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M. Palanikumar
Annamalai University

K. Arulmozhi
Annamalai University

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Type-I Generalized Spherical Interval Valued Fuzzy Soft Sets In Medical Diagnosis for Decision Making

[1] *M. Palanikumar and [2]K. Arulmozhi

[1]Department of Mathematics
Saveetha School of Engineering
Saveetha Institute of Medical and Technical Sciences
Chennai-602105, India
palanimaths86@gmail.com

[2]Department of Mathematics
Bharath Institute of Higher Education and Research
Chennai-600073
Tamilnadu, India
arulmozhiems@gmail.com

*Corresponding Author

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Abstract

In the present communication, we introduce the concept of Type-I generalized spherical interval valued fuzzy soft set and define some operations. It is a generalization of the interval valued fuzzy soft set and the spherical fuzzy soft set. The spherical interval valued fuzzy soft set theory satisfies the condition that the sum of its degrees of positive, neutral, and negative membership does not exceed unity and that these parameters are assigned independently. We also propose an algorithm to solve the decision making problem based on a Type-I generalized soft set model. We introduce a similarity measure based on the Type-I generalized soft set model for two Type-I generalized spherical interval valued fuzzy soft sets and discuss its application in a medical diagnosis problem. Illustrative examples are mentioned to show that they can be successfully used to solve problems with uncertainties.

Keywords: Soft set; FSS; SFS; PFS; IVFSS; Type-I GSIVFSS; Decision making problem

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1. Introduction

Uncertainty can be seen everywhere in most real problems. In order to cope with the uncertain, the theory of fuzzy sets was introduced by Zadeh (1965). A fuzzy set is a set where each element of the universe belongs to it but with some grade or degree of belongingness that lies between zero and one, and such grades are called the membership value of an element in that set. Atanassov (1986) introduces the concept of intuitionistic fuzzy set logic, which is defined by the condition that the sum of its membership degree and non-membership degree value is less than or equal to one. Sometimes, we may face a problem in decision-making, the sum of the membership degree and non-membership degree value is greater than one. So, Yager (2014) has introduced the new notion of Pythagorean fuzzy set logic, which is a generalization of the intuitionistic fuzzy set and characterized by the square sum of its membership degree and non-membership degree, for which the value is less than or equal to one. Akram et al. (2019) has discussed the various applications based on the Pythagorean fuzzy set (PFS). Ashraf and Abdullah (2018) discussed the notion of spherical fuzzy set (SFS), which is an advanced tool of the fuzzy sets, intuitionistic fuzzy sets, and picture fuzzy sets. The idea behind a spherical fuzzy set is to let decision makers generalize other extensions of fuzzy sets by defining a membership function on a spherical surface and independently assigning the parameters of that membership function to a larger domain. Gundoodu and Kahraman (2019) discussed the logic for spherical interval valued fuzzy sets.

Molodtsov (1999) proposed the notion of soft set theory. In comparison with other uncertain theories, soft sets more accurately reflect the objectivity and complexity of decision making in actual situations. Moreover, the combination of soft sets with other mathematical models is also a critical research area. Maji et al. (2001a) proposed the concept of a fuzzy soft set (FSS). Maji et al. (2001b) discussed the logic for intuitionistic fuzzy soft set. These two theories are applied to solve various decision making problems. Xiao et al. (2013) initiated the concept of interval valued fuzzy soft set (IVFSS). Yager (2014) proposed the logic for the Pythagorean fuzzy set. Yang et al. (2015) have discussed the concept of picture fuzzy soft set. In recent years, Peng et al. (2015) have discussed the notion of a fuzzy soft set to a Pythagorean fuzzy soft set. This model solves a class of multi attribute decision making consists of the sum of the degree of membership and non membership values that exceeds unity but the square sum of its membership degree and non-membership degree for which the value is less than or equal to one. Majumdar and Samantab (2010) proposed the logic for generalized fuzzy soft sets. Alkhazaleh and Salleh (2012) discussed the new concept of generalized interval valued fuzzy soft sets.

The purpose of this paper is to extend the concept of generalized interval valued fuzzy soft set to parameterization of Type-I generalized spherical interval valued fuzzy set using a generalized soft set model. We shall further establish a similarity measure based on a generalized soft set model. In Type-I generalized spherical interval valued fuzzy soft sets are defined, their properties are studied, and as an application, a decision making problem is solved.

2. Type-I GSIVFSS

In this section, we give a definition of the Type-I generalized spherical interval valued fuzzy soft set.

Definition 2.1.

Let $X = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. Suppose that $\overline{\widetilde{\mathfrak{U}}} : E \to \overline{S\widetilde{\mathfrak{U}}(X)}$ and $\overline{p} : E \to \overline{[0,1]}$, where $\overline{S\widetilde{\mathfrak{U}}(X)}$ denotes the collection of all spherical interval valued subsets of X. If $\overline{\widetilde{\mathfrak{U}}_p} : E \to \overline{S\widetilde{\mathfrak{U}}(X)} \times \overline{[0,1]}$ is a function defined as $\overline{\widetilde{\mathfrak{U}}_p(e)} = \left(\overline{\widetilde{\mathfrak{U}}(e)(x)}, \overline{p(e)}\right), x \in X$, then $\overline{\widetilde{\mathfrak{U}}_p}$ is called a Type-I generalized spherical interval valued fuzzy soft set (Type-I GSIVFSS) on (X, E) and $\overline{[0,1]}$ is a spherical fuzzy subset of E. For each parameter e_i, $\overline{\widetilde{\mathfrak{U}}_p(e_i)} = \left(\overline{\widetilde{\mathfrak{U}}(e_i)(x)}, \overline{p(e_i)}\right)$ indicates not only the degree of belongingness of the elements of X in $\overline{\widetilde{\mathfrak{U}}(e_i)}$ but also the degree of possibility of such belongingness which is represented by $\overline{p(e_i)}$. So we can write $\overline{\widetilde{\mathfrak{U}}_p(e_i)}$ as follows:

$$\overline{\widetilde{\mathfrak{U}}_p(e_i)} = \left(\left\{\frac{x_1}{(\varepsilon_{\widetilde{\mathfrak{U}}(e_i)}(x_1), \zeta_{\widetilde{\mathfrak{U}}(e_i)}(x_1), \tau_{\widetilde{\mathfrak{U}}(e_i)}(x_1))}, ..., \frac{x_n}{(\varepsilon_{\widetilde{\mathfrak{U}}(e_i)}(x_n), \zeta_{\widetilde{\mathfrak{U}}(e_i)}(x_n), \tau_{\widetilde{\mathfrak{U}}(e_i)}(x_n))}\right\}, (p_1(e_i), p_2(e_i), p_3(e_i))\right),$$

where $\varepsilon_{\widetilde{\mathfrak{U}}(e_i)}(x)$ is the degree of positive membership, $\zeta_{\widetilde{\mathfrak{U}}(e_i)}(x)$ is the degree of neutral membership and $\tau_{\widetilde{\mathfrak{U}}(e_i)}(x)$ is the degree of negative membership function respectively with the condition, $(\varepsilon_{\widetilde{\mathfrak{U}}(e_i)}(x))^2 + (\zeta_{\widetilde{\mathfrak{U}}(e_i)}(x))^2 + (\tau_{\widetilde{\mathfrak{U}}(e_i)}(x))^2 \leq 1$.

To demonstrate the Definition 2.1, we provide a numerical example as follows.

Example 2.1.

Let $X = \{x_1, x_2, x_3\}$ represent a group of cold infection patients and $E = \{e_1, e_2, e_3\}$ represent a group of symptoms that stand for runny or stuffy nose, sore throat, cough, and so on. $\overline{\widetilde{\mathfrak{U}}_p} : E \to \overline{S\widetilde{\mathfrak{U}}(X)} \times \overline{[0,1]}$ is given by

$$\overline{\widetilde{\mathfrak{U}}_p(e_1)} = \left(\left\{\begin{array}{c}\frac{x_1}{([0.40,0.45],[0.10,0.15],[0.50,0.55])} \\ \frac{x_2}{([0.55,0.55],[0.10,0.20],[0.45,0.50])} \\ \frac{x_3}{([0.35,0.40],[0.25,0.30],[0.45,0.65])}\end{array}\right\}, (0.45, 0.55, 0.30)\right),$$

$$\overline{\widetilde{\mathfrak{U}}_p(e_2)} = \left(\left\{\begin{array}{c}\frac{x_1}{([0.35,0.40],[0.15,0.25],[0.55,0.65])} \\ \frac{x_2}{([0.30,0.40],[0.25,0.35],[0.45,0.55])} \\ \frac{x_3}{([0.35,0.45],[0.15,0.20],[0.50,0.65])}\end{array}\right\}, (0.50, 0.25, 0.35)\right),$$

$$\overbracket{\widetilde{\mathcal{U}}_p(e_3)} = \left(\left\{ \begin{array}{c} \frac{x_1}{([0.20,0.25],[0.35,0.40],[0.50,0.65])} \\ \frac{x_2}{([0.30,0.35],[0.45,0.50],[0.35,0.55])} \\ \frac{x_3}{([0.10,0.15],[0.30,0.40],[0.40,0.50])} \end{array} \right\}, (0.30, 0.35, 0.65) \right).$$

Definition 2.2.

Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\overbracket{\widetilde{\mathcal{U}}_p}$ and $\overbracket{\widetilde{\mathcal{V}}_q}$ are two Type-I GSIVFSSs on (X, E). Now, $\overbracket{\widetilde{\mathcal{U}}_p}$ is a Type-I generalized spherical interval valued fuzzy soft subset of $\overbracket{\widetilde{\mathcal{V}}_q}$ (denoted by $\overbracket{\widetilde{\mathcal{U}}_p} \sqsubseteq \overbracket{\widetilde{\mathcal{V}}_q}$) if and only if

(i) $\overbracket{\widetilde{\mathcal{U}}(e)(x)} \sqsubseteq \overbracket{\widetilde{\mathcal{V}}(e)(x)}$ if $\omega_{\widetilde{\mathcal{U}}(e)}(x) \leq \omega_{\widetilde{\mathcal{V}}(e)}(x),\ \ \delta_{\widetilde{\mathcal{U}}(e)}(x) \leq \delta_{\widetilde{\mathcal{V}}(e)}(x),\ \ \pi_{\widetilde{\mathcal{U}}(e)}(x) \geq \pi_{\widetilde{\mathcal{V}}(e)}(x)$,

(ii) $\overbracket{p(e)} \leq \overbracket{q(e)}, \forall\, e \in E$ and $\forall\, x \in X$.

3. Method for Similarity Measure

In this section, a measure of similarity between two Type-I GSIVFSS has been given. The set theoretic approach has been taken in this regard because it is easier for calculation and is a popular method too.

Let $X = \{x_1, x_2, ..., x_m\}$ be a universal set and $E = \{e_1, e_2, ..., e_n\}$ be a set of parameters. Suppose that $\overbracket{\widetilde{\mathcal{U}}_p}$ and $\overbracket{\widetilde{\mathcal{V}}_q}$ are two Type-I GSIVFSSs on (X, E). The similarity measure between two Type-I GSIVFSSs defined as $Sim(\overbracket{\widetilde{\mathcal{U}}_p}, \overbracket{\widetilde{\mathcal{V}}_q}) = \varphi(\overbracket{\widetilde{\mathcal{U}}}, \overbracket{\widetilde{\mathcal{V}}}) \cdot \psi(p, q)$, where $\psi(p, q) = 1 - \frac{\sum_{i=1}^{n} \left| p(e_i) - q(e_i) \right|}{\sum_{i=1}^{n} \left| p(e_i) + q(e_i) \right|}$, the degree of possibility belongingness $p(e_i)$, $q(e_i)$ of $\widetilde{\mathcal{U}}(e_i)$ and $\widetilde{\mathcal{V}}(e_i)$, respectively, where

$$\varphi(\overbracket{\widetilde{\mathcal{U}}}, \overbracket{\widetilde{\mathcal{V}}}) = \frac{1}{m} \sum_{j=1}^{m} \left[\begin{array}{l} \min \left\{ T_1^- \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right), T_2^- \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right), S^- \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right) \right\}, \\ \max \left\{ T_1^+ \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right), T_2^+ \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right), S^+ \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right) \right\} \end{array} \right],$$

$$\widetilde{T}_1 \left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_j)} \right)$$

$$= \left[\frac{\sum_{i=1}^{n} \left(\varepsilon^-_{\mathcal{U}(e_i)}(x_j) \cdot \varepsilon^-_{\mathcal{V}(e_i)}(x_j) \right)}{\sum_{i=1}^{n} \left(1 - \sqrt{\left(1 - \varepsilon^{2-}_{\mathcal{U}(e_i)}(x_j)\right) \cdot \left(1 - \varepsilon^{2-}_{\mathcal{V}(e_i)}(x_j)\right)} \right)},\ \frac{\sum_{i=1}^{n} \left(\varepsilon^+_{\mathcal{U}(e_i)}(x_j) \cdot \varepsilon^+_{\mathcal{V}(e_i)}(x_j) \right)}{\sum_{i=1}^{n} \left(1 - \sqrt{\left(1 - \varepsilon^{2+}_{\mathcal{U}(e_i)}(x_j)\right) \cdot \left(1 - \varepsilon^{2+}_{\mathcal{V}(e_i)}(x_j)\right)} \right)} \right],$$

$$\widetilde{T}_2\left(\overline{\widetilde{\mathcal{U}}(e)(x_j)}, \overline{\widetilde{\mathcal{V}}(e)(x_j)}\right)$$

$$= \left[\frac{\sum_{i=1}^{n}\left(\zeta^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\zeta^{2-}_{\mathcal{V}(e_i)}(x_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\zeta^{4-}_{\mathcal{U}(e_i)}(x_j)\right)\cdot\left(1-\zeta^{4-}_{\mathcal{V}(e_i)}(x_j)\right)}\right)},\ \frac{\sum_{i=1}^{n}\left(\zeta^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\zeta^{2+}_{\mathcal{V}(e_i)}(x_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\zeta^{4+}_{\mathcal{U}(e_i)}(x_j)\right)\cdot\left(1-\zeta^{4+}_{\mathcal{V}(e_i)}(x_j)\right)}\right)}\right],$$

$$\widetilde{S}\left(\overline{\widetilde{\mathcal{U}}(e)(x_j)}, \overline{\widetilde{\mathcal{V}}(e)(x_j)}\right)$$

$$= \left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)-\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\right)},\ \frac{\sum_{i=1}^{n}\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)-\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\right)}\right]\right|}\right].$$

Theorem 3.1.
Let $\overline{\widetilde{\mathcal{U}}_p}$, $\overline{\widetilde{\mathcal{V}}_q}$ and $\overline{\widetilde{\mathcal{W}}_r}$ be the any three Type-I GSIVFSSs over (X, E). If $\overline{\widetilde{\mathcal{U}}_p} \sqsubseteq \overline{\widetilde{\mathcal{V}}_q} \sqsubseteq \overline{\widetilde{\mathcal{W}}_r}$, then prove that $Sim(\overline{\widetilde{\mathcal{U}}_p}, \overline{\widetilde{\mathcal{W}}_r}) \leq Sim(\overline{\widetilde{\mathcal{V}}_q}, \overline{\widetilde{\mathcal{W}}_r})$.

Proof:
Let $\overline{\widetilde{\mathcal{U}}_p}$, $\overline{\widetilde{\mathcal{V}}_q}$ and $\overline{\widetilde{\mathcal{W}}_r}$ be the any three Type-I GSIVFSSs over (X, E). Now,

$$\overline{\widetilde{\mathcal{U}}_p} \sqsubseteq \overline{\widetilde{\mathcal{V}}_q} \implies \left\{\begin{array}{l}\left[\varepsilon^{-}_{\mathcal{U}(e_i)}(x_j), \varepsilon^{+}_{\mathcal{U}(e_i)}(x_j)\right] \leq \left[\varepsilon^{-}_{\mathcal{V}(e_i)}(x_j), \varepsilon^{+}_{\mathcal{V}(e_i)}(x_j)\right], \\ \left[\zeta^{-}_{\mathcal{U}(e_i)}(x_j), \zeta^{+}_{\mathcal{U}(e_i)}(x_j)\right] \leq \left[\zeta^{-}_{\mathcal{V}(e_i)}(x_j), \zeta^{+}_{\mathcal{V}(e_i)}(x_j)\right] \\ \left[\tau^{-}_{\mathcal{U}(e_i)}(x_j), \tau^{+}_{\mathcal{U}(e_i)}(x_j)\right] \geq \left[\tau^{-}_{\mathcal{V}(e_i)}(x_j), \tau^{+}_{\mathcal{V}(e_i)}(x_j)\right], \\ \left[p^{-}(e_i), p^{+}(e_i)\right] \leq \left[q^{-}(e_i), q^{+}(e_i)\right]\end{array}\right\},$$

$$\overline{\widetilde{\mathcal{V}}_q} \sqsubseteq \overline{\widetilde{\mathcal{W}}_r} \implies \left\{\begin{array}{l}\left[\varepsilon^{-}_{\mathcal{V}(e_i)}(x_j), \varepsilon^{+}_{\mathcal{V}(e_i)}(x_j)\right] \leq \left[\varepsilon^{-}_{\mathcal{W}(e_i)}(x_j), \varepsilon^{+}_{\mathcal{W}(e_i)}(x_j)\right], \\ \left[\zeta^{-}_{\mathcal{V}(e_i)}(x_j), \zeta^{+}_{\mathcal{V}(e_i)}(x_j)\right] \leq \left[\zeta^{-}_{\mathcal{W}(e_i)}(x_j), \zeta^{+}_{\mathcal{W}(e_i)}(x_j)\right] \\ \left[\tau^{-}_{\mathcal{V}(e_i)}(x_j), \tau^{+}_{\mathcal{V}(e_i)}(x_j)\right] \geq \left[\tau^{-}_{\mathcal{W}(e_i)}(x_j), \tau^{+}_{\mathcal{W}(e_i)}(x_j)\right], \\ \left[q^{-}(e_i), q^{+}(e_i)\right] \leq \left[r^{-}(e_i), r^{+}(e_i)\right]\end{array}\right\}. \quad (1)$$

From the Equation (1), we get

$$\left\{\begin{array}{l}\left[\left(\varepsilon^{-}_{\mathcal{U}(e_i)}(x_j)\cdot\varepsilon^{-}_{\mathcal{W}(e_i)}(x_j)\right), \left(\varepsilon^{+}_{\mathcal{U}(e_i)}(x_j)\cdot\varepsilon^{+}_{\mathcal{W}(e_i)}(x_j)\right)\right] \leq \\ \left[\left(\varepsilon^{-}_{\mathcal{V}(e_i)}(x_j)\cdot\varepsilon^{-}_{\mathcal{W}(e_i)}(x_j)\right), \left(\varepsilon^{+}_{\mathcal{V}(e_i)}(x_j)\cdot\varepsilon^{+}_{\mathcal{W}(e_i)}(x_j)\right)\right]\end{array}\right\},$$

$$\left\{\begin{array}{l}\left[\sum_{i=1}^{n}\left(\varepsilon^{-}_{\mathfrak{U}(e_i)}(x_j)\cdot\varepsilon^{-}_{\mathfrak{W}(e_i)}(x_j)\right),\sum_{i=1}^{n}\left(\varepsilon^{+}_{\mathfrak{U}(e_i)}(x_j)\cdot\varepsilon^{+}_{\mathfrak{W}(e_i)}(x_j)\right)\right]\leq\\ \left[\sum_{i=1}^{n}\left(\varepsilon^{-}_{\mathfrak{V}(e_i)}(x_j)\cdot\varepsilon^{-}_{\mathfrak{W}(e_i)}(x_j)\right),\sum_{i=1}^{n}\left(\varepsilon^{+}_{\mathfrak{V}(e_i)}(x_j)\cdot\varepsilon^{+}_{\mathfrak{W}(e_i)}(x_j)\right)\right]\end{array}\right\}. \tag{2}$$

From the Equation (1) and for $j = 1, 2, ..., m$, we get

$$\begin{aligned}\left[1-\varepsilon^{2-}_{\mathfrak{U}(e_i)}(x_j),\ 1-\varepsilon^{2+}_{\mathfrak{U}(e_i)}(x_j)\right] &\geq \left[1-\varepsilon^{2-}_{\mathfrak{V}(e_i)}(x_j),\ 1-\varepsilon^{2+}_{\mathfrak{V}(e_i)}(x_j)\right]\\ &\geq \left[1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j),\ 1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right],\end{aligned}$$

$$\left\{\begin{array}{l}\left[\sqrt{\left(\left(1-\varepsilon^{2-}_{\mathfrak{U}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j)\right)\right)},\ \sqrt{\left(\left(1-\varepsilon^{2+}_{\mathfrak{U}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right]\geq\\ \left[\sqrt{\left(\left(1-\varepsilon^{2-}_{\mathfrak{V}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j)\right)\right)},\ \sqrt{\left(\left(1-\varepsilon^{2+}_{\mathfrak{V}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right]\end{array}\right\},$$

$$\left\{\begin{array}{l}\left[1-\sqrt{\left(\left(1-\varepsilon^{2+}_{\mathfrak{U}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right)\right)},\right.\\ \left.1-\sqrt{\left(\left(1-\varepsilon^{2-}_{\mathfrak{U}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right]\leq\\ \left[1-\sqrt{\left(\left(1-\varepsilon^{2+}_{\mathfrak{V}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right)\right)},\right.\\ \left.1-\sqrt{\left(\left(1-\varepsilon^{2-}_{\mathfrak{V}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right]\end{array}\right\},$$

$$\left\{\begin{array}{l}\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varepsilon^{2+}_{\mathfrak{U}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right),\right.\\ \left.\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varepsilon^{2-}_{\mathfrak{U}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right)\right]\leq\\ \left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varepsilon^{2+}_{\mathfrak{V}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2+}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right),\right.\\ \left.\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varepsilon^{2-}_{\mathfrak{V}(e_i)}(x_j)\right)\cdot\left(1-\varepsilon^{2-}_{\mathfrak{W}(e_i)}(x_j)\right)\right)}\right)\right]\end{array}\right\}. \tag{3}$$

Equation (2) is divided by (3), and we get

$$\left\{ \begin{array}{c} \dfrac{\left[\sum_{i=1}^{n} \left(\varepsilon^{-}_{\mathcal{U}(e_i)}(x_j) \cdot \varepsilon^{-}_{\mathcal{W}(e_i)}(x_j) \right),\ \sum_{i=1}^{n} \left(\varepsilon^{+}_{\mathcal{U}(e_i)}(x_j) \cdot \varepsilon^{+}_{\mathcal{W}(e_i)}(x_j) \right) \right]}{\left[\sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2+}_{\mathcal{U}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2+}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right), \sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2-}_{\mathcal{U}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2-}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right) \right]} \leq \\ \dfrac{\left[\sum_{i=1}^{n} \left(\varepsilon^{-}_{\mathcal{V}(e_i)}(x_j) \cdot \varepsilon^{-}_{\mathcal{W}(e_i)}(x_j) \right),\ \sum_{i=1}^{n} \left(\varepsilon^{+}_{\mathcal{V}(e_i)}(x_j) \cdot \varepsilon^{+}_{\mathcal{W}(e_i)}(x_j) \right) \right]}{\left[\sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2+}_{\mathcal{V}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2+}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right), \sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2-}_{\mathcal{V}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2-}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right) \right]} \end{array} \right\},$$

$$\left\{ \begin{array}{c} \left[\dfrac{\left[\sum_{i=1}^{n} \left(\varepsilon^{-}_{\mathcal{U}(e_i)}(x_j) \cdot \varepsilon^{-}_{\mathcal{W}(e_i)}(x_j) \right) \right]}{\left[\sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2-}_{\mathcal{U}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2-}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right) \right]}, \dfrac{\left[\sum_{i=1}^{n} \left(\varepsilon^{+}_{\mathcal{U}(e_i)}(x_j) \cdot \varepsilon^{+}_{\mathcal{W}(e_i)}(x_j) \right) \right]}{\left[\sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2+}_{\mathcal{U}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2+}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right) \right]} \right] \leq \\ \left[\dfrac{\left[\sum_{i=1}^{n} \left(\varepsilon^{-}_{\mathcal{V}(e_i)}(x_j) \cdot \varepsilon^{-}_{\mathcal{W}(e_i)}(x_j) \right) \right]}{\left[\sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2-}_{\mathcal{V}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2-}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right) \right]}, \dfrac{\left[\sum_{i=1}^{n} \left(\varepsilon^{+}_{\mathcal{V}(e_i)}(x_j) \cdot \varepsilon^{+}_{\mathcal{W}(e_i)}(x_j) \right) \right]}{\left[\sum_{i=1}^{n} \left(1 - \sqrt{\left(\left(1 - \varepsilon^{2+}_{\mathcal{V}(e_i)}(x_j) \right) \cdot \left(1 - \varepsilon^{2+}_{\mathcal{W}(e_i)}(x_j) \right) \right)} \right) \right]} \right] \end{array} \right\}.$$

Therefore,

$$\widetilde{T}_1 \left(\overline{\widetilde{\mathcal{U}}(e)(x_j)}, \overline{\widetilde{\mathcal{W}}(e)(x_j)} \right) \leq \widetilde{T}_1 \left(\overline{\widetilde{\mathcal{V}}(e)(x_j)}, \overline{\widetilde{\mathcal{W}}(e)(x_j)} \right). \tag{4}$$

From the Equation (1), we get

$$\left\{ \begin{array}{c} \left[\sum_{i=1}^{n} \left(\zeta^{2-}_{\mathcal{U}(e_i)}(x_j) \cdot \zeta^{2-}_{\mathcal{W}(e_i)}(x_j) \right), \sum_{i=1}^{n} \left(\zeta^{2+}_{\mathcal{U}(e_i)}(x_j) \cdot \zeta^{2+}_{\mathcal{W}(e_i)}(x_j) \right) \right] \leq \\ \left[\sum_{i=1}^{n} \left(\zeta^{2-}_{\mathcal{V}(e_i)}(x_j) \cdot \zeta^{2-}_{\mathcal{W}(e_i)}(x_j) \right), \sum_{i=1}^{n} \left(\zeta^{2+}_{\mathcal{V}(e_i)}(x_j) \cdot \zeta^{2+}_{\mathcal{W}(e_i)}(x_j) \right) \right] \end{array} \right\}. \tag{5}$$

From the Equation (1), we get

$$\begin{aligned} \left[1 - \zeta^{4-}_{\mathcal{U}(e_i)}(x_j),\ 1 - \zeta^{4+}_{\mathcal{U}(e_i)}(x_j) \right] &\geq \left[1 - \zeta^{4-}_{\mathcal{V}(e_i)}(x_j),\ 1 - \zeta^{4+}_{\mathcal{V}(e_i)}(x_j) \right] \\ &\geq \left[1 - \zeta^{4-}_{\mathcal{W}(e_i)}(x_j),\ 1 - \zeta^{4+}_{\mathcal{W}(e_i)}(x_j) \right], \end{aligned}$$

$$\left\{\begin{array}{l}\Big[1-\sqrt{\Big(\Big(1-\zeta^{4+}_{\mathcal{U}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)},\\ 1-\sqrt{\Big(\Big(1-\zeta^{4-}_{\mathcal{U}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big]\leq\\ \Big[1-\sqrt{\Big(\Big(1-\zeta^{4+}_{\mathcal{V}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)},\\ 1-\sqrt{\Big(\Big(1-\zeta^{4-}_{\mathcal{V}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big]\end{array}\right\},$$

$$\left\{\begin{array}{l}\Big[\sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4+}_{\mathcal{U}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big),\\ \sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4-}_{\mathcal{U}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big)\Big]\leq\\ \Big[\sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4+}_{\mathcal{V}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big),\\ \sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4-}_{\mathcal{V}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big)\Big]\end{array}\right\}. \tag{6}$$

Equation (5) is divided by (6), and we get

$$\left\{\begin{array}{l}\dfrac{\Big[\sum_{i=1}^{n}\Big(\zeta^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\zeta^{2-}_{\mathcal{W}(e_i)}(x_j)\Big),\ \sum_{i=1}^{n}\Big(\zeta^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\zeta^{2+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big]}{\Big[\sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4+}_{\mathcal{U}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big),\sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4-}_{\mathcal{U}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big)\Big]}\leq\\ \dfrac{\Big[\sum_{i=1}^{n}\Big(\zeta^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\zeta^{2-}_{\mathcal{W}(e_i)}(x_j)\Big),\ \sum_{i=1}^{n}\Big(\zeta^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\zeta^{2+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big]}{\Big[\sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4+}_{\mathcal{V}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big),\sum_{i=1}^{n}\Big(1-\sqrt{\Big(\Big(1-\zeta^{4-}_{\mathcal{V}(e_i)}(x_j)\Big)\cdot\Big(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\Big)\Big)}\Big)\Big]}\end{array}\right\},$$

$$\left\{\begin{array}{l}\left[\frac{\left[\sum_{i=1}^{n}\left(\zeta^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\zeta^{2-}_{\mathcal{W}(e_i)}(x_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\zeta^{4-}_{\mathcal{U}(e_i)}(x_j)\right)\cdot\left(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\right)\right)}\right)\right]}, \frac{\left[\sum_{i=1}^{n}\left(\zeta^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\zeta^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\zeta^{4+}_{\mathcal{U}(e_i)}(x_j)\right)\cdot\left(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\right)\right)}\right)\right]}\right]\leq \\ \left[\frac{\left[\sum_{i=1}^{n}\left(\zeta^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\zeta^{2-}_{\mathcal{W}(e_i)}(x_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\zeta^{4-}_{\mathcal{V}(e_i)}(x_j)\right)\cdot\left(1-\zeta^{4-}_{\mathcal{W}(e_i)}(x_j)\right)\right)}\right)\right]}, \frac{\left[\sum_{i=1}^{n}\left(\zeta^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\zeta^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\zeta^{4+}_{\mathcal{V}(e_i)}(x_j)\right)\cdot\left(1-\zeta^{4+}_{\mathcal{W}(e_i)}(x_j)\right)\right)}\right)\right]}\right]\end{array}\right\}.$$

Therefore,

$$\widetilde{T}_2\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_j)},\overbrace{\widetilde{\mathcal{W}}(e)(x_j)}\right)\leq\widetilde{T}_2\left(\overbrace{\widetilde{\mathcal{V}}(e)(x_j)},\overbrace{\widetilde{\mathcal{W}}(e)(x_j)}\right). \quad (7)$$

From the Equation (1), we get

$$\left|\left[\tau^{2-}_{\mathcal{U}(e_i)}(x_j),\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\right]-\left[\tau^{2-}_{\mathcal{W}(e_i)}(x_j),\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right]\right|\geq\left|\left[\tau^{2-}_{\mathcal{V}(e_i)}(x_j),\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\right]-\left[\tau^{2-}_{\mathcal{W}(e_i)}(x_j),\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right]\right|,$$

$$\left\{\begin{array}{l}\left|\left[\tau^{2-}_{\mathcal{U}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\ ,\ \tau^{2+}_{\mathcal{U}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right]\right|\geq \\ \left|\left[\tau^{2-}_{\mathcal{V}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\ ,\ \tau^{2+}_{\mathcal{V}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right]\right|\end{array}\right\},$$

$$\left\{\begin{array}{l}\left|\left[\sum_{i=1}^{n}\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\ ,\ \sum_{i=1}^{n}\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)\right]\right|\geq \\ \left|\left[\sum_{i=1}^{n}\left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\ ,\ \sum_{i=1}^{n}\left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)\right]\right|\end{array}\right\}. \quad (8)$$

From the Equation (1), we get

$$\left\{\begin{array}{l}\left|\left[\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right),\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\right]\right|\geq \\ \left|\left[\left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right),\left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\right]\right|\end{array}\right\},$$

$$\left\{\begin{array}{l}\left|\left[\sum_{i=1}^{n}1+\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right),\sum_{i=1}^{n}1+\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\right]\right|\geq \\ \left|\left[\sum_{i=1}^{n}1+\left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right),\sum_{i=1}^{n}1+\left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)\right]\right|\end{array}\right\}. \quad (9)$$

Equation (8) is divided by (9), we get

$$\left\{ \begin{array}{l} \left| \left[\frac{\sum_{i=1}^n \left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)} \; , \; \frac{\sum_{i=1}^n \left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)} \right] \right| \geq \\ \left| \left[\frac{\sum_{i=1}^n \left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)} \; , \; \frac{\sum_{i=1}^n \left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)} \right] \right| \end{array} \right\},$$

$$\left\{ \begin{array}{l} \sqrt{\left| \left[\frac{\sum_{i=1}^n \left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)} \; , \; \frac{\sum_{i=1}^n \left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)} \right] \right|} \geq \\ \sqrt{\left| \left[\frac{\sum_{i=1}^n \left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)} \; , \; \frac{\sum_{i=1}^n \left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)} \right] \right|} \end{array} \right\},$$

$$\left\{ \begin{array}{l} \left[1-\sqrt{\left| \left[\frac{\sum_{i=1}^n \left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)} \; , \; \frac{\sum_{i=1}^n \left(\tau^{2+}_{\mathcal{U}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2-}_{\mathcal{U}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)} \right] \right|} \right] \leq \\ \left[1-\sqrt{\left| \left[\frac{\sum_{i=1}^n \left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)-\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2+}_{\mathcal{W}(e_i)}(x_j)\right)} \; , \; \frac{\sum_{i=1}^n \left(\tau^{2+}_{\mathcal{V}(e_i)}(x_j)-\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)}{\sum_{i=1}^n 1+\left(\tau^{2-}_{\mathcal{V}(e_i)}(x_j)\cdot\tau^{2-}_{\mathcal{W}(e_i)}(x_j)\right)} \right] \right|} \right] \end{array} \right\},$$

$$\widetilde{S}\left(\overbracket{\widetilde{\mathcal{U}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{W}}(e)(x_j)}\right) \leq \widetilde{S}\left(\overbracket{\widetilde{\mathcal{V}}(e)(x_j)}, \overbracket{\widetilde{\mathcal{W}}(e)(x_j)}\right). \tag{10}$$

From the Equations (4), (7) and (10), we get

$$\varphi(\overbracket{\widetilde{\mathcal{U}}}, \overbracket{\widetilde{\mathcal{W}}}) \leq \varphi(\overbracket{\widetilde{\mathcal{V}}}, \overbracket{\widetilde{\mathcal{W}}}). \tag{11}$$

From the Equation (1), we get

$$\left|p(e)-r(e)\right| \geq \left|q(e)-r(e)\right| \text{ and } \left|p(e)+r(e)\right| \leq \left|q(e)+r(e)\right|.$$

Hence, $\sum_{i=1}^n \left|p(e_i)-r(e_i)\right| \geq \sum_{i=1}^n \left|q(e_i)-r(e_i)\right|$. Thus,

$$-\sum_{i=1}^n \left|p(e_i)-r(e_i)\right| \leq -\sum_{i=1}^n \left|q(e_i)-r(e_i)\right|, \tag{12}$$

$$\sum_{i=1}^n \left|p(e_i)+r(e_i)\right| \leq \sum_{i=1}^n \left|q(e_i)+r(e_i)\right|. \tag{13}$$

Equation (12) is divided by (13), and we get

$$-\frac{\sum_{i=1}^{n}\left|p(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i)+r(e_i)\right|} \leq -\frac{\sum_{i=1}^{n}\left|q(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|q(e_i)+r(e_i)\right|},$$

$$1-\frac{\sum_{i=1}^{n}\left|p(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i)+r(e_i)\right|} \leq 1-\frac{\sum_{i=1}^{n}\left|q(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|q(e_i)+r(e_i)\right|},$$

$$\psi(p,r) \leq \psi(q,r). \tag{14}$$

Equations (11) and (14) give $\varphi(\overbracket{\widetilde{\mathcal{U}}}, \overbracket{\widetilde{\mathcal{W}}}) \cdot \psi(p,r) \leq \varphi(\overbracket{\widetilde{\mathcal{V}}}, \overbracket{\widetilde{\mathcal{W}}}) \cdot \psi(q,r)$.

Hence, $Sim(\overbracket{\widetilde{\mathcal{U}}_p}, \overbracket{\widetilde{\mathcal{W}}_r}) \leq Sim(\overbracket{\widetilde{\mathcal{V}}_q}, \overbracket{\widetilde{\mathcal{W}}_r})$. ■

Example 3.1.

Consider the following two Type-I GSIVFSS $\overbracket{\widetilde{\mathcal{U}}_p}$ and $\overbracket{\widetilde{\mathcal{V}}_q}$ where $X = \{x_1$: severe, x_2: mild, x_3 : no$\}$ and $E = \{e_1, e_2, e_3\}$.

Table 1

$\overbracket{\mathcal{U}_p(e)}$	e_1	e_2	e_3
$\overbracket{\mathcal{U}(e)(x_1)}$	$[0.35, 0.45]$ $[0.25, 0.35]$ $[0.55, 0.6]$	$[0.55, 0.6]$ $[0.35, 0.5]$ $[0.45, 0.5]$	$[0.45, 0.55]$ $[0.35, 0.4]$ $[0.4, 0.55]$
$\overbracket{\mathcal{U}(e)(x_2)}$	$[0.35, 0.55]$ $[0.25, 0.35]$ $[0.4, 0.65]$	$[0.55, 0.65]$ $[0.3, 0.4]$ $[0.3, 0.55]$	$[0.25, 0.4]$ $[0.3, 0.5]$ $[0.5, 0.6]$
$\overbracket{\mathcal{U}(e)(x_3)}$	$[0.35, 0.45]$ $[0.4, 0.55]$ $[0.3, 0.5]$	$[0.3, 0.45]$ $[0.3, 0.45]$ $[0.4, 0.55]$	$[0.1, 0.3]$ $[0.5, 0.65]$ $[0.5, 0.6]$
$p(e)$	$(0.45, 0.45, 0.35)$	$(0.35, 0.55, 0.25)$	$(0.55, 0.4, 0.15)$
$\overbracket{\mathcal{V}_q(e)}$	e_1	e_2	e_3
$\overbracket{\mathcal{V}(e)(x_1)}$	$[0.25, 0.35]$ $[0.5, 0.55]$ $[0.25, 0.5]$	$[0.35, 0.45]$ $[0.15, 0.45]$ $[0.3, 0.4]$	$[0.25, 0.3]$ $[0.45, 0.6]$ $[0.25, 0.3]$
$\overbracket{\mathcal{V}(e)(x_2)}$	$[0.4, 0.45]$ $[0.4, 0.5]$ $[0.25, 0.35]$	$[0.3, 0.55]$ $[0.6, 0.7]$ $[0.15, 0.2]$	$[0.2, 0.45]$ $[0.5, 0.6]$ $[0.35, 0.4]$
$\overbracket{\mathcal{V}(e)(x_3)}$	$[0.5, 0.65]$ $[0.3, 0.65]$ $[0.15, 0.25]$	$[0.4, 0.6]$ $[0.35, 0.5]$ $[0.25, 0.35]$	$[0.3, 0.5]$ $[0.4, 0.6]$ $[0.35, 0.4]$
$q(e)$	$(0.4, 0.5, 0.4)$	$(0.5, 0.1, 0.6)$	$(0.6, 0.2, 0.5)$

Now,

$$T_1\left(\overbracket{\widetilde{\mathcal{U}}(e)(x_1)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_1)}\right) = \left[\tfrac{0.3925}{0.44598}, \tfrac{0.5925}{0.652337}\right] = \left[0.880075, 0.908273\right],$$

$$T_2\left(\overbracket{\widetilde{\mathcal{U}}(e)(x_1)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_1)}\right) = \left[\tfrac{0.043188}{0.069523}, \tfrac{0.145281}{0.18491}\right] = \left[0.621197, 0.785686\right] \text{ and}$$

$$S\left(\overbracket{\widetilde{\mathcal{U}}(e)(x_1)}, \overbracket{\widetilde{\mathcal{V}}(e)(x_1)}\right) = 1 - \sqrt{\left|\left[\tfrac{0.165}{3.157225}, \tfrac{0.6975}{3.047131}\right]\right|} = \left[0.521561, 0.771393\right].$$

Hence,

$$\begin{bmatrix} \min\left\{T_1^-\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_1)},\overbrace{\widetilde{\mathcal{V}}(e)(x_1)}\right),T_2^-\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_1)},\overbrace{\widetilde{\mathcal{V}}(e)(x_1)}\right),S^-\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_1)},\overbrace{\widetilde{\mathcal{V}}(e)(x_1)}\right)\right\}, \\ \max\left\{T_1^+\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_1)},\overbrace{\widetilde{\mathcal{V}}(e)(x_1)}\right),T_2^+\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_1)},\overbrace{\widetilde{\mathcal{V}}(e)(x_1)}\right),S^+\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_1)},\overbrace{\widetilde{\mathcal{V}}(e)(x_1)}\right)\right\} \end{bmatrix}$$
$$= \big[0.521561, 0.908273\big].$$

Similarly,

$$T_1\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right) = \left[\tfrac{0.355}{0.396075},\tfrac{0.785}{0.80103}\right] = \big[0.896295, 0.979989\big],$$

$$T_2\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right) = \left[\tfrac{0.0649}{0.12133},\tfrac{0.199025}{0.275227}\right] = \big[0.534904, 0.72313\big], \text{ and}$$

$$S\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right) = 1-\sqrt{\left|\left[\tfrac{0.1775}{3.121456},\tfrac{0.8775}{3.04265}\right]\right|} = \big[0.462971, 0.761537\big].$$

Hence,

$$\begin{bmatrix} \min\left\{T_1^-\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right),T_2^-\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right),S^-\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right)\right\}, \\ \max\left\{T_1^+\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right),T_2^+\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right),S^+\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_2)},\overbrace{\widetilde{\mathcal{V}}(e)(x_2)}\right)\right\} \end{bmatrix}$$
$$= \big[0.462971, 0.979989\big].$$

Similarly,

$$T_1\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_3)},\overbrace{\widetilde{\mathcal{V}}(e)(x_3)}\right) = \left[\tfrac{0.325}{0.365294},\tfrac{0.7125}{0.780799}\right] = \big[0.889695, 0.912527\big],$$

$$T_2\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_3)},\overbrace{\widetilde{\mathcal{V}}(e)(x_3)}\right) = \left[\tfrac{0.065425}{0.072676},\tfrac{0.330531}{0.342321}\right] = \big[0.900226, 0.965558\big], \text{ and}$$

$$S\left(\overbrace{\widetilde{\mathcal{U}}(e)(x_3)},\overbrace{\widetilde{\mathcal{V}}(e)(x_3)}\right) = 1-\sqrt{\left|\left[\tfrac{0.155}{3.110281},\tfrac{0.705}{3.04265}\right]\right|} = \big[0.518642, 0.776763\big],$$

$$\left[\begin{array}{l}\min\left\{T_1^-\left(\overparen{\widetilde{\mathcal{U}}(e)(x_3)},\overparen{\widetilde{\mathcal{V}}(e)(x_3)}\right),T_2^-\left(\overparen{\widetilde{\mathcal{U}}(e)(x_3)},\overparen{\widetilde{\mathcal{V}}(e)(x_3)}\right),S^-\left(\overparen{\widetilde{\mathcal{U}}(e)(x_3)},\overparen{\widetilde{\mathcal{V}}(e)(x_3)}\right)\right\},\\ \max\left\{T_1^+\left(\overparen{\widetilde{\mathcal{U}}(e)(x_3)},\overparen{\widetilde{\mathcal{V}}(e)(x_3)}\right),T_2^+\left(\overparen{\widetilde{\mathcal{U}}(e)(x_3)},\overparen{\widetilde{\mathcal{V}}(e)(x_3)}\right),S^+\left(\overparen{\widetilde{\mathcal{U}}(e)(x_3)},\overparen{\widetilde{\mathcal{V}}(e)(x_3)}\right)\right\}\end{array}\right]$$

$$= \big[0.518642, 0.965558\big],$$

$$\varphi(\overparen{\widetilde{\mathcal{U}}},\overparen{\widetilde{\mathcal{V}}}) = \left[\tfrac{0.521561+0.462971+0.518642}{3}, \tfrac{0.908273+0.979989+0.965558}{3}\right] = \big[0.501058,\ 0.951273\big],$$

$$\psi(p,q) = 1 - 0.232877 = 0.767123,$$

$$Sim(\overparen{\widetilde{\mathcal{U}}_p},\overparen{\widetilde{\mathcal{V}}_q}) = \big[0.501058\times 0.767123,\ 0.951273\times 0.767123\big] = \big[0.3843732,\ 0.729744\big].$$

4. An application for medical diagnosis

In this section, we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from dengue hemorrhagic fever. We first construct a Type-I GSIVFSS model for dengue hemorrhagic fever and the Type-I GSIVFSS of symptoms for the ill person. We find the similarity measure between these two sets. If they are significantly similar, then we conclude that the person is possibly suffering from dengue hemorrhagic fever. In this application, we present a method for a medical diagnosis problem based on the proposed similarity measure of Type-I GSIVFSS's. This technique of similarity measure between two Type-I GSIVFSS can be applied to detect whether an ill person is suffering from a disease or not. We first give the following note.

Note: Let $\overparen{\widetilde{\mathcal{U}}_p}$ and $\overparen{\widetilde{\mathcal{V}}_q}$ be two Type-I GSIVFSS's over (X,E). We call the two Type-I GSIVFSS's to be significantly similar if $Sim^+(\overparen{\widetilde{\mathcal{U}}_p},\overparen{\widetilde{\mathcal{V}}_q}) \geq 0.70$.

4.1. Algorithms

Step 1. Input the Type-I GSIVFSS in tabular form.

Step 2. Input the set of choice parameters $U \subseteq E$.

Step 3. Compute the values of $T_1(x_j)$, $T_2(x_j)$ and $S(x_j)$ and $1 \leq j \leq m$.

Step 4. Calculate $\varphi = \frac{1}{m}\sum_{j=1}^{m}\Big[\min\{T_1^-(x_j),T_2^-(x_j),S^-(x_j)\},\max\{T_1^+(x_j),T_2^+(x_j),S^+(x_j)\}\Big]$.

Step 5. Determine the value $\psi(p,q) = 1 - \frac{\sum_{i=1}^{n}\left|p(e_i)-q(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i)+q(e_i)\right|}$.

Step 6. Compute the similarity measure = $\varphi \cdot \psi$.

Step 7. Select similarity measure, when suitable criteria for significantly similar.

Step 8. Finally, decision to the problem.

Step 9. End.

4.2. Data Analysis

Assume there are five patients in a hospital with dengue hemorrhagic fever symptoms: $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4$ and $\mathcal{P}_5$. Let the universal set contain only three elements. That is, $X = \{x_1$: severe, x_2: mild, x_3 : no$\}$, where the set of parameters E is the set of certain symptoms of dengue hemorrhagic fever is represented by $E = \{e_1$: severe abdominal pain, e_2: persistent vomiting, e_3 : rapid breathing, e_4 : bleeding gums, e_5: restlessness and blood in vomit$\}$. We first construct a Type-I GSIVFSS for the illness with the help of a medical person and a Type-I GSIVFSS for the ill person. Then, we calculate the similarity measure between two Type-I GSIVFSS's. If they are significantly similar, then we infer that the person may have the disease, and otherwise not.

Table 2 is represented by the dengue hemorrhagic fever prepared with the help of a medical person.

$L(e)(x_1)$ indicates that the person requires medical attention.

$L(e)(x_2)$ denotes that the person occasionally requires the assistance of a doctor.

$L(e)(x_3)$ indicates that the person does not require medical attention.

$\overbracket{\mathcal{P}_i(e)(x_1)}$ is used to identify the i^{th} person who has severe dengue hemorrhagic fever.

$\overbracket{\mathcal{P}_i(e)(x_2)}$ is used to identify the i^{th} person who has mild dengue hemorrhagic fever.

$\overbracket{\mathcal{P}_i(e)(x_3)}$, $i = 1, 2, 3, 4, 5$; this means that the i^{th} person is not affected by dengue hemorrhagic fever.

Table 2. Type-I GSIVFSS model for pneumonia (dengue hemorrhagic fever)

$\overline{\mathcal{L}_{p(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{L}(e)(x_1)}$	$[0.35, 0.65]$ $[0.45, 0.55]$ $[0.45, 0.55]$	$[0.25, 0.55]$ $[0.30, 0.35]$ $[0.45, 0.65]$	$[0.35, 0.45]$ $[0.20, 0.25]$ $[0.55, 0.60]$	$[0.25, 0.35]$ $[0.35, 0.45]$ $[0.35, 0.45]$	$[0.20, 0.30]$ $[0.50, 0.55]$ $[0.45, 0.55]$
$\overline{\mathcal{L}(e)(x_2)}$	$[0.45, 0.50]$ $[0.30, 0.40]$ $[0.40, 0.60]$	$[0.55, 0.60]$ $[0.35, 0.50]$ $[0.35, 0.50]$	$[0.60, 0.65]$ $[0.30, 0.40]$ $[0.25, 0.45]$	$[0.35, 0.45]$ $[0.50, 0.65]$ $[0.35, 0.45]$	$[0.50, 0.55]$ $[0.40, 0.55]$ $[0.40, 0.55]$
$\overline{\mathcal{L}(e)(x_3)}$	$[0.50, 0.65]$ $[0.30, 0.45]$ $[0.30, 0.35]$	$[0.40, 0.60]$ $[0.35, 0.55]$ $[0.40, 0.55]$	$[0.35, 0.45]$ $[0.40, 0.60]$ $[0.35, 0.45]$	$[0.45, 0.60]$ $[0.35, 0.40]$ $[0.20, 0.35]$	$[0.45, 0.55]$ $[0.60, 0.65]$ $[0.40, 0.50]$
$\overline{p(e)}$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$

We construct the Type-I GSIVFSSs for five patients are as follows:

Table 3. Type-I GSIVFSS model for the ill person $\mathcal{P}_1$

$\overline{\mathcal{P}1_{p_1(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{P}_1(e)(x_1)}$	$[0.2, 0.45]$ $[0.2, 0.7]$ $[0.3, 0.35]$	$[0.2, 0.35]$ $[0.4, 0.6]$ $[0.35, 0.4]$	$[0.25, 0.4]$ $[0.25, 0.35]$ $[0.4, 0.5]$	$[0.15, 0.35]$ $[0.4, 0.5]$ $[0.2, 0.25]$	$[0.1, 0.25]$ $[0.5, 0.6]$ $[0.3, 0.4]$
$\overline{\mathcal{P}_1(e)(x_2)}$	$[0.3, 0.4]$ $[0.4, 0.55]$ $[0.25, 0.35]$	$[0.25, 0.3]$ $[0.5, 0.65]$ $[0.15, 0.3]$	$[0.3, 0.35]$ $[0.2, 0.3]$ $[0.2, 0.25]$	$[0.4, 0.55]$ $[0.35, 0.45]$ $[0.2, 0.3]$	$[0.35, 0.45]$ $[0.45, 0.6]$ $[0.3, 0.35]$
$\overline{\mathcal{P}_1(e)(x_3)}$	$[0.25, 0.35]$ $[0.25, 0.55]$ $[0.1, 0.25]$	$[0.1, 0.2]$ $[0.3, 0.65]$ $[0.3, 0.35]$	$[0.3, 0.55]$ $[0.25, 0.3]$ $[0.2, 0.3]$	$[0.2, 0.45]$ $[0.6, 0.65]$ $[0.1, 0.15]$	$[0.2, 0.25]$ $[0.55, 0.65]$ $[0.25, 0.35]$
$\overline{p_1(e)}$	$(0.5, 0.55, 0.6)$	$(0.55, 0.45, 0.5)$	$(0.4, 0.55, 0.45)$	$(0.5, 0.35, 0.45)$	$(0.6, 0.45, 0.55)$

Table 4. Type-I GSIVFSS model for the ill person $\mathcal{P}_2$

$\overline{\mathcal{P}2_{p_2(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{P}_2(e)(x_1)}$	$[0.1, 0.3]$ $[0.2, 0.45]$ $[0.1, 0.15]$	$[0.15, 0.35]$ $[0.4, 0.6]$ $[0.2, 0.3]$	$[0.25, 0.4]$ $[0.2, 0.3]$ $[0.1, 0.25]$	$[0.2, 0.3]$ $[0.4, 0.5]$ $[0.15, 0.2]$	$[0.15, 0.25]$ $[0.5, 0.55]$ $[0.25, 0.3]$
$\overline{\mathcal{P}_2(e)(x_2)}$	$[0.4, 0.55]$ $[0.4, 0.5]$ $[0.25, 0.3]$	$[0.3, 0.35]$ $[0.5, 0.6]$ $[0.2, 0.25]$	$[0.4, 0.6]$ $[0.2, 0.3]$ $[0.15, 0.2]$	$[0.5, 0.6]$ $[0.4, 0.5]$ $[0.25, 0.3]$	$[0.4, 0.7]$ $[0.5, 0.6]$ $[0.3, 0.35]$
$\overline{\mathcal{P}_2(e)(x_3)}$	$[0.55, 0.7]$ $[0.2, 0.5]$ $[0.15, 0.25]$	$[0.25, 0.5]$ $[0.25, 0.6]$ $[0.25, 0.35]$	$[0.5, 0.6]$ $[0.3, 0.35]$ $[0.2, 0.25]$	$[0.4, 0.5]$ $[0.6, 0.7]$ $[0.1, 0.15]$	$[0.3, 0.4]$ $[0.5, 0.6]$ $[0.3, 0.35]$
$\overline{p_2(e)}$	$(0.5, 0.25, 0.4)$	$(0.6, 0.35, 0.3)$	$(0.35, 0.5, 0.2)$	$(0.4, 0.3, 0.45)$	$(0.6, 0.3, 0.5)$

Table 5. Type-I GSIVFSS model for the ill person $\mathcal{P}_3$

$\overline{\mathcal{P}3}_{p_3(e)}$	e_1	e_2	e_3
$\overline{\mathcal{P}_3(e)(x_1)}$	$[0.25, 0.55]$ $[0.2, 0.4]$ $[0.3, 0.4]$	$[0.25, 0.6]$ $[0.2, 0.3]$ $[0.25, 0.35]$	$[0.1, 0.5]$ $[0.3, 0.5]$ $[0.4, 0.45]$
$\overline{\mathcal{P}_3(e)(x_2)}$	$[0.25, 0.45[0.45, 0.5]$ $[0.3, 0.35]$	$[0.3, 0.5[0.55, 0.7]$ $[0.25, 0.3]$	$[0.45, 0.6[0.25, 0.3]$ $[0.15, 0.2]$
$\overline{\mathcal{P}_3(e)(x_3)}$	$[0.1, 0.7[0.2, 0.35]$ $[0.15, 0.2]$	$[0.15, 0.3]$ $[0.25, 0.5]$ $[0.3, 0.35]$	$[0.45, 0.6[0.3, 0.35]$ $[0.25, 0.3]$
$\overline{p_3(e)}$	$(0.65, 0.5, 0.55)$	$(0.5, 0.55, 0.65)$	$(0.6, 0.5, 0.6)$

$\overline{\mathcal{P}3}_{p_3(e)}$	e_4	e_5
$\overline{\mathcal{P}_3(e)(x_1)}$	$[0.2, 0.3]$ $[0.4, 0.7]$ $[0.2, 0.25]$	$[0.1, 0.2]$ $[0.3, 0.6]$ $[0.25, 0.3]$
$\overline{\mathcal{P}_3(e)(x_2)}$	$[0.3, 0.6[0.45, 0.5][0.25, 0.3]$	$[0.4, 0.5[0.55, 0.6]$ $[0.3, 0.35]$
$\overline{\mathcal{P}_3(e)(x_3)}$	$[0.2, 0.55[0.6, 0.65]$ $[0.1, 0.15]$	$[0.15, 0.4[0.5, 0.65]$ $[0.3, 0.35]$
$\overline{p_3(e)}$	$(0.5, 0.7, 0.5)$	$(0.6, 0.5, 0.6)$

Table 6. Type-I GSIVFSS model for the ill person $\mathcal{P}_4$

$\overline{\mathcal{P}4}_{p_4(e)}$	e_1	e_2	e_3
$\overline{\mathcal{P}_4(e)(x_1)}$	$[0.2, 0.4]$ $[0.25, 0.4]$ $[0.3, 0.4]$	$[0.2, 0.3]$ $[0.45, 0.55]$ $[0.35, 0.4]$	$[0.15, 0.45]$ $[0.25, 0.3]$ $[0.4, 0.5]$
$\overline{\mathcal{P}_4(e)(x_2)}$	$[0.15, 0.3]$ $[0.45, 0.55]$ $[0.2, 0.25]$	$[0.2, 0.35]$ $[0.55, 0.65]$ $[0.1, 0.3]$	$[0.15, 0.3]$ $[0.2, 0.35]$ $[0.15, 0.2]$
$\overline{\mathcal{P}_4(e)(x_3)}$	$[0.25, 0.35]$ $[0.2, 0.5]$ $[0.1, 0.15]$	$[0.1, 0.3]$ $[0.25, 0.4]$ $[0.2, 0.25]$	$[0.3, 0.6]$ $[0.3, 0.35]$ $[0.15, 0.25]$
$\overline{p_4(e)}$	$(0.6, 0.5, 0.45)$	$(0.55, 0.6, 0.5)$	$(0.4, 0.5, 0.6)$

$\overline{\mathcal{P}4}_{p_4(e)}$	e_4	e_5
$\overline{\mathcal{P}_4(e)(x_1)}$	$[0.15, 0.25]$ $[0.4, 0.5]$ $[0.2, 0.3]$	$[0.1, 0.25]$ $[0.5, 0.6]$ $[0.35, 0.4]$
$\overline{\mathcal{P}_4(e)(x_2)}$	$[0.3, 0.35]$ $[0.4, 0.45]$ $[0.15, 0.25]$	$[0.4, 0.45]$ $[0.5, 0.55]$ $[0.1, 0.25]$
$\overline{\mathcal{P}_4(e)(x_3)}$	$[0.35, 0.45]$ $[0.5, 0.65]$ $[0.1, 0.15]$	$[0.1, 0.4]$ $[0.4, 0.55]$ $[0.3, 0.35]$
$\overline{p_4(e)}$	$(0.5, 0.55, 0.6)$	$(0.6, 0.5, 0.55)$

Table 7. Type-I GSIVFSS model for the ill person $\mathcal{P}_5$

$\overline{\mathcal{P}5}_{p_5(e)}$	e_1	e_2	e_3
$\overline{\mathcal{P}_5(e)(x_1)}$	$[0.1, 0.25]$ $[0.3, 0.4]$ $[0.25, 0.4]$	$[0.2, 0.3]$ $[0.4, 0.6]$ $[0.3, 0.35]$	$[0.2, 0.35]$ $[0.5, 0.6]$ $[0.45, 0.5]$
$\overline{\mathcal{P}_5(e)(x_2)}$	$[0.2, 0.25]$ $[0.4, 0.5]$ $[0.25, 0.35]$	$[0.15, 0.2]$ $[0.5, 0.65]$ $[0.15, 0.3]$	$[0.4, 0.45]$ $[0.2, 0.3]$ $[0.15, 0.2]$
$\overline{\mathcal{P}_5(e)(x_3)}$	$[0.2, 0.7]$ $[0.25, 0.35]$ $[0.2, 0.25]$	$[0.1, 0.3]$ $[0.25, 0.5]$ $[0.25, 0.35]$	$[0.35, 0.6]$ $[0.2, 0.35]$ $[0.2, 0.25]$
$\overline{p_5(e)}$	$(0.65, 0.4, 0.5)$	$(0.7, 0.5, 0.35)$	$(0.6, 0.65, 0.25)$

$\overline{\mathcal{P}5}_{p_5(e)}$	e_4	e_5
$\overline{\mathcal{P}_5(e)(x_1)}$	$[0.2, 0.3]$ $[0.4, 0.7]$ $[0.25, 0.3]$	$[0.1, 0.35]$ $[0.3, 0.6]$ $[0.3, 0.4]$
$\overline{\mathcal{P}_5(e)(x_2)}$	$[0.4, 0.5]$ $[0.4, 0.5]$ $[0.2, 0.25]$	$[0.3, 0.35]$ $[0.5, 0.6]$ $[0.3, 0.35]$
$\overline{\mathcal{P}_5(e)(x_3)}$	$[0.45, 0.55]$ $[0.6, 0.7]$ $[0.1, 0.15]$	$[0.35, 0.4]$ $[0.55, 0.65]$ $[0.25, 0.35]$
$\overline{p_5(e)}$	$(0.4, 0.5, 0.6)$	$(0.6, 0.4, 0.5)$

The Type-I generalized spherical interval valued fuzzy values in Table 3 to Table 7 are provided by the experts, depending on their assessment of the alternatives against the criteria under consideration. We should calculate the similarity measure of Type-I GSIVFSSs in Table 3 to Table 7 with the one in Table 2. The following is the formula for calculating the similarity measure for $\mathcal{P}_1$ to $\mathcal{P}_5$ ill people:

Table 8. Similarity measure for $\mathcal{P}_1$ to $\mathcal{P}_5$

	$T_1(x_1)$	$T_2(x_1)$	$S(x_1)$
$\overbracket{(\mathcal{L}, \mathcal{P}_1)}$	$[0.9004, 0.938117]$	$[0.863977, 0.865789]$	$[0.538884, 0.770218]$
$\overbracket{(\mathcal{L}, \mathcal{P}_2)}$	$[0.838408, 0.872224]$	$[0.864766, 0.872809]$	$[0.463942, 0.622412]$
$\overbracket{(\mathcal{L}, \mathcal{P}_3)}$	$[0.850357, 0.983529]$	$[0.658158, 0.783483]$	$[0.519859, 0.724731]$
$\overbracket{(\mathcal{L}, \mathcal{P}_4)}$	$[0.835859, 0.899095]$	$[0.868963, 0.889508]$	$[0.546118, 0.799245]$
$\overbracket{(\mathcal{L}, \mathcal{P}_5)}$	$[0.809728, 0.823702]$	$[0.651175, 0.703714]$	$[0.5401, 0.781923]$

	$T_1(x_2)$	$T_2(x_2)$	$S(x_2)$
$\overbracket{(\mathcal{L}, \mathcal{P}_1)}$	$[0.846649, 0.889397]$	$[0.838395, 0.860706]$	$[0.540608, 0.834875]$
$\overbracket{(\mathcal{L}, \mathcal{P}_2)}$	$[0.917218, 0.951179]$	$[0.861827, 0.918049]$	$[0.545669, 0.791361]$
$\overbracket{(\mathcal{L}, \mathcal{P}_3)}$	$[0.909933, 0.981084]$	$[0.81735, 0.86935]$	$[0.55704, 0.822021]$
$\overbracket{(\mathcal{L}, \mathcal{P}_4)}$	$[0.697739, 0.859319]$	$[0.807296, 0.864217]$	$[0.508215, 0.753084]$
$\overbracket{(\mathcal{L}, \mathcal{P}_5)}$	$[0.797022, 0.835324]$	$[0.861827, 0.898827]$	$[0.536789, 0.807445]$

	$T_1(x_3)$	$T_2(x_3)$	$S(x_3)$
$\overbracket{(\mathcal{L}, \mathcal{P}_1)}$	$[0.742599, 0.811477]$	$[0.827085, 0.838911]$	$[0.604346, 0.827016]$
$\overbracket{(\mathcal{L}, \mathcal{P}_2)}$	$[0.952962, 0.970603]$	$[0.791017, 0.818239]$	$[0.607543, 0.811962]$
$\overbracket{(\mathcal{L}, \mathcal{P}_3)}$	$[0.666442, 0.941046]$	$[0.791017, 0.843519]$	$[0.620683, 0.814643]$
$\overbracket{(\mathcal{L}, \mathcal{P}_4)}$	$[0.748984, 0.877362]$	$[0.762618, 0.793868]$	$[0.594191, 0.764943]$
$\overbracket{(\mathcal{L}, \mathcal{P}_5)}$	$[0.855075, 0.941046]$	$[0.806992, 0.807236]$	$[0.604908, 0.811962]$

	φ	ψ	$Similarity$
$\overbracket{(\mathcal{L}, \mathcal{P}_1)}$	$[0.561279, 0.888808]$	0.663697	$[0.3725193, 0.5898995]$
$\overbracket{(\mathcal{L}, \mathcal{P}_2)}$	$[0.539051, 0.931531]$	0.571429	$[0.3080293, 0.5323032]$
$\overbracket{(\mathcal{L}, \mathcal{P}_3)}$	$[0.565861, 0.968553]$	0.723404	$[0.409346, 0.700655]$
$\overbracket{(\mathcal{L}, \mathcal{P}_4)}$	$[0.549508, 0.880225]$	0.695652	$[0.3822663, 0.6123303]$
$\overbracket{(\mathcal{L}, \mathcal{P}_5)}$	$[0.560599, 0.887858]$	0.672566	$[0.3770402, 0.5971437]$

4.3. Results

From the above information, the upper similarity measure of first two and last two patients are < 0.70, but the upper similarity measure of third patient $\mathcal{P}_3$ is $\overbracket{(\mathcal{L}, \mathcal{P}_3)} = \mathbf{0.700655} \geq 0.70$. Hence, these two Type-I GSIVFSS's are significantly similar. Therefore, we conclude that the patient $\mathcal{P}_3$ is suffering from dengue hemorrhagic fever.

5. Conclusion

The main goal of this work is to present a Type-I GSIVFSS and study some of its properties. A similarity measure of two Type-I GSIVFSS is discussed and an application of this to medical diagnosis has been shown. In the future, we will apply the generalized spherical cubic soft sets and generalized bipolar spherical fuzzy soft sets theories.

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