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Stability of Predator-Prey Model for Worm Attack in Wireless Sensor Networks

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Abstract

In this paper, we propose a predator-prey mathematical model for analyzing the dynamical behaviors of the system. This system is an epidemic model, and it is capable of ascertaining the worm's spreading at the initial stage and improving the security of wireless sensor networks. We investigate different fixed points and examine the stability of the projected model.

Keywords: Susceptible Nodes Stability; Infectious Nodes; Predator-Prey; Worms

MSC 2020 No.: Primary: 92D30, 60H10 Secondary: 34D23

1. Introduction

A wireless sensor network is a collection of sensing devices that are used to monitor and control real-time physical or environmental conditions such as noise, motion, temperature, humidity, and the reliability of the system. WSNs are being created by a large number of small sensor nodes. Sensor nodes are tiny devices that assemble units like sensing, memory, processing energy sources, an analog-to-digital converter, a transceiver, communication, etc. The most important problem in WSN is energy consumption and security. In this way, epidemic models are playing a wide role as a tool, such as improving energy efficiency (Rathna and Sivasubramanian 2012), virus contagion (Lu. Yanling and Jiang 2014), and knowledge transmission (Cao et al. 2016).

It is very important to detect worm propagation behavior in WSNs. Many researchers have developed worm propagation models to detect and monitor worm attacks in WSNs. Castaneda et al. (2004) discussed four different techniques for detection and prevention based on the anti-worm predation method. In some studies, by Kephart and White (1991), (1993), Yang et al. (2008), (2012), Mishra and Pandey (2011), and Wei et al. (2011), They defined different types of interactions between worms in epidemiological models, such as worms with vertical transmission, improving sensor network immunity under worm attacks, and virus propagation models and their dynamics. In ecological studies, we examine predator-prey correlations among different species that are common. An eco-epidemiology is based on predator-prey models along with susceptible-infected-susceptible. It was expended by Lotka-Volterra dynamics in Hethcote et al. (2004), Mishra and Keshri (2014), and Greenhalgh et al. (2017), where two predator-prey models on the attack of malicious objects on networks were proposed. Predator-prey models on wireless sensor networks (WSN) were proposed by Srivastva et al. (2016). The basic concept of the computer worm is to gain access to another device so that it can replicate itself on the new device and reproduce further. We have to use antivirus software for controlling the attacks of computer worms in devices, though the attacking behaviors of different worms are different, so we study the literature on the attacking behaviors of worms in wireless sensor networks (Mishra and Ansari (2009), Yang and Yang (2012), Bera et al. (2015)). It is based on predator-prey mathematical models. These models introduce the various challenges being faced due to worm attacks in the WSN network, which is based on the predator-prey concept. As per the above information, we have created a mathematical model that is based on the predator-prey concept and that makes it possible to survive and examine the feasible conditions of WSN for security improvement. Hence, we further investigate the various fixed or equilibrium points and explore the conditions of stability for fixed points.

2. Model Formulation

In this model, $X(t)$, $Y(t)$, $Z(t)$, $L(t)$ $R(t)$ and denotes the number of nodes in the classes of prey susceptible, infected prey susceptible, predator susceptible, infected predator susceptible, and recover susceptible, respectively, at time t. Then the transmission representation of a mathematical model for worm attacks is shown in Figure 1.

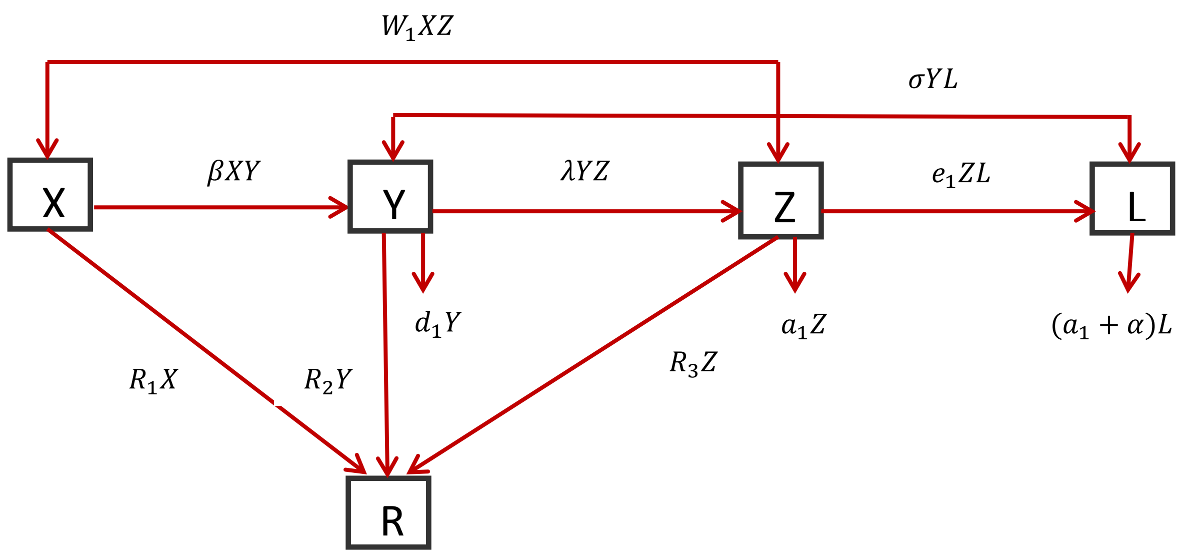

Figure 1. Predator-Prey Model

The above model can be exposed by the following set of non-linear differential equations, such as:

$$\left.\begin{aligned}
\frac{dX}{dt} &= rX\left(1-\frac{X}{k}\right)-\beta XY - w_1XZ - R_1X,\\
\frac{dY}{dt} &= \beta XY - d_1Y - \lambda YZ - \sigma YL - R_2Y,\\
\frac{dZ}{dt} &= b_1\omega_1XZ + b_2\lambda YZ - e_1ZL - a_1Z - R_3Z,\\
\frac{dL}{dt} &= n_1YL + e_1ZL - (a_1+\alpha)L,\\
\frac{dR}{dt} &= R_1X + R_2Y + R_3Z,
\end{aligned}\right\} \tag{1}$$

Here for all t,

$$X(t)+Y(t)+Z(t)+L(t)+R(t)=N(t);$$

Table 2. The nomenclature used in the model

Parameters/Variables	Description
β	The infection rate of class X
W_1	Predation rate of class X to class Z
k	Carrying capacity of prey X
r	The growth rate of class X in the absence of predator without infection
d_1	The death rate of infected class Y
λ	The death rate of the infected class X to Y
σ	The death rate of the infected class Y to L
b_1	Conversion factor rate for class X to class Z
b_2	Conversion factor rate for class Y to class Z
e_1	The infection rate of the predator population L
a_1	The natural death rate for the predator population
n_1	Conversion factor rate for class Y to class L
α	The death rate of the infected class L due to infection
R_1, R_2, R_3	Rate of recovering of the class X, Y, Z, respectively

3. Fixed Points of the System

(i) Fixed point

$P_0(0,0,0,0)$, always exists;

(ii) Fixed point

$P_1\left(\frac{k(r-R_1)}{r},0,0,0\right)$, exist if $r > R_1$;

(iii) Fixed point

$P_2\left(\frac{d_1+R_2}{\beta},\frac{r-R_1}{\beta}-\frac{r(d_1+R_2)}{r\beta^2},0,0\right)$, exist if $(d_1+R_2)>0$ and $r\beta(r-\lambda_1)+r(d_1+R_2)>0$;

(iv) Fixed point

$P_3\left(0,\frac{a_1+R_3}{b_2\lambda},\frac{(-d_1-R_2)}{\lambda},0\right)$, exist if $(a_1+R_3)>0$ and $(-d_1-R_2)>0$;

(v) Fixed point

$P_4\left(0,0,\frac{a_1+\alpha}{e_1},\frac{(-a_1-R_3)}{e_1}\right)$, exist if $(a_1+\alpha)>0$ and $(-a_1-R_3)>0$;

(vi) Fixed point

$P_5\left(\frac{a_1+R_3}{b_1w_1},0,\frac{kb_1w_1^2(r-R_1)-r(a_1+R_3)}{kb_1w_1^2},0\right)$, exist if $(a_1+R_3)>0$ and $kb_1w_1^2(r-R_1)-r(a_1+R_3)>0$;

(vii) Fixed point

$$P_6\left(\frac{(r-R_1)k}{r},0,0,\frac{kb_1w_1(r-R_1)-r(a_1+R_3)}{re_1}\right),$$ exist if $r>R_1$ and $kb_1w_1(r-R_1)-r(a_1+R_3)>0$;

(viii) Fixed point

$$P_7\left(0,\frac{(a_1+\alpha)}{n_1},0,\frac{b_2\lambda(a_1+\alpha)-n_1(a_1-R_3)}{n_1e_1}\right),$$ exist if $a_1+\alpha>0$ and $b_2\lambda(a_1+\alpha)-n_1(a_1-R_3)>0$;

(ix) Fixed point

$$P_8\left(\begin{array}{l}\left[\dfrac{n_1(a_1+R_3)-b_2\lambda(a_1+\alpha)-e_1b_2(d_1+R_2)}{n_1b_1w_1-e_1b_2\beta}\right],\\ \dfrac{(r-R_1)}{\beta}-\dfrac{r}{k\beta}\left[\dfrac{n_1(a_1+R_3)-b_2\lambda(a_1+\alpha)-e_1b_2(d_1+R_2)}{n_1b_1w_1-e_1b_2\beta}\right]\\ -\dfrac{w_1}{\lambda\beta}\left[\dfrac{n_1\beta(a_1+R_3)-n_1b_1w_1(d_1+R_2)}{n_1b_1w_1-e_1b_2\beta}\right],\\ \left[\dfrac{n_1\beta(a_1+R_3)-n_1b_1w_1(d_1+R_2)}{\lambda n_1b_1w_1-\lambda e_1b_2\beta}\right],0\end{array}\right),$$

exist if $r-R_1>0$, and $n_1(a_1+R_3)-b_2\lambda(a_1+\alpha)-e_1b_2(d_1+R_2)>0$, $n_1\beta(a_1+R_3)-n_1b_1w_1(d_1+R_2)>0$;

(x) Fixed point

$$P_9\left(\begin{array}{l}\left[\dfrac{e_1k(r-R_1)-w_1k(a_1+\alpha)}{re_1}\right],0,\dfrac{(a_1+\alpha)}{e},\\ \left[\dfrac{\lambda w_1b_1(a_1+\alpha)+b_1w_1(d_1+R_2)-\beta(a_1+R_3)}{e_1\beta-\sigma b_1w_1}\right]\end{array}\right),$$

exist if $e_1k(r-R_1)-w_1k(a_1+\alpha)>0$ $(a_1+\alpha)>0$ and $\lambda w_1b_1(a_1+\alpha)+b_1w_1(d_1+R_2)-\beta(a_1+R_3)>0$;

(xi) Fixed point

$$P_{10}\left(0,\left[\frac{e_1(r-R_1)+w_1(a_1+\alpha)}{\beta e_1+n_1w_1}\right],\left[\frac{\beta(a_1+\alpha)-n_1(r-R_1)}{\beta e_1+n_1w_1}\right],\right.$$
$$\left.\left[\frac{\lambda n_1(r-R_1)-\lambda\beta(a_1+\alpha)-(\beta e_1+n_1w_1)(R_2+d_1)}{\sigma(\beta e_1+n_1w_1)}\right]\right),$$

exist if $e_1(r-R_1)+w_1(a_1+\alpha)>0$ and $\beta(a_1+\alpha)-n_1(r-R_1)>0$, $\lambda n_1(r-R_1)-\lambda\beta(a_1+\alpha)-(\beta e_1+n_1w_1)(R_2+d_1)>0$;

(xii) Fixed point

$$P_{11}\left(\begin{array}{l}\left[\dfrac{kn_1(r-R_1)+k\beta(a_1+\alpha)}{n_1r}\right],\left[\dfrac{(a_1+\alpha)}{n_1}\right],0,\\ \left[\dfrac{n_1\beta k(r-R_1)+k\beta^2(a_1+\alpha)-n_1r(R_2+d_1)}{\sigma n_1r}\right]\end{array}\right),$$

exist if $kn_1(r-R_1)+k\beta(a_1+\alpha)>0$, $(a_1+\alpha)>0$ and

$n_1\beta k(r-R_1)+k\beta^2(a_1+\alpha)-n_1 r(R_2+d_1)>0$;

(xiii) Fixed point $P_{12}(X,Y,Z,L)$ at that point all classes are in active mode. Therefore, Equation (1)

$$rX\left(1-\frac{X}{k}\right)-\beta XY-w_1XZ-R_1X=0 \qquad \Rightarrow \frac{rX}{k}+\beta Y+w_1Z=r-R_1, \tag{2}$$

$$\beta XY-d_1Y-\lambda YZ-\sigma YL-R_2Y=0 \qquad \Rightarrow \beta X-\lambda Z-\sigma L=d_1+R_2, \tag{3}$$

$$b_1\omega_1XZ+b_2\lambda YZ-e_1ZL-a_1Z-R_3Z=0 \qquad \Rightarrow b_1\omega_1X+b_2\lambda Y-e_1L=a_1+R_3, \tag{4}$$

$$n_1YL+e_1ZL-(a_1+\alpha)L=0 \qquad \Rightarrow n_1Y+e_1Z=a_1+\alpha, \tag{5}$$

From Equation (2) and (3), we have

$$\beta^2Y+(w_1\beta+\lambda\frac{r}{k})Z-\frac{\sigma r}{k}L=\beta(r-R_1)+\frac{r}{k}(d_1+R_2), \tag{6}$$

From Equation (3) and (4), we have

$$\beta b_2\lambda Y+\lambda b_1w_1Z+(\sigma b_1w_1-e_1\beta)L=\beta(a_1+R_3)-b_1w_1(d_1+R_2), \tag{7}$$

From Equation (6) and (7), we have

$$\begin{aligned}&\beta\lambda(\beta b_1w_1-b_2)Y-\left[\frac{2\sigma r\lambda b_1w_1}{k}-\frac{e_1\lambda r\beta}{k}+\beta\sigma b_1w_1^2-e_1w_1\beta^2\right]L\\&=\lambda\beta b_1w_1(r-R_1)+\left[b_1w_1^2\beta+\frac{2r\lambda b_1w_1}{k}\right](d_1+R_2)\\&-\left[w_1\beta^2+\frac{\lambda r\beta}{k}\right](a_1+R_3),\end{aligned} \tag{8}$$

From Equation (6) and (5), we have

$$\begin{aligned}&(e_1\beta^2-\frac{n_1\lambda r}{k}-n_1w_1\beta)Y-\left(\frac{e_1\sigma r}{k}\right)L\\&=\beta e_1(r-R_1)+\left(\frac{re_1}{k}\right)(d_1+R_2)-\left[w_1\beta+\frac{\lambda r}{k}\right](a_1+\alpha),\end{aligned} \tag{9}$$

From Equation (8) and (9), we have

$$\begin{aligned}F_1(Y,L)&=\beta\lambda(\beta b_1w_1-b_2)Y-\left[\frac{2\sigma r\lambda b_1w_1}{k}-\frac{e_1\lambda r\beta}{k}+\beta\sigma b_1w_1^2-e_1w_1\beta^2\right]L\\&-\left[b_1w_1^2\beta+\frac{2r\lambda b_1w_1}{k}\right](d_1+R_2)-\lambda\beta b_1w_1(r-R_1)+\left[w_1\beta^2+\frac{\lambda r\beta}{k}\right](a_1+R_3)\end{aligned}$$

$$\begin{aligned}F_2(Y,L)&=(e_1\beta^2-\frac{n_1\lambda r}{k}-n_1w_1\beta)Y-\left(\frac{e_1\sigma r}{k}\right)L-\beta e_1(r-R_1)\\&-\left(\frac{re_1}{k}\right)(d_1+R_2)+\left[w_1\beta+\frac{\lambda r}{k}\right](a_1+\alpha),\end{aligned}$$

In Equation (8), when $L = 0$ then

$$Y_e = Y = \frac{\lambda\beta b_1 w_1 (r - R_1) + \left[b_1 w_1^2 \beta + \frac{2r\lambda b_1 w_1}{k}\right](d_1 + R_2) - \left[w_1 \beta^2 + \frac{\lambda r \beta}{k}\right](a_1 + R_3)}{\beta\lambda(\beta b_1 w_1 - b_2)},$$

We note that $Y_e > 0$ if

$$\lambda\beta b_1 w_1 (r - R_1) + \left[b_1 w_1^2 \beta + \frac{2r\lambda b_1 w_1}{k}\right](d_1 + R_2) - \left[w_1 \beta^2 + \frac{\lambda r \beta}{k}\right](a_1 + R_3) > 0,$$

In Equation (8)

$$\frac{dY}{dL} = \frac{\frac{2\sigma r \lambda b_1 w_1}{k} - \frac{e_1 \lambda r \beta}{k} + \beta\sigma b_1 w_1^2 - e_1 w_1 \beta^2}{\beta\lambda(\beta b_1 w_1 - b_2)} = \frac{A_1}{B_1}\ (say),$$

Hence, $\frac{dY}{dL} < 0$ if

(a) $A_1 < 0 \,\&\, B_1 > 0$ (b) $A_1 > 0 \,\&\, B_1 < 0$.

Similarly, in Equation (9) when $L = 0$, then

$$Y_e = Y = \frac{\beta e_1 (r - R_1) + \left(\frac{r e_1}{k}\right)(d_1 + R_2) - \left[w_1 \beta + \frac{\lambda r}{k}\right](a_1 + \alpha)}{(e_1 \beta^2 - \frac{n_1 \lambda r}{k} - n_1 w_1 \beta)},$$

We note that $Y_e > 0$ if

$$\beta e_1 (r - R_1) + \left(\frac{r e_1}{k}\right)(d_1 + R_2) - \left[w_1 \beta + \frac{\lambda r}{k}\right](a_1 + \alpha) > 0,$$

$$\frac{dY}{dL} = \frac{\left(\frac{e_1 \sigma r}{k}\right)}{(e_1 \beta^2 - \frac{n_1 \lambda r}{k} - n_1 w_1 \beta)} = \frac{A_2}{B_2}\ (say),$$

Hence, $\frac{dY}{dL} < 0,$ if

(a) $A_2 < 0 \,\&\, B_2 > 0$ (b) $A_2 > 0 \,\&\, B_2 < 0$,

Hence, the fixed point $P_{13}(X, Y, Z, L)$ exists.

4. Stability Analysis

The Jacobian linearization of fixed points is:

$$J(P)=\begin{pmatrix} \begin{Bmatrix} r-\frac{2Xr}{k}-\beta Y- \\ \omega_1 Z-R_1 \end{Bmatrix} & -\beta X & -\omega X & 0 \\ \beta Y & \begin{Bmatrix} \beta X-d_1-\lambda Z- \\ \sigma L-R_2 \end{Bmatrix} & -\lambda Y & -\sigma Y \\ b_1\omega_1 Z & b_2\lambda Z & \begin{Bmatrix} b_1\omega_1 X+b_2\lambda Y-e_1 L \\ -a_1-R_3 \end{Bmatrix} & -e_1 Z \\ 0 & n_1 L & e_1 L & \begin{Bmatrix} n_1 Y+n_1 Z \\ -(a_1+\alpha) \end{Bmatrix} \end{pmatrix},$$

4.1. Stability of Jacobian matrix at the point $P_0(0,0,0,0)$

$$J(P_0)=\begin{pmatrix} r-R_1 & 0 & 0 & 0 \\ 0 & -d_1-R_2 & 0 & 0 \\ 0 & 0 & -a_1-R_3 & 0 \\ 0 & 0 & 0 & -a_1-\alpha \end{pmatrix},$$

Now, the latent roots of the above matrix are denoted as

$$w_1=(r-R_1), w_2=-(d_1+R_2), w_3=-(a_1+R_3), w_4=-(a_1+\alpha),$$

Observation 1:

With the support of latent roots, we observed that the fixed point $P_0(0,0,0,0)$, is locally asymptotically stable in a steady state, if $R_1>r$, $R_2>d_1$, $R_3>a_1$ and the point P_0 is unstable in a steady state, if $R_1<r$. Therefore, the point P_0 is called the saddle point.

4.2. Stability of Jacobian matrix at the point $P_1\left(\frac{(r-\lambda_1)k}{r},0,0,0\right)$

$$J_1(P)=\begin{pmatrix} -2(r-R_1) & -\frac{k\beta}{r}(r-R_1) & 0 & 0 \\ 0 & \frac{k\beta}{r}(r-R_1)-(d_1+R_2) & 0 & 0 \\ 0 & 0 & \frac{kb_1w_1}{r}(r-R_1)-(a_1+R_3) & 0 \\ 0 & 0 & 0 & -(a_1+\alpha) \end{pmatrix},$$

Observation 2:

Here, the trace of the Jacobian matrix is negative and the determinant is positive, therefore, all latent roots are negative. Hence, in a steady state, the fixed point $P_1\left(\frac{(r-\lambda_1)k}{r},0,0,0\right)$ is locally asymptotically stable. Similarly, check the stability of all remaining points.

5. Conclusion

The proposed mathematical model has explained the controlling and spreading behavior of worms in WSN. It also creates ordinary nonlinear differential equations for the study of latent periods effects on the transmission of worms in WSN, and we examine the behavior of different classes of the model that is the attack of worms in WSN. When the infected class recovers, it becomes minimal. Therefore, this epidemic model is capable of ascertaining the worm's spreading at the initial stage and improving the security of wireless sensor networks. Here we have calculated the various fixed or equilibrium points, explored the conditions of fixed points, and then analyzed their local stability. In the future, we will discuss the effects of different parameters on some constraints.

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