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Effect of Climate Change on Brain Tumor

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Abstract

In this paper, we introduce a new dynamical model addressing the variation in climate condition due the presence of microorganisms. We also introduce a new dynamical model of cancer growth which includes three interactive cell populations with drug free environment, namely tumor cells, healthy host cells, and immune effector cells. In this, we considered the super growth of tumor cells. For the choice of certain parameters, both of the systems exhibit chaotic behavior. The aim of this work is to design the controller to control the chaos and to provide sufficient conditions which achieve synchronization of two non-identical systems, which is based on Lyapunov stability theory. To verify synchronization is achieved between the systems, we performed the numerical simulation.

Keywords: Cancer; Tumor growth; Bacteria; Virus; Anthrobacter; Proliferation; Lyapunov exponent; Attractor; Chaos synchronization; Climate conditions

MSC 2010 No.: 34H10, 34D06, 34D20, 34C60, 92B05

1. Introduction

The majority of partial seizure research has focused on determining what was wrong with aberrant section of the brain. Larter et al. (1999) were able to look at the difficult problem of how bad tissues manages with healthy tissues, and by using a computer, this behavior was replicated, with thousands of the brain's nerve cells called neurons. They joined approximately a thousand neurons together to simulate abnormally acting portion of the brain, and in the beginning they took, typical non-linear equations that described the behavior of individual neurons.

Simulations of Cell-to-Cell and Drug-to-Cell interactions by using a massive system of linear differential equations studied by De Pillis and Randunskaya (2003). The differential equations of Kirschner and Panneta (1998) are used to represent well-known medications such as metha-trexate, which has been used to treat leukemia for over 30 years.

A tumor in the brain is characterized as a dynamical system in which cells grow rapidly and behave abnormally. By depriving good tissue of nutrition and oxygen, these aberrant cells cause these cells to die. When aberrant cells are present in the brain, tumors develop in the brain or outside of it as described in Murray (2003). The two most common types of tumors are malignant and benign tumors, in which malignant tumors are cancerous tumors. Tumors in the brain may be characterized as primary and secondary brain tumors and this has been studied by several researchers, namely Burgess et al. (1997), Coldman and Murray (2002), and Boondirek et al. (2006). Primary brain tumors begin in the brain but secondary tumors originated from the other organ in the body and spread from into the brain. Kirschner and Panneta (1998) developed the mathematical model of brain tumor. Further De Pillis and Randunskaya (2003) improved this model by integrate it with chemotherapy. With the specific set of parameters, they analyzed the attractors and measured the equilibria in the chemotherapy-free environment.

To comprehend chaotic attractors, you must first understand the concept of phase space, which is considerably older. The concept of an attractor has long been used to analyze the behavior of a system in phase space. Although there are highly rigorous definitions of attractors (based on the existence of a dense orbit and a basin of attraction), a simpler definition has been used for the purposes of this work, that is, a point or set of points towards which a dynamical system evolves. That is said to be an attractor which is given in Oestreicher (2007). However, not all attractors allow for accurate prediction of long-term behavior of chaotic dynamical system. Strange attractors can also be either periodic or aperiodic in nature.

In this work, we observed a chaotic attractor which might be defined as an aperiodic attractor and a small change in the initial conditions has a significant impact on the long-term behavior of chaotic attractors. Itik and Banks (2010) took a more rigorous mathematical approach in the model provided by De Pillis and Randunskaya (2003) and observed the chaos in this model. They calculated the Lyapunov exponents for a given set of parameters and they officially verify the existence of chaos in the fundamental model.

In the recent years, prognoses of many type of cancer have been improved and many research is

done to help to improve diagnoses and therapy, in both adults and children. The application of mathematical modelling is used to accelerate medical discoveries, and only by combining various medications, cancer has been brought under control. The primary method of determining which pharmaceuticals to combine, and how to do so efficiently, through the use of mathematical models did not require each drug permutation to be tested in clinical trials beforehand, for this the synchronization of two dynamical systems has been substantially studied in recent years. A wide range of synchronization methods between two or more identical and non-similar chaotic systems has been studied by Boccaletti et al. (2002) and Yassen (2005).

Climate change prediction in the twenty-first century is a huge difficulty for scientists because the factors are anthropogenic. The combustion of fossil fuels results in the rise in CO_2 in the atmosphere and a change in the concentration of tiny gaseous species that govern the ozone concentration, which effects the temperature in the environmental. Therefore, climate change in the atmosphere can be characterized by a set of distributed factors: temperature, pressure, humidity, wind velocity, and gas concentration. Recently, the effect of climate change due to the presence of microorganisms in the atmosphere studied by Cavicchioli et al. (2019). In their research, they have been described change in temperature in the environment, theoretically.

Most authors have not taken into account the changes in climate conditions caused by microorganisms, present in the environment, which play a key role in the carbon and nutrient cycles for animal health. In this paper, we are concentrating upon the changes in climate conditions caused by these microorganisms, and for this we have designing a model that exhibits the dissipating behavior in the environment. By synchronizing it with a model of the tumor cells population, we can understand the behavior of tumors in the presence of microscopic organisms. To synchronize our models, we use active control scheme proposed by Bai and Lonngren (1997) for synchronization two non-identical chaotic systems. A numerical simulation is performed to verify the synchronization scheme, by which we may regulate the tumor cells chaotic behavior, and shows how temperature in the environment synchronized with the tumor cells.

2. Mathematical Model

In recent years many model have been used to characterize the evolution of tumor cells. These model can be organized by the number of compartments. In this paper, we consider only non-metastatic malignancies, as localized tumors for which interaction of cells population is considered, by using the system of ordinary differential equations given in Taylor (2011). To this aim, we introduce a new chaotic dynamical system, which categorised cell's population into three classes and each variable represents a cell population, namely: $p(t)$, the tumor cells; $q(t)$, the healthy host cells near the tumor and $r(t)$, the effective immune cells, and with the following postulates:

P1: The growth of the tumor is non-linear.

P2: Interaction of tumor cells with immune cells reduce the activity of host cells.

P3: The presence of host and immune cells kills tumor cells.

With these postulates, the mathematical model comprises three compartments, two Proliferation and one Non-Proliferation as shown in Figure 1.

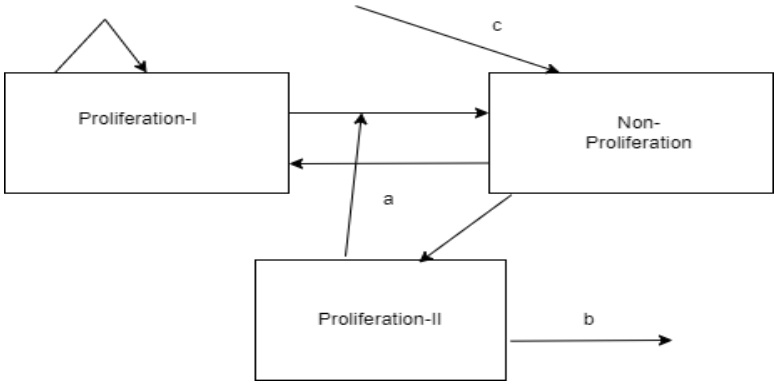

Figure 1. Compartmental diagram of Tumor Model

The model consists of system of three ordinary differential equations categorising the changes in cells population given as follows:

$$\begin{aligned} \frac{dp}{dt} &= -apr - q + p^2, \\ \frac{dq}{dt} &= -pr + c, \\ \frac{dr}{dt} &= pq - br, \end{aligned} \tag{1}$$

with initial conditions $p(0) = p_0, q(0) = q_0, r(0) = r_0$.

Based on the study of Sparrow (1982), we introduce a new dynamical system which characterised the interaction of microorganism with the atmosphere. In particular, we consider the interaction between the population of virus and bacteria in an appropriate environment. This model is based on the following assumptions:

A1: Particularly, we include the population of anthrobacter.

A2: Initially the temperature is high.

Based on these assumptions, we consider the variables $p(t)$, the temperature, $q(t)$, the bacteria population, and $r(t)$, the virus population. The model is designed as a system of ordinary differential equations, described as follows:

$$\begin{aligned} \frac{dp}{dt} &= -dp + q, \\ \frac{dq}{dt} &= \varepsilon q - pr, \\ \frac{dr}{dt} &= dpq - fr, \end{aligned} \tag{2}$$

with initial conditions $p(0) = p_0, q(0) = q_0$, and $r(0) = r_0$.

2.1. Existence of Chaos

Lyapunov exponents are a measure of sensitivity to tiny changes in initial circumstances and these can be used to define chaotic behavior in a dynamical system. Also, many trials over small time steps at different positions on the attractor are required for the numerical calculation of Lyapunov exponents. A Lyapunov exponent can be determined using the time step (t) and the initial separation (d_0) as follows:

$$\lambda = \lim_{n\to\infty} \left(\frac{1}{n\Delta t} \sum_{i=1}^{n} \ln \left| \frac{d_i}{d_0)} \right| \right). \tag{3}$$

At step i, distance di is the distance between neighboring points given in Otto and Day (2007). When the beginning conditions of two orbits are relatively close together, this equation is a measure of the pace at which their distances increase. A Lyapunov exponent exists for each dimension of a dynamical system, which can be arranged in descending order ($\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots$) to give λ_1, the maximal Lyapunov exponent (MLE). Only the MLE has been used to recognize chaos in this paper, as chaos can be recognized by the criteria that "MLE > 0 implies chaotic behavior". Abernethy and Gooding (2018) observed that MLE will dominate the divergence between neighboring orbits for big enough n. Therefore, Equation 3 can be understood as an expression for the MLE.

For System (1), we noticed that with the parameters $a = 5, b = 2, c = 27$, the Lyapunov exponents are $(0.0000415982, -5.40242, -18.9973)$. Therefore, the System (1) exhibits the chaotic behavior as shown in Figure 2.

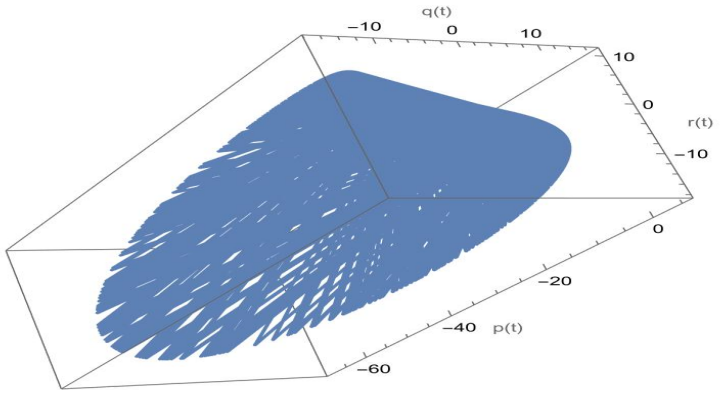

Figure 2. Chaotic Attractor of Tumor Model is $p(0) = 3, q(0) = 2, r(0) = 1$

For System (2), we noticed that with the parameters when $d = 36, \varepsilon = 13, f = 4$, the Lyapunov exponents are $(0.0561504, -0.0713626, -26.8027)$. Therefore, the System (2) exhibits the chaotic behavior as shown in Figure 3.

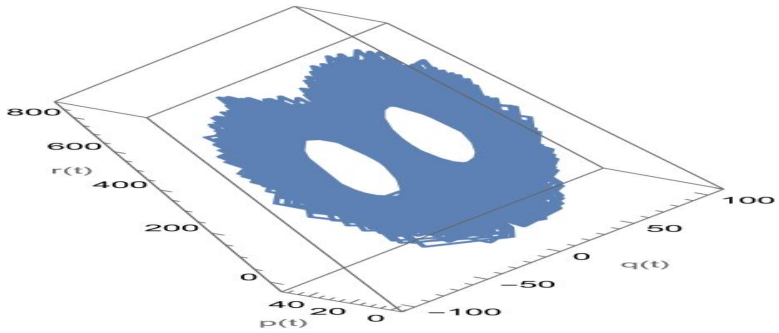

Figure 3. Chaotic Attractor of environmental Model is $p(0) = 50, q(0) = 10, r(0) = 10$

3. Chaos Synchronization Between the Systems (1) and (2)

If System (2) is considered as a drive system and Systems (1) as a response system, then the drive system and the response systems are described as follows:

$$\begin{cases} \dot{p_1} = q_1 - d\,p_1, \\ \dot{q_1} = \varepsilon\, q_1 - p_1 r_1, \\ \dot{r_1} = d\,p_1 q_1 - f r_1, \end{cases} \tag{4}$$

$$\begin{cases} \dot{p}_2 = -a p_2 r_2 - q_2 + p_2^2 + u_1(\mathrm{t}), \\ \dot{q}_2 = -p_2 r_2 + c + u_2(\mathrm{t}), \\ \dot{r}_2 = p_2 q_2 - b r_2 + u_3(\mathrm{t}). \end{cases} \tag{5}$$

The control functions $u_1(\mathrm{t}), u_2(\mathrm{t})$ and $u_3(\mathrm{t})$ are introduced in System (5). To determine the control function $u_1(\mathrm{t}), u_2(\mathrm{t})$ and $u_3(\mathrm{t})$, we subtract System (4) from System (5). The state error variables between the Systems (4) and (5) are described as

$$\begin{cases} e_1 = p_2 - p_1, \\ e_2 = q_2 - q_1, \\ e_3 = r_2 - r_1. \end{cases} \tag{6}$$

Then, the error dynamical System correspond to Systems (4) and (5) is

$$\begin{cases} \dot{e}_1 = -a p_2 r_2 + p_2^2 - q_2 + d p_1 - q_1 + u_1(t), \\ \dot{e}_2 = -p_2 r_2 + c - \varepsilon q_1 + p_1 r_1 + u_2(t), \\ \dot{e}_3 = p_2 q_2 - b r_2 + c r_1 - d p_1 q_1 + u_3(t). \end{cases} \tag{7}$$

We define active control functions $u_1(\mathrm{t}), u_2(\mathrm{t})$ and $u_3(\mathrm{t})$ as follows

$$\begin{cases} u_1(\mathrm{t}) = v_1(\mathrm{t}) + a p_2 r_2 - p_2^2 + b q_1 - d p_1, \\ u_2(\mathrm{t}) = v_2(\mathrm{t}) + a p_2 r_2 - c - q_2 - p_1 r_1 + (\varepsilon + 1) q_1, \\ u_3(\mathrm{t}) = v_3(\mathrm{t}) - p_2 q_2 - (f - 2) r_1 + d p_1 q_1. \end{cases} \tag{8}$$

Where $v_1(\mathrm{t}), v_2(\mathrm{t})$ and $v_3(\mathrm{t})$ are control input defined as a function of error state variables e_1, e_2 and e_3.

Using System (8), System (7) reduces to the following form

$$\begin{cases} \dot{e}_1 = -e_2 + v_1(\mathrm{t}), \\ \dot{e}_2 = -e_2 + v_2(\mathrm{t}), \\ \dot{e}_3 = -be_3 + v_3(\mathrm{t}). \end{cases} \tag{9}$$

The error System (9) to be controlled is a linear system with control input are $v_1(\mathrm{t}), v_2(\mathrm{t})$ and $v_3(\mathrm{t})$. When the System (9) is stabilized by control inputs $v_1(\mathrm{t}), v_2(\mathrm{t})$ and $v_3(\mathrm{t})$, then e_1, e_2 and e_3 will converge to zero as time $t \to \infty$, which implies that the Systems (1) and (2) are synchronized. To achieve this goal, we choose the control inputs as

$$\begin{bmatrix} v_1(\mathrm{t}) \\ v_2(\mathrm{t}) \\ v_3(\mathrm{t}) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{10}$$

Where A is 3×3 constant matrix. For System (10) to be asymptotically stable, its system characteristic matrix must have all its eigenvalues with negative real parts. There are many choices of A but we choose the matrix A in the following form

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \times 10^5 \end{bmatrix}.$$

With the above choice, all the three eigenvalues of closed loop System (10) are negative. This particular choice will lead to $\lim_{t\to\infty} ||e(t)|| = 0$, which implies that the synchronization between Systems (1) and (2) is achieved.

4. Numerical Simulation

The analysis of numerical study is carried out by the software Mathematica. We have considered the parameters of the drive System (4) and response System (5) as $d = 36, \varepsilon = 13, f = 4$, and $a = 1, b = 2, c = 27$. Further, we take the initial values of the Systems (4) and (5) as $p_1(0) = 50, q_1(0) = 10, r_1(0) = 10$ and $p_2(0) = 3, q_2(0) = 2, r_2(0) = 1$. As a result, the initial states of the error system (9) are $e_1(0) = -47, e_2(0) = -8$, and $e_3(0) = -9$. Figures 4, 5 and 6 depicts the dynamics of synchronization for the state variables for the Systems (4) and (5) under control Law (8). From Figure 7, it is observed that the synchronization errors converge rapidly to zero as time tends to infinity.

5. Conclusion

This research work demonstrates that some variation in temperature may control the growth of tumor cells by synchronizing it with the help of active control design scheme, and we observe mathematically that temperature may regulate the proliferation of tumor cells. The solution to this synchronization problem of a brain tumor may aid oncologists in determining the tumor cells can be increased or decreased, how much resistance has the patient and disease, as well as which climate conditions are best for tumor therapy.

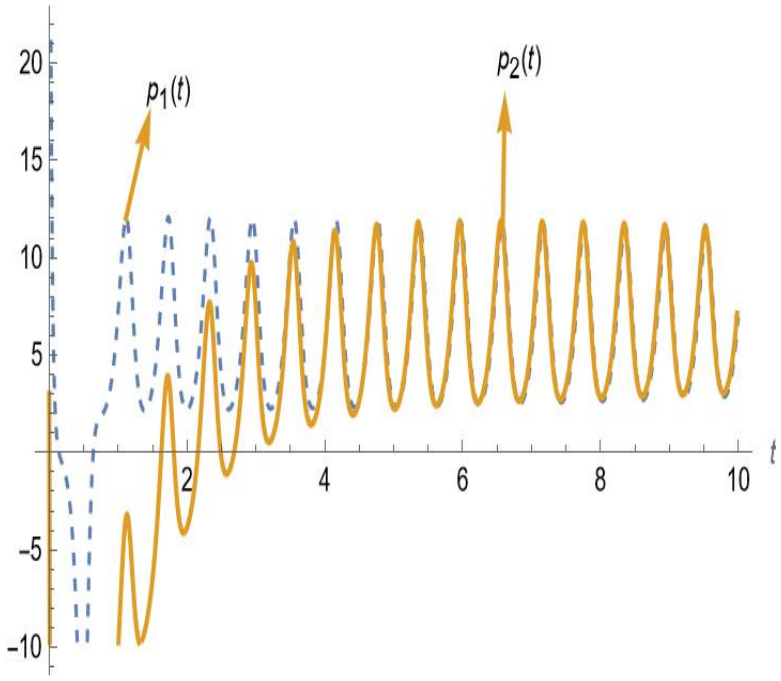

Figure 4. Time Series of p_1 and p_2

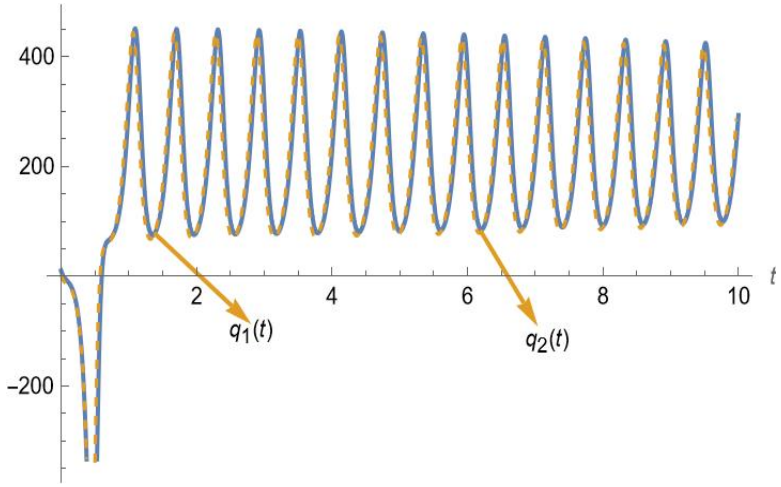

Figure 5. Time Series of q_1 and q_2

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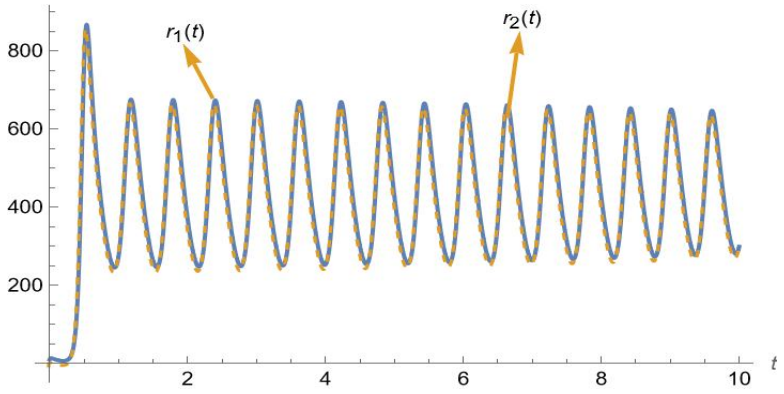

Figure 6. Time Series of r_1 and r_2

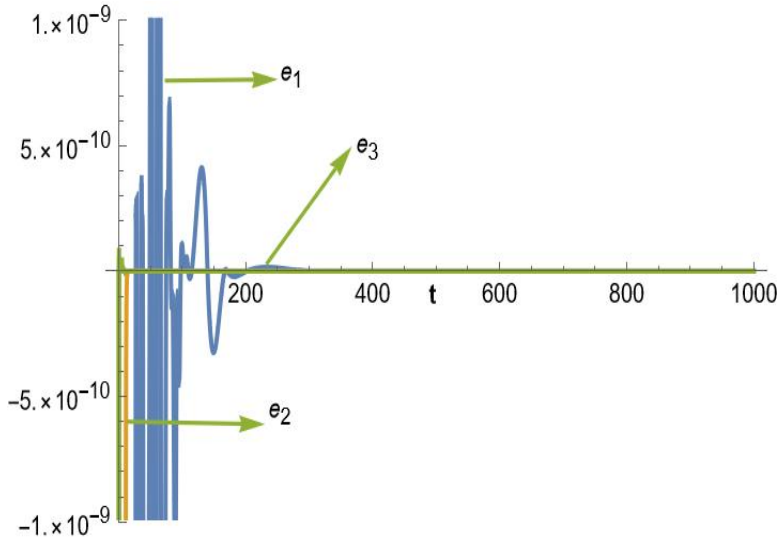

Figure 7. Dynamics of Synchronization of error states (e_1, e_2, e_3) for Systems (1) and (2)

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