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Circular Restricted Three-Body Interaction Problem With Various Perturbations

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Abstract

The motion properties of the infinitesimal body is studied under the forces due to kerr-like oblate heterogeneous primary, continuation fractional potential for secondary, solar sail, three-body interactions, Coriolis and centrifugal forces in the circular restricted three-body problem. The equations of motion of infinitesimal body are evaluated under the above-said perturbations. Using these equations of motion, we illustrate the locations of equilibrium points, their stability, the periodic orbits and Poincaré surfaces of section. This study will applicable on the motion of the artificial satellite.

Keywords: Perturbation; Solar sail; Kerr-like oblate heterogeneous body; Continuation fractional potential

MSC 2010 No.: 70F15, 70F07, 37N05

1. Introduction

The restricted three-body problem is an application based problem in applied mathematics with various perturbations. These perturbations separately play an important role on the motion of the infinitesimal body. Researchers have investigated their problems by assuming various types of perturbations. Some of those researchers who have studied these perturbations are as follows.

Bhatnagar and Hallan (1978) have investigated the effects of Coriolis and centrifugal forces on the stability of equilibrium points in the restricted 3-body problem. McInnes and Simmons (1992a) and McInnes and Simmons (1992b) have investigated the motion properties for geocentric and heliocentric cases with solar sail which perform minimum and families of linearly stable trajectories identified. After patching these halo orbits, new complex trajectories were found. McInnes et al. (1994) have evaluated the stationary solutions to the restricted 3-body problem for the solar sail spacecraft for various systems. They observed that stationary solutions are unstable, therefore, they developed a simple closed-loop control scheme to ensure asymptotic stability.

McInnes (2000) investigated the solar sail mission design in the circular restricted 3-body problem with many strategies. McInnes et al. (2001) have explored the geomagnetic tail using the small solar sail, because solar sails do not require reaction mass, a geomagnetic tail mission can be configured that provides a continuous science return by permanently stationing a science pay load within the geomagnetic tail. Macdonald et al. (2007) have explored the solar sail magneto-tail mission concept. Using a parametric analysis, they identify the key sail technology requirements. Mengali et al. (2007) have proposed a refined mathematical model for describing the acceleration exerted by solar sail. They also elaborated that how the main variable affects to the force coefficients.

Farres (2009) investigated the effects of solar sail on the motion properties (like families of equilibrium points, stability of equilibrium points, station keeping strategies and periodic orbits) of infinitesimal bodies in the restricted three bodies. Farres and Jorba (2010) have considered the Earth-Sun restricted three-body model to investigate the effects of solar sail. For the different values of the orientations of the solar sail, they have explained the periodic and quasi-periodic motions of a solar sail close to equilibrium points that lies between the Earth and the Sun.

Gong and Li (2014) have investigated the solar sail with a reflection control device in the Heliocentric elliptic displaced orbits. They found the stable orbits in the pulsating rotating frame and image the polar region of the planet with an orbit that is highly elliptic.

Heiligers et al. (2015) have investigated the time-optimal solar sail transfers trajectories in two cases, i.e., (1.) Libration point orbits (LPOs) of Sun-Earth L_2-Halo orbit to Sun-Mars L_1-Halo orbit, and (2.) Libration point orbits (LPOs) of Sun-Earth L_1-Halo orbit to Sun-Mercury L_2-Halo orbit.

Peloni et al. (2016) have presented a method to find sequences of encounters for multiple near-Earth-Asteroid rendezvous missions by solar sailing. They also have demonstrated the novel coplanar shape-based approach for solar sailing to obtain excellent results both within the sequence search and as an initial-guess solution for 3D direct optimization.

Bosanac et al. (2016) have explored the effects of natural autonomous force (interactions between bodies) in the restricted 3-body problem. Ragos et al. (2020) and Ragos (2022) have investigated the long and short periodic orbits in the classical restricted 3-body problem and in the Sitnikov restricted 3-body problem with the 3-body interaction effects.

Singh et al. (2022) have studied the effects of modified Newtonian force on the restricted 3-body configuration in the non-linear sense. Sood and Howell (2019) have re-evaluated the concept of a mission to $L_{4,5}$ in the restricted three-body problem with solar sail dynamics. Some more researchers are as follows: Huang et al. (2020) and Zhao et al. (2023). The Hill problem is the new kind of configuration which is investigated by many mathematicians and physicists with various factors. Some of them are as follow: Szebehely (1967), Abouelmagd and El-Shaboury (2012), Abouelmagd et al. (2021), Abouelmagd et al. (2022), Ershkov et al. (2023), Doshi et al. (2023), and Alshaery and Abouelmagd (2020).

Ansari (2017), Ansari (2018), Ansari et al. (2019), Ansari et al. (2022), Sahdev and Ansari (2020), Albidah and Abdullah (2023c), Albidah and Abdullah (2023a), Bouaziz and Ansari (2021) and Abdullah (2023) have studied the effect of mass variation in the restricted system. They have illustrated these models numerically for location of equilibrium points, their stability, regions of motion, Poincaré surfaces of section, periodic orbits and basins of attraction.

This paper is distributed in various sections and subsections. The brief history of the literature of the problem is given in Section 1. The model presentation and the equations of motion are given in Section 2. Section 3 represents the numerical investigations of the problem with many sub-sections. The paper ends with the conclusion in Section 4.

2. Model Presentation and Equations of Motion

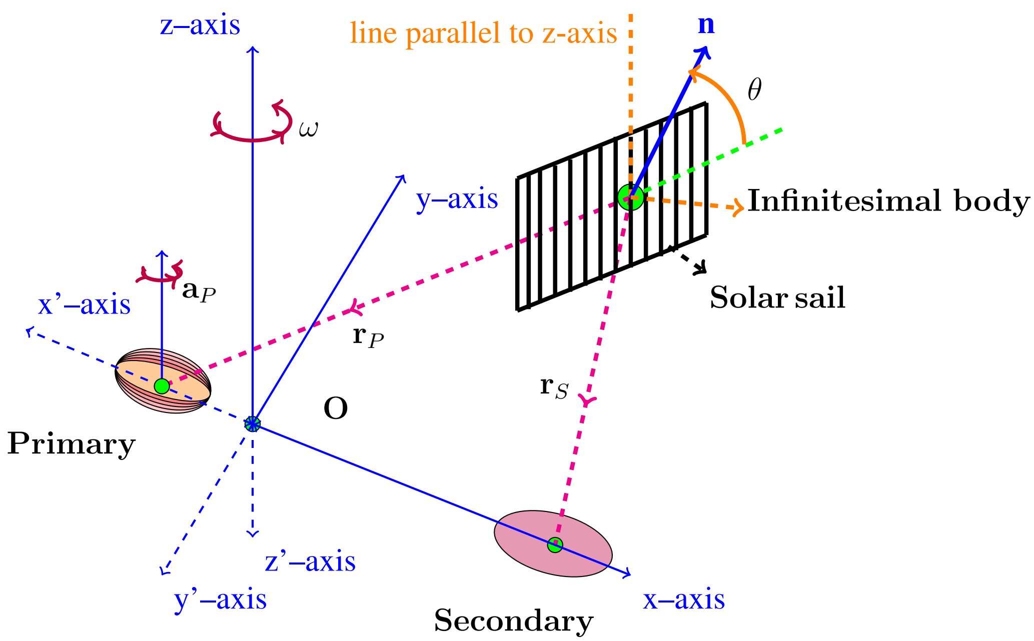


Figure 1. Perturbed restricted three-body interaction problem with solar sail effect

Let there be three masses m_1, m_2 and m of kerr-like oblate heterogeneous primary (when the transition parameter ϵ_1 and heterogeneous density parameter ρ_{11}), having continuation fractional potential secondary with fractional parameter ϵ_2 and the third infinitesimal body with solar sail effects (where S_1, S_2 and S_3 are the forces due to solar sail in the x, y and z directions respectively). The primary and secondary are placed at x-axis on either side of the origin and moving around their common center of mass which is taken as origin. The system have angular velocity $\omega = n\,\hat{k}$. We also supposed the effects of three-body interactions and Coriolis as well as centrifugal forces with parameters K and α as well as β respectively. The complete view can obtain in the Figure (1). Using the procedures given by Abouelmagd (2018), Farres et al. (2019), Ansari and Abouelmagd (2020), De et al. (2023) and Albidah and Abdullah (2023b), one can write the equations of motion of the infinitesimal body as:

$$\begin{aligned}
\ddot{x} - 2\,\alpha\,n\,\dot{y} &= \frac{\partial\,\mathbf{\Omega}}{\partial\,x} + S_1,\\
\ddot{y} + 2\,\alpha\,n\,\dot{x} &= \frac{\partial\,\mathbf{\Omega}}{\partial\,y} + S_2,\\
\ddot{z} \qquad\qquad &= \frac{\partial\,\mathbf{\Omega}}{\partial\,z} + S_3,
\end{aligned} \tag{1}$$

with

$$\mathbf{\Omega} = \frac{\beta\,n^2}{2}(x^2+y^2) + \frac{\mu_P}{r_P} + \frac{\rho_{11}}{2\,r_P^3} - \frac{\epsilon_1\,\mu_P}{r_P^2}\,a_P\,Cos\,\theta_P - \frac{\epsilon_1^2\,\mu_P}{2\,r_P^3}\,\{\mu_P^2 + a_P^2\,(3\,Cos^2\,\theta_P - 1)\}$$

$$+ \frac{\mu_S\,r_S}{(r_S^2 + \epsilon_2)} + \frac{K}{r_P\,r_S},$$

$$S_1 = B\left[\frac{x+\mu}{r_P}\,Cos\,\theta - \frac{(x+\mu)\,z}{r_1\,r_P}\,Sin\,\theta\cdot Cos\,\phi + \frac{y}{r_1}\,Sin\,\theta\cdot Sin\,\phi\right],$$

$$S_2 = B\left[\frac{y}{r_P}\,Cos\,\theta - \frac{y\,z}{r_1\,r_P}\,Sin\,\theta\cdot Cos\,\phi - \frac{x+\mu}{r_1}\,Sin\,\theta\cdot Sin\,\phi\right],$$

$$S_3 = B\left[\frac{z}{r_P}\,Cos\,\theta + \frac{r_1}{r_P}\,Sin\,\theta\cdot Cos\,\phi\right],$$

$$B = \frac{q_0\,(1-\mu)}{r_P^2}\cdot Cos^2\,\theta, \quad r_P^2 = (x+\mu)^2 + y^2 + z^2,$$

$$r_S^2 = (x+\mu-1)^2 + y^2 + z^2, \quad r_1^2 = (x+\mu)^2 + y^2,$$

$$\mu = \frac{m_2}{m_1+m_2} = \mu_S, \quad \mu_P = 1 - \mu,$$

$$n^2 = 1 - 3\,\epsilon_2 - \epsilon_1\,a_P + \frac{3}{2}\frac{\rho_{11}}{(1-\mu)} + \frac{K}{\mu\,(1-\mu)}, \quad \epsilon_2 = \text{fractional parameter},$$

ρ_{11} = the density parameter, ϵ_1 = transition parameter,

θ = orientation of the solar panel, q_0 = the radiation parameter,

ϕ = clock angle, α and β = Coriolis and centrifugal forces parameters, respectively.

3. Numerical investigations

In this section, we will investigate numerically the dynamical behaviours of the infinitesimal body as the locations of equilibrium points, their stability, periodic orbits and Poincaré surfaces of section in five different cases with the help of well known software Mathematica. These cases are described as:

(1) Unperturbed case,
(2) Perturbed case I (Solar sail, Coriolis and centrifugal forces effects),
(3) Perturbed case II (Perturbed case I and fractional potential effects),
(4) Perturbed case III (Perturbed case II and kerr-like heterogeneous effects),
(5) Perturbed case IV (Perturbed case III and interaction effects).

3.1. Equilibrium points

The locations of equilibrium points can be obtained by putting zero to all the derivatives with respect to time in the Equation (1), and hence we get

$$\begin{aligned} \mathbf{\Omega_x} + S_1 &= 0, \\ \mathbf{\Omega_y} + S_2 &= 0, \\ \mathbf{\Omega_z} + S_3 &= 0. \end{aligned} \tag{2}$$

If we solve first two equations of Equation (2), we will get the in-plane equilibrium points, i.e., in the x-y-plane, while if we solve first and last equations of Equation (2), we will get out-of-plane equilibrium points, i.e., in x-z-plane. Here, we will illustrate only the in-plane equilibrium points. When we contour plot with the first two equations of Equation (2) numerically with numerical values $z = 0$, $\mu = 0.01215$, $\alpha = \beta = 1.2$, $\rho_{11} = 0.001$, $\theta = \pi/4$, $\phi = \pi/4$, $q = 0.2$, $\epsilon_1 = 0.2$, $\epsilon_2 = 0.2$, $a_P = 0.2$ and K = 0.2, we obtain the locations of in-plane equilibrium points.

Here, the in-plane equilibrium points are illustrated in the above said five cases. These equilibrium points are given in the sub-figures 2(a), 2(b), 2(c), 2(d) and 2(e) for cases 1, 2, 3, 4 and 5, respectively. The sub-figure 2(a) is for the unperturbed case, which complies to the classical case where three collinear and two triangular equilibrium points exist. The sub-figure 2(b) is for the perturbed case I where the effects of solar sail, Coriolis and centrifugal forces are taken in consideration. In this case we found only three equilibrium points. The sub-figure 2(c) is for the perturbed case II where the effects of solar sail, Coriolis, centrifugal forces and continuous fractional potential are taken in consideration. In this case no equilibrium point exists. The sub-figure 2(d) is for the perturbed case III where the effects of solar sail, Coriolis, centrifugal forces, continuous fractional potential and kerr-like heterogeneous body are taken in consideration. In this case only three equilibrium points exist. The sub-figure 2(e) is for the perturbed case IV where the effects of solar sail, Coriolis, centrifugal forces, continuous fractional potential, kerr-like heterogeneous body and interaction between bodies are taken in consideration. In this case only two equilibrium points exist. In this way, the considered perturbations have excellent influences on the motion of the infinitesimal body.

3.2. Stability states

We can rewrite the equations of motion as

$$\begin{aligned} \ddot{x} - 2\,\alpha\, n\, \dot{y} &= \Pi_x, \\ \ddot{y} + 2\,\alpha\, n\, \dot{x} &= \Pi_y, \\ \ddot{z} \qquad\qquad &= \Pi_z, \end{aligned} \tag{3}$$

where $\Pi_x = \mathbf{\Omega}_x + S_1$, $\Pi_y = \mathbf{\Omega}_y + S_2$ and $\Pi_z = \mathbf{\Omega}_z + S_3$.

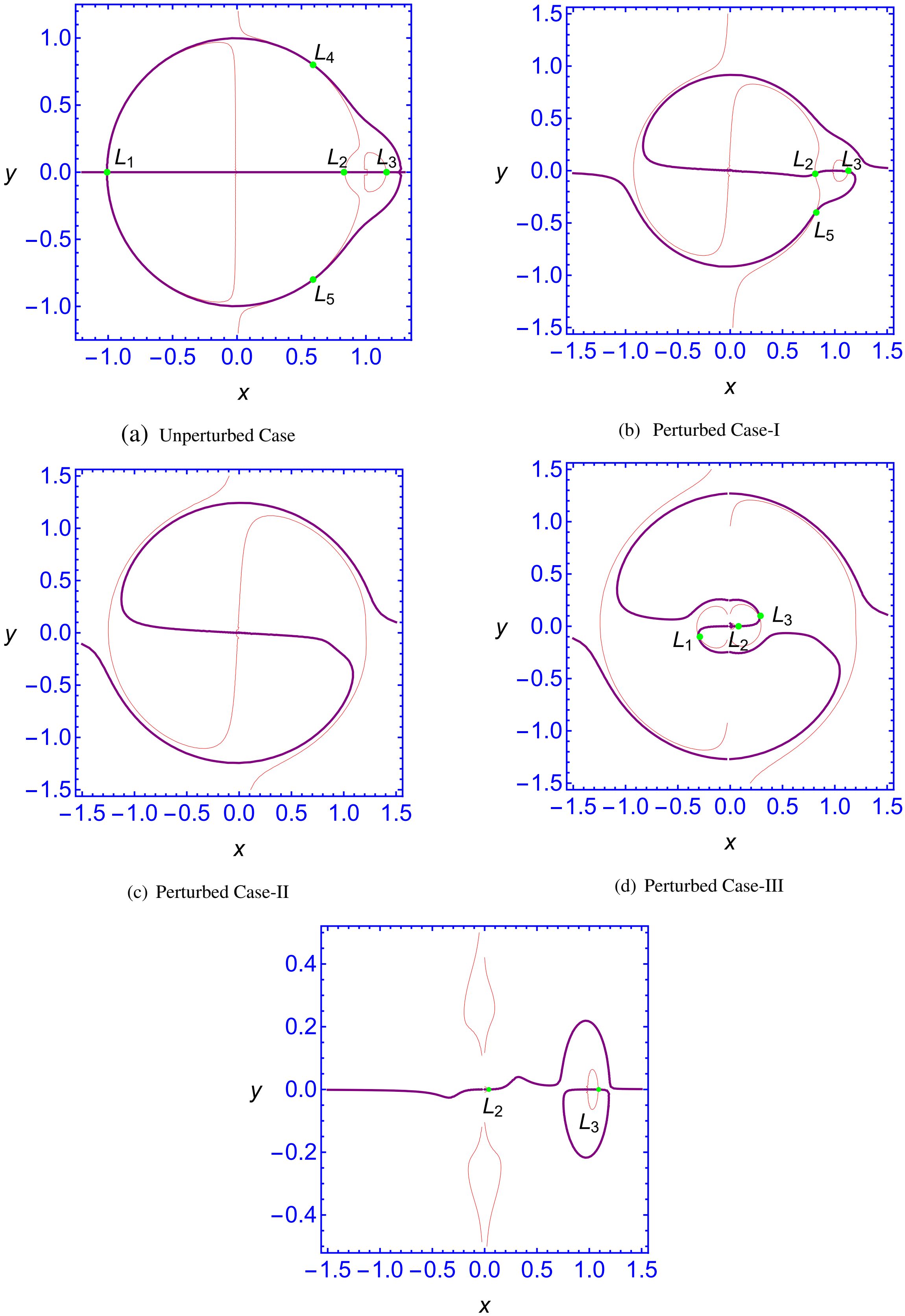
To examine the stability of equilibrium points, we will shift the equilibrium point $(x_0,\, y_0,\, z_0)$ to the point $(x_1,\, y_1,\, z_1)$. With the use of this shift, Equation (3) can be written as

$$\begin{aligned} \ddot{x}_1 - 2\,\alpha\, n\, \dot{y}_1 &= \Pi^0_{x\,x}\, x_1 + \Pi^0_{x\,y}\, y_1 + \Pi^0_{x\,z}\, z_1, \\ \ddot{y}_1 + 2\,\alpha\, n\, \dot{x}_1 &= \Pi^0_{y\,x}\, x_1 + \Pi^0_{y\,y}\, y_1 + \Pi^0_{y\,z}\, z_1, \\ \ddot{z}_1 \qquad\qquad &= \Pi^0_{z\,x}\, x_1 + \Pi^0_{z\,y}\, y_1 + \Pi^0_{z\,z}\, z_1. \end{aligned} \tag{4}$$

Hence, the characteristic polynomial corresponding to Equation (4) can be written as:

$$f(\lambda) = \lambda^6 + P_4\, \lambda^4 + P_2\, \lambda^2 + P_0, \tag{5}$$

where,



(a) Unperturbed Case

(b) Perturbed Case-I

(c) Perturbed Case-II

(d) Perturbed Case-III

(e) Perturbed Case-IV

Figure 2. Locations of equilibrium points for various cases

$$P_4 = 4\,\alpha^2\,n^2 - (\Pi^0_{x\,x} + \Pi^0_{y\,y} + \Pi^0_{z\,z}),$$

$$\begin{aligned} P_2 = & -4\,\alpha^2\,n^2\,\Pi^0_{z\,z} - (\Pi^0_{x\,y})^2 - (\Pi^0_{x\,z})^2 - (\Pi^0_{y\,z})^2 \\ & + \Pi^0_{x\,x}\,\Pi^0_{y\,y} + \Pi^0_{x\,x}\,\Pi^0_{z\,z} + \Pi^0_{y\,y}\,\Pi^0_{z\,z}, \end{aligned} \tag{6}$$

$$\begin{aligned} P_0 = & (\Pi^0_{x\,z})^2\,\Pi^0_{y\,y} - 2\,\Pi^0_{x\,y}\,\Pi^0_{x\,z}\,\Pi^0_{y\,z} + \Pi^0_{x\,x}\,(\Pi^0_{y\,z})^2 \\ & + (\Pi^0_{x\,y})^2\,\Pi_{z\,z} - \Pi^0_{x\,x}\,\Pi^0_{y\,y}\,\Pi_{z\,z}. \end{aligned}$$

We have numerically solved the equation (5) corresponding to equilibrium points for the perturbed cases and given in the Tables 1, 2 and 3. From these tables, we got at least one root as either positive real value or positive real part of the complex roots. Hence, these equilibrium points are unstable.

Table 1. The nature of equilibrium points in the Perturbed case I

$Equilibrium\,points$	$Roots$	$Nature$
L_2	Positive real value	$Unstable$
L_3	Positive real value	$Unstable$
L_5	Positive real part	$Unstable$

Table 2. The nature of equilibrium points in the Perturbed case III

$Equilibrium\,points$	$Roots$	$Nature$
L_1	Positive real value	$Unstable$
L_2	Positive real value	$Unstable$
L_3	Positive real value	$Unstable$

3.3. Periodic orbits

To allocate the path of the infinitesimal body during the motion, we have to perform the periodic orbits by solving the evaluated equations of motion. For which firstly, we have to write the equa-

Table 3. The nature of equilibrium points in the Perturbed case IV

$Equilibrium\,points$	$Roots$	$Nature$
L_2	Positive real value	$Unstable$
L_3	Positive real value	$Unstable$

tions of motion in phase plane and then with the proper choice of the initial values as well as by well known software, we will determine the solutions of the equations of motion. With the use of these solutions, we shall plot the periodic orbits for the above defined cases. In the unperturbed case, the initial value is x[0] = 0.5595, y[0] = 0, u[0] = 0, v[0] = 0.5595 for which the time period is 18.93 unit. In the perturbed case I, the initial value is x[0] = 1.7, y[0] = 0, u[0] = 1.51, v[0] = 0 for which the time period is 19 unit. In the perturbed case II, the initial value is x[0] = 2.045, y[0] = 0, u[0] = 2.2, v[0] = 0 for which the time period is 29.9 unit. In the perturbed case III, the initial value is x[0] = 0.01, y[0] = 0, u[0] = 0, v[0] = 0.01 for which the time period is 10.64 unit. In the perturbed case IV, the initial value is x[0] = 1.778, y[0] = 0, u[0] = 0, v[0] = 1.778 for which the time period is 27.95 unit. These orbits are presented in Figure (3) from where we observed that these periodic orbits are not simply periodic.

3.4. Poincaré surfaces of section

To observe the chaos, we have illustrated the Poincaré surfaces of section for the different five cases. To perform the Poincaré surfaces of section, we have to evaluate the value of coordinate (x, y) and velocity $(\dot{x}, \dot{y})$ of the infinitesimal body in phase space. Then, draw the graph between $(x, \dot{x})$ at $y = 0$, whenever the path intersects the plane for $\dot{y} > 0$ and similarly can draw for $(y,\, \dot{y})$. The Poincaré surfaces of sections are performed in Figure (4) from where we observed that the surfaces are symmetrical about x-axis and there is no chaos.

4. Conclusion

The motion properties of the infinitesimal body have investigated in the perturbed circular restricted three-body interaction problem. Where mathematically the effects of the perturbations are clearly visible. The numerical studies are studied in the five cases. In these five cases, the number of equilibrium points varies while except unperturbed case, for the other four cases (perturbed cases), the equilibrium points are unstable. We also have drawn the periodic orbits for these five cases and found periodic orbits in each case with different time periods. Further, we have illustrated the Poincaré surfaces of section in five cases where we observed that there are no chaos and surfaces are symmetrical about x-axis. Our study will be useful for the space agencies.

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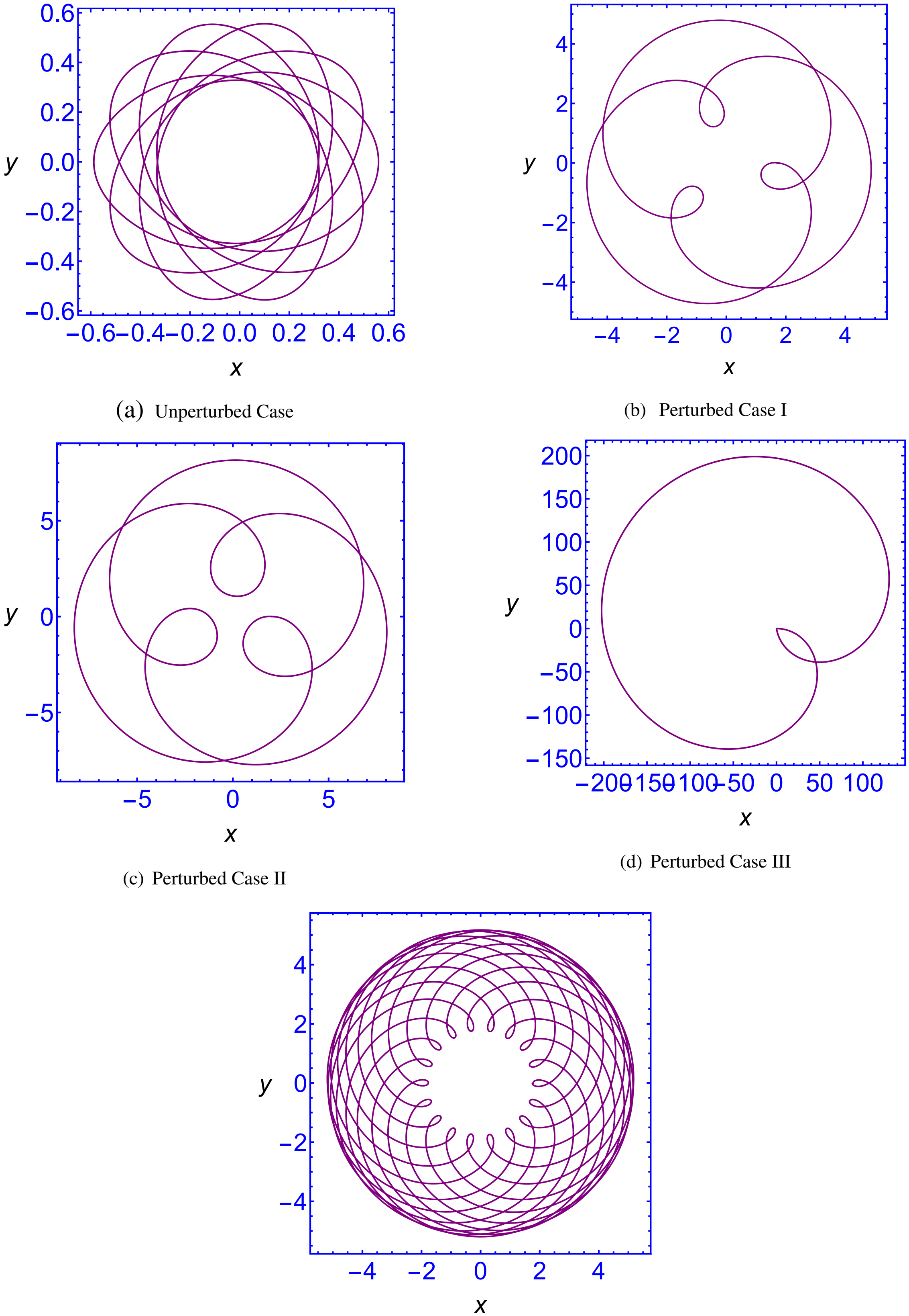
Appendix

Figure 3 and Figure 4 follow on the next pages.



(a) Unperturbed Case

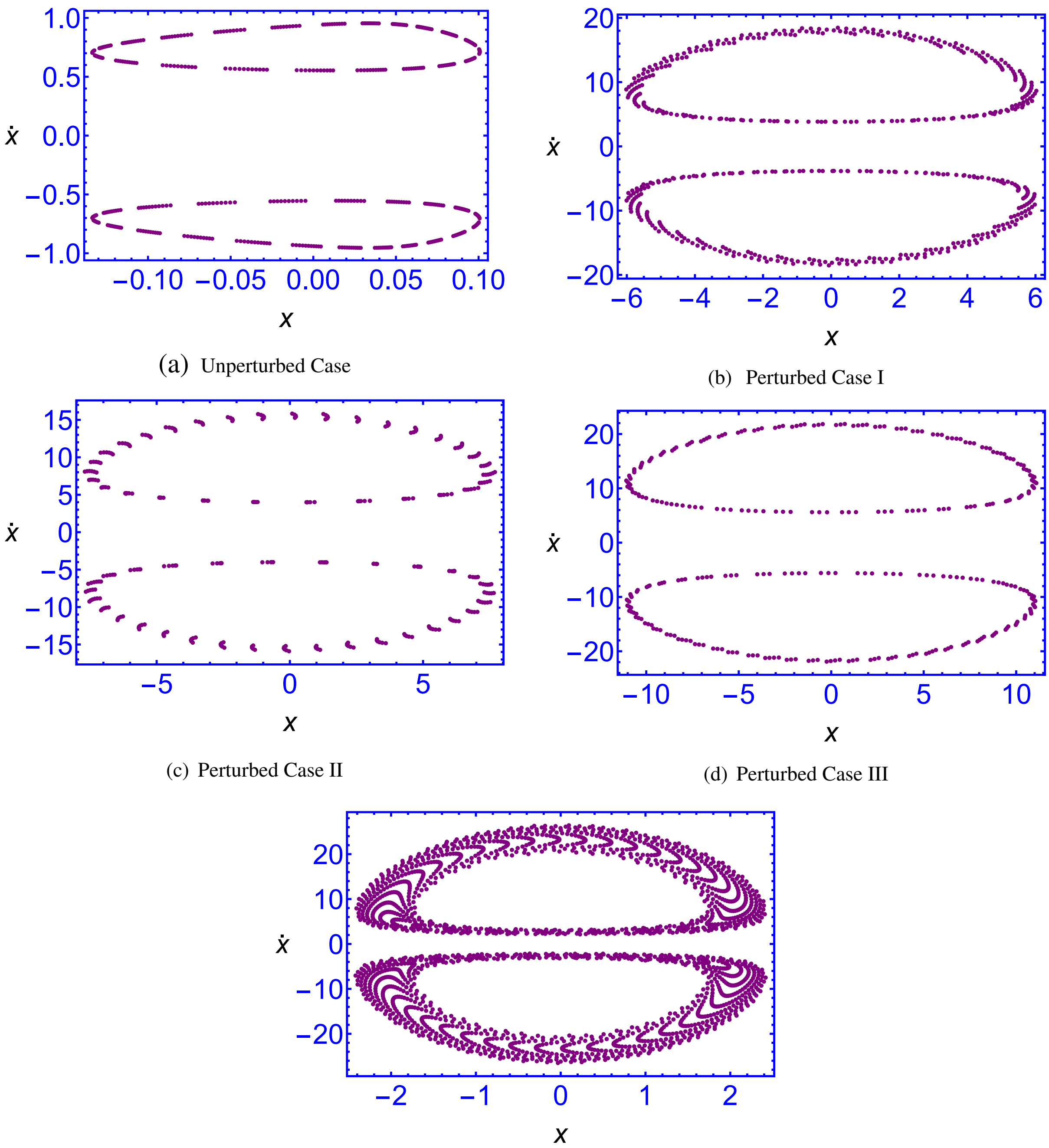
(b) Perturbed Case I

(c) Perturbed Case II

(d) Perturbed Case III

(e) Perturbed Case IV

Figure 3. Periodic orbits in various cases



(a) Unperturbed Case

(b) Perturbed Case I

(c) Perturbed Case II

(d) Perturbed Case III

(e) Perturbed Case IV

Figure 4. Poincar e surfaces of section for five defined cases