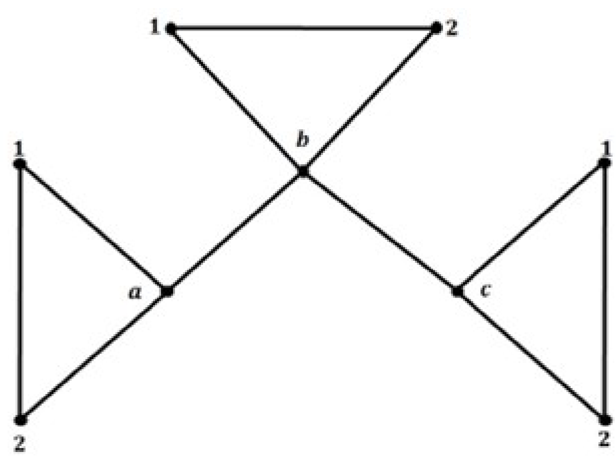

Figure 1. Corona Product of Graphs G and H, GoH

In this example, we have considered two graphs G and H of orders 3 and 2, respectively. Hence, by following the definition, we have one copy of G and 3 (order of G) copies of H in the corona product of G and H, GoH.

The objective of this paper is to further extend and enrich the concept of the corona graph using the existing body of knowledge and to verify its various paradigms, both functional and non-functional in order to fully comprehend the concept in reality.

We have considered the analytical and theoretical perspective while writing this paper where we have singularly implemented the concept of graph product. We have considered seven widely known graphs such as the complete graph, star graph, null graph, double star graph, wheel graph, and sunlet graph.

3. Results

Lemma 3.1.

The corona product of K_1 and K_r, $K_1 o\, K_r$, where $r = 1, 2, \cdots, n$, is K_{r+1} or the r^{th} corona product of K_1, $K_1 o^r\, K_1$ is K_{r+1}.

Proof:

Let G_1 be K_1 and G_2 be K_r. Since we are taking the corona $K_1 o\, K_r$, therefore, by definition of the corona product of two graphs, we will take one copy of K_1 and $\mid V(K_1) \mid$, that is, one copy of K_r. Now, by definition of the corona product, join the single node of K_1 to each node of K_r. Then, we will get a node which is adjacent to each node of K_r. That is how we have constructed a complete graph of size $r + 1$, (for $r = 1, 2, ..., n$), i.e, K_{r+1}. Alternatively, let us prove this by induction method. Let us assume that this is true for $n = k$. So, now we have to show that it is also true for $n = k + 1$. $K_1(\cdots((K_1 o\, K_1)\cdots)o\, K_1)o\, K_1 = K_k$, for k copies of K_1. So, now $K_k o\, K_1$ consists of $k + 1$ copies of K_1. Hence, $K_k o\, K_1 = K_{k+1}$.

Now let us take the corona product of K_1 with K_1 itself.

$K_1 o\, K_1$ will be K_2 as shown in the Figure 2.

Figure 2. Corona product K_2 as $K_1\, oK_1$

$(K_1 o\, K_1)o\, K_1$ will be K_3.

$((K_1 o\, K_1)o\, K_1)o\, K_1$ will be K_4.

Lemma 3.2.

The corona product of complete graph K_1 and null graph with cardinality n, N_n, $K_1 o\, N_n$, is the star graph S_n.

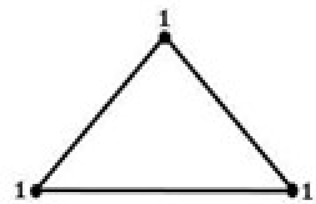

Figure 3. Corona product K_3 as $K_1\, o(K_1\, oK_1)$

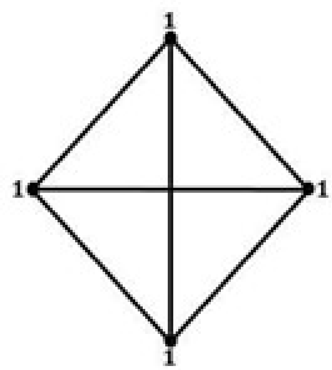

Figure 4. Corona product K_4 as $K_1\, o((K_1\, oK_1)oK_1$

Proof:

Let G_1 be K_1 and G_2 be disconnected null graph N_n. Now let us take the corona product of K_1 and N_n, $K_1 o\, N_n$. Let u be the single vertex of K_1 and $v_1, v_2, v_3, \cdots, v_n$ be the vertices of null graph N_n. As shown in Figure 5, place the vertex u in the center and join it to each vertex of N_n. Thus, we will have a graph with center vertex u and n edges, i.e., $K_{1,n}$. Now take a star graph S_n with center vertex s_1 and n leaves as shown in Figure 6.

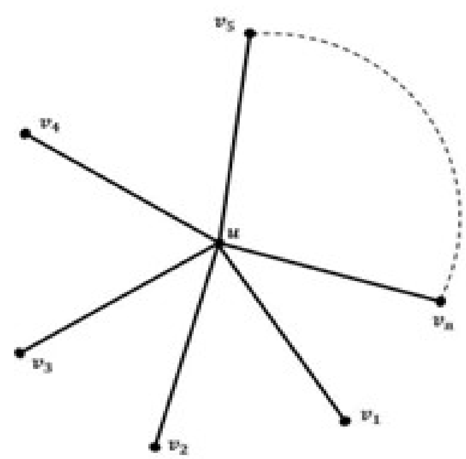

Figure 5. Corona product $K_1\, oN_n$

Now, let us prove the isomorphism between $K_1 o\, N_n$ and S_n. Let us assume a function f such that $f(u) = s_1, f(v_1) = s_2, f(v_2) = s_3, \cdots, f(v_n) = s_{n+1}$ is a one-to-one map. We can easily check the adjacency tenacity between vertices $u, v_1, v_2, v_3, \cdots, v_n$ and $s_1, s_2, s_3, \cdots, s_n, s_{n+1}$ via the map f. ■

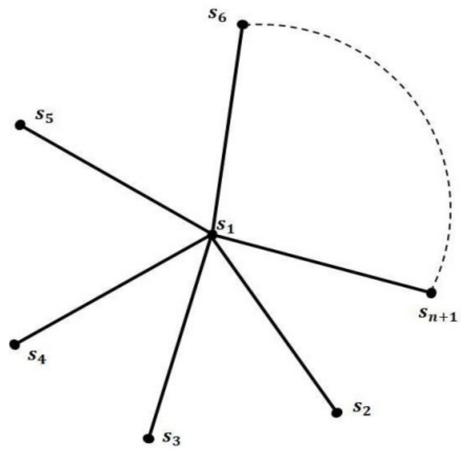

Figure 6. Star Graph S_n

Lemma 3.3.

The corona product of null graph with cardinality n, N_n and complete graph K_1, $N_n o\ K_1$, is the disconnected graph with $n\ K_2$ copies.

Proof:

Let G_1 be null graph with cardinality n, N_n and G_2 be K_1. Now, for the corona product $N_n o\ K_1$, we will have n copies of K_1 joined to each singleton vertex of N_n. Thus, we will get n disconnected edges as shown in Figure 7. This is nothing but a disconnected graph with $2n$ vertices and n edges, that is, $n\ K_2$ copies. ■

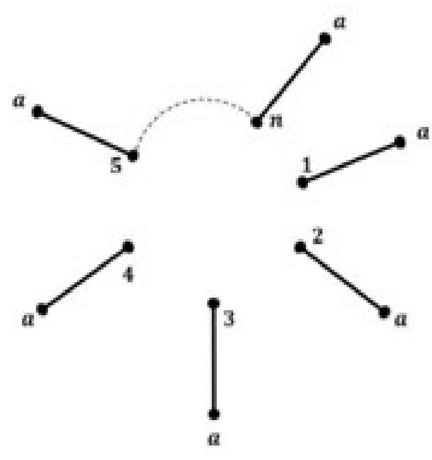

Figure 7. n copies of K_2

Lemma 3.4.

The corona product of complete graph K_2 and null graph of cardinality $n\ N_n$, $K_2 o\, N_n$, is the double star graph with $n\ D_{n,n}$ copies.

Proof:

Let G_1 be K_2 and G_2 be disconnected null graph N_n. Now, let us take the corona product of K_2 and N_n, $K_2 o\, N_n$. Let u_1 and u_2 be the vertices of K_2 and $v_1, v_2, v_3, \cdots, v_n$ be the vertices of null graph N_n. As shown in Figure 8, join one copy of N_n to each vertex of K_2. Thus, we will have a tree with two non pendant vertices u_1 and u_2 with degree n.

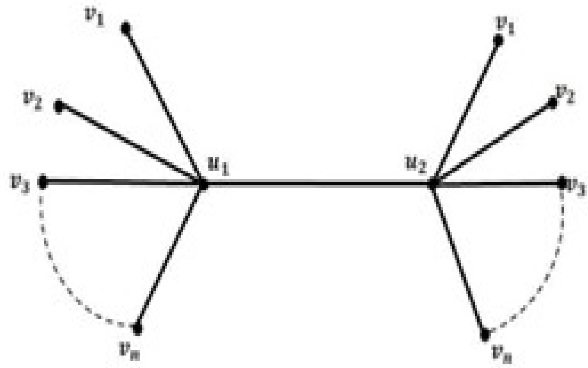

Figure 8. Corona Product $K_2\, oN_n$

Now, take a double star graph $D_{n,n}$ with vertices $d_1, d_2, d_3, \cdots, d_n, \cdots, d_{2n}, d_{2n+1}, d_{2n+2}$ as shown in the Figure 9.

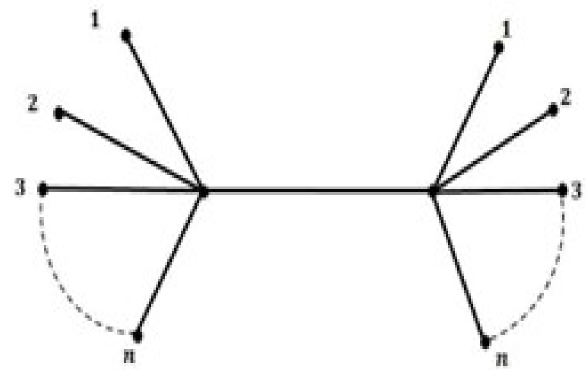

Figure 9. Double star Graph $D_{n,n}$

Now, let us prove the isomorphism between $K_2 o\, N_n$ and $D_{n,n}$. Let us assume a function g such that $g(u_1) = d_1, g(u_2) = d_2, g(v_1) = d_3, \cdots, g(v_n) = d_{n+2}, g(v_1^{'}) = d_{n+3}, g(v_2^{'}) = d_{n+4}, \cdots, g(v_n^{'}) = d_{2n+2}$, is a one-to-one map. We can easily check the adjacency tenacity between vertices $u, v_1, v_2, v_3, \cdots, v_n$ and $s_1, s_2, s_3, \cdots, s_n, s_{n+1}$ via the map g. ■

Lemma 3.5.

The corona product of complete graph K_1 and cycle graph of cardinality n C_n, $K_1 o\, C_n$, is the wheel graph W_n.

Proof:

Let G_1 be K_1 and G_2 be cycle graph C_n. Now, let us take the corona product of K_1 and C_n, $K_1 o\, C_n$. Let a be the single vertex of K_1 and $v_1, v_2, v_3, \cdots, v_n$ be the vertices of cycle graph C_n. As shown in Figure 10, place the vertex a in the center and join it to each vertex of C_n. Thus we will have a graph with center vertex a and n edges, i.e., $K_{1,n}$. Now take a wheel graph W_n with internal vertex w_1 as shown in the Figure 11.

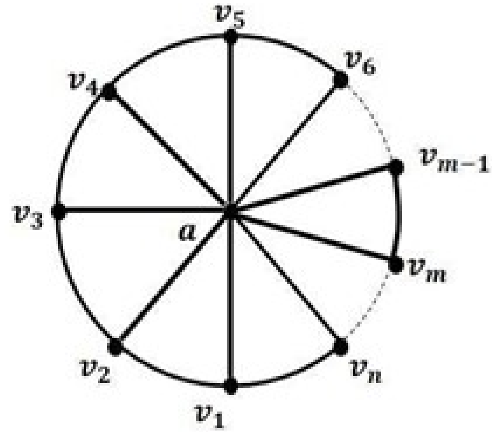

Figure 10. Corona product of K_1 and $C_n(K_1\, oC_n)$

Now let us prove the isomorphism between $K_1 o\, C_n$ and W_n. Let us assume a function f such that $f(a) = w_1, f(v_1) = w_2, f(v_2) = w_3, \cdots, f(v_n) = w_{n+1}$ is a one-to-one map. We can easily check the adjacency between vertices $a, v_1, v_2, v_3, \cdots, v_n$ and $w_1, w_2, w_3, \cdots, w_n, w_{n+1}$ via the map f.■

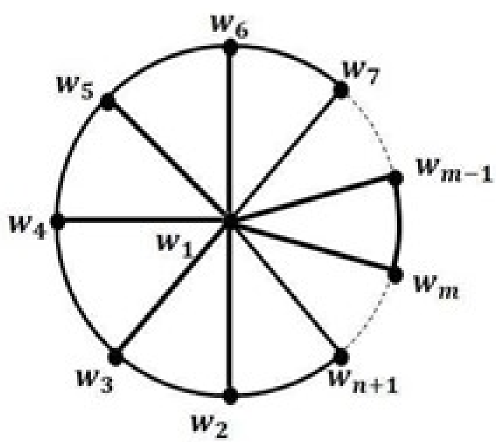

Figure 11. Wheel Graph W_n

Lemma 3.6.

The corona product of cycle graph of cardinality n, C_n and complete graph K_1, $C_n o\, K_1$, is the n-sunlet graph.

Proof:

Let G_1 be cycle graph C_n and G_2 be K_1. Now let us take the corona product of C_n and K_1, $C_n o\, K_1$.

Let a be the single vertex of K_1 and $v_1, v_2, v_3, \cdots, v_n$ be the vertices of cycle graph C_n. Place the vertex a outside the circle C_n join the vertex a to each vertex of C_n as shown in Figure 12. Thus, we will have a graph with circle C_n with n pendant vertices.

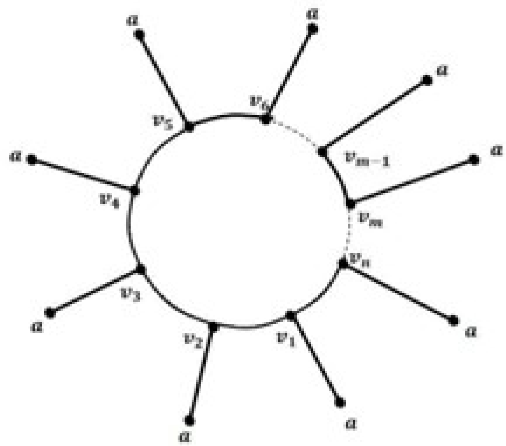

Figure 12. Corona Product $C_n\ oK_1$

We can easily show that this resultant graph is isomorphic to any n-sunlet graph in terms of adjacency. ∎

We can also study the corona product of graphs with some special types of graphs and digraphs that are used as modeling tools in different disciplines where we model with the help of Petri Nets (Jangid and Singh (2023); Singh (2014); Singh et al. (2022); Singh et al. (2023)).

4. Conclusions and Future Scope

This paper primarily dealt with the technique to generate a new graph in the most convenient theoretical approach to understand the new graph. Here we employed the basic definition of a corona product to perceive some of the existing and well-known graphs. This paper will surely help to understand the larger graph along with its components by means of a product graph.

The main motive behind writing this paper was to bring out the notion of corona product of two graphs in understanding known graphs, which later leads us to study the known larger graph as multiplication of two smaller graphs, but not as a bigger graph. Hence, this paper will certainly help the researchers to understand the idea of product graph more explicitly, especially the corona product which is already existing as known graphs of bigger size. The purpose of this study is not to define anything novel, but to analyze the anatomy of an existing concept. In addition to the structural properties, the adjacency properties of these graphs can also be understood in term of corona product of two graphs. Some of the open challenges for future work are given below:
1. Is it possible to define the corona product in Petri Nets and its reachability tree?
2. Does there exist the class of Petri Nets whose reachability tree generates the results discussed in the paper?

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