Applications and Applied Mathematics: An International Journal (AAM)

Volume 16 | Issue 2 | Article 19

12-2021

# (R1506) Generalized cr3b Problem with Heterogeneous Primary and Secondary as Finite Straight Segment

Abdullah A. Ansari
*International Center for Advanced Interdisciplinary Research (ICAIR)*

K. R. Meena
*University of Delhi*

K. Shalini
*University of Delhi*

Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 16, Issue 2 (December 2021), pp. 1120 – 1129

# Generalized cr3b Problem with Heterogeneous Primary and Secondary as Finite Straight Segment

[1*]**Abdullah A. Ansari**, [2]**K. R. Meena, and** [3]**K. Shalini**

[1]International Center for Advanced Interdisciplinary Research (ICAIR)
Sangam Vihar, New Delhi, India
icairndin@gmail.com

[2]Department of Mathematics
Acharya Narendra Dev College
University of Delhi
Delhi, India
kmeena.iitr@gmail.com

[3]Department of Mathematics
Zakir Hussain Delhi College
University of Delhi
Delhi, India
thakurshalini2000@gmail.com

[*] Corresponding Author

Received: April 4, 2021; Accepted: August 4, 2021

## Abstract

The existence and stability of stationary points are investigated under the effects of heterogeneous primary having N-layers with different densities, radiating finite straight segment and the Coriolis as well as centrifugal forces in the frame of cr3bp. The equations of motion are determined with the help of which we evaluate five stationary points analytically as well as graphically, and examine their stability.

**Keywords:** Restricted three-body problem; Heterogeneous body; Finite straight segment; Radiation pressure; Stationary points

**MSC 2010 No.:** 70F15, 70F07, 70F05

# 1. Introduction

The restricted 3-body problem of many perturbations has been a critical subject for several past decades in celestial mechanics and space dynamics. The restricted 3-body problem explains the motion of the smallest body moving under the mutual gravitational effects of two finite bodies, known as primaries, moving around their common center of mass in circular orbits due to their mutual gravitational attraction, and the motion of the primaries are not influenced by the smallest body.

Numerous researchers have examined the restricted 3-body problem with various disturbances namely oblateness, radiation pressure, albedo effects, the Yarkovsky effects, the Coriolis and centrifugal forces, atmospheric drag, solar wind and the Poynting-Robertson drag etc. Poynting (1903), Sharma and SubbaRao (1975), Douskos and Markellos (2006), Abouelmagd (2013), Abouelmagd et al. (2019), Bouaziz and Ansari (2021). About relevant research of perturbed restricted 3-body problem under different perturbations are carried by Chernikov (1970), Simmons et al. (1985), Kushvah (2008), Ansari (2017), Ansari (2018), Ansari (2020), Sahdev and Ansari (2021b), Sahdev et al. (2021a), etc.

In addition, the analysis by Arribas and Elipe (2001) and Riaguas et al. (2001) has been expanded by Jain and Sinha (2014) to the restricted 3-body problem and studied the stationary solutions and their stability as well as regions of motion by classifying both primaries as a form of finite straight segments. In comparative analysis to a classical case of the restricted 3-body problem, they discovered that the Jacobian constant decreases. Ansari et al. (2019) used the Jeans law to evaluate equations of motion and determined the Jacobian integral of circular restricted 3-body problem with the assumptions that two primaries are finite-straight segments. Authors illustrated the equilibrium points in various planes, regions of possible motion, zero velocity curves, surfaces and the basins of convergence. Singh and Haruna (2020) investigated equilibrium points and their stability in the photo-gravitational restricted 3-body problem with oblateness under a heterogeneous spheroid, in which the larger primary is a radiating body and the smaller is a heterogeneous oblate spheroid with 3-layers of different densities while the test particle is taken as oblate in shape. The points of equilibrium are discovered and their linear stability are also analyzed.

The paper is organized as follows. In section 1 of the introduction, there is a literature review. The potential between the bodies are introduced in the gravitational potential section 2. We have evaluated the equations of motion under the effects of various perturbations in section 3. Then in the stationary points section 4, we have determined the locations of stationary points analytically as well as graphically into three subsections 4.1, 4.2, and 4.3. Also examined is the stability of the stationary points in section 5. Section 6 presents the conclusion.

# 2. Gravitational Potential

Using the formula with same notation from Ansari and Abouelmagd (2020), the potential between the point mass $m$ and the heterogeneous primary $m_1$ can be written as

$$U_1 = -\frac{G\, m_1\, m}{r_1} - \frac{G\, m}{2\, r_1^3}\left[h_{11} - \frac{3}{r_1^2}\,(h_{12}\, y^2 + h_{13}\, z^2)\right], \tag{1}$$

where, $h_{11}$, $h_{12}$ and $h_{13}$ are the density parameters. Also the potential between the finite straight segment of mass $m_2$ with finite length $s_1$ and point mass $m$ can be written as (see Ansari and Abouelmagd (2020)):

$$U_2 = -\frac{G\, m_2\, m}{r_2}\left(1 + \frac{s_1^2}{3\, r_2^2}\right), \tag{2}$$

where G is the universal gravitational constant and $r_1$ and $r_2$ are distances between the point mass as well as primary and secondary respectively.

## 3. Equations of motion

Let $m_1$ be a heterogeneous body with $N-$layers having different densities $\rho_N$, and $m_2$ is a radiating finite straight segment with radiation factor $q$ having length $2s_1$ which is parallel to $x-$axis. Both bodies are placed at x-axis and are orbiting in the circular orbits around the origin $O$ which is also common center of mass. We also have taken the effect of the Coriolis and centrifugal forces with the parameters $\phi_1$ and $\phi_2$ respectively. And the third smallest body of mass $m$ orbiting in space and follow the synodic coordinate system which is rotating with angular velocity $n$.

- The total gravitational potential exerted by both the bodies (primary and secondary) on the infinitesimal body will be:

$$V = -\frac{G\, m_1\, m}{r_1} - \frac{G\, m}{2\, r_1^3}\left(h_{1\,1} - \frac{3}{r_1^2}\, h_{1\,2}\, y^2\right) - \frac{G\, m_2\, m}{r_2}\left(1 + \frac{s_1^2}{3\, r_2^2}\right).$$

To fix the unit, let us assume that $m_1 + m_2 = 1$, $G = 1$ and the separation distance between the primaries $R = 1$ and also $\mu = \dfrac{m_2}{(m_1 + m_2)}$. Hence $m_1 = 1 - \mu$ and $J_j$ is the dimensionless quantities of $h_{1j}$ for j = 1, 2. Hence, the equations of motion of the infinitesimal body in the synodic coordinate will be:

$$\ddot{x} - 2\, n\, \phi_1\, \dot{y} = \Omega_x, \quad \ddot{y} + 2\, n\, \phi_1\, \dot{x} = \Omega_y, \tag{3}$$

with

$$n^2 = 1 + s_1^2 + \frac{3}{2}\left(\frac{J_1}{1-\mu}\right),$$

$$\Omega = \frac{n^2\, \phi_2}{2}(x^2 + y^2) + \frac{(1-\mu)}{r_1} + \frac{1}{2\, r_1^3}\left[J_1 - \frac{3}{r_1^2}\, J_2\, y^2\right] + \frac{q\, \mu}{r_2}\left(1 + \frac{s_1^2}{3\, r_2^2}\right), \tag{4}$$

$$r_1 = \sqrt{(x + \mu)^2 + y^2}, \quad \text{and} \quad r_2 = \sqrt{(x - 1 + \mu)^2 + y^2}. \tag{5}$$

# 4. Stationary points

The system (3) will produce the stationary points when we will put zero to all the derivatives with respect to time, hence,

$$
\begin{aligned}
n^2\,\phi_2\,x \;-\; \frac{q\,\mu\,(x+\mu-1)}{r_2^3}\,(1+\frac{s_1^2}{r_2^2}) \;-\; \frac{(1-\mu)(x+\mu)}{r_1^3} \\
-\,\frac{3\,(x+\mu)}{2\,r_1^5}\left[J_1-\frac{5}{r_1^2}\,J_2\,y^2\right]=0,
\end{aligned}
\tag{6}
$$

$$
\left(n^2\,\phi_2 \;-\; \frac{q\,\mu}{r_2^3}\,(1+\frac{s_1^2}{r_2^2}) \;-\; \frac{1-\mu}{r_1^3} \;-\; \frac{3}{2\,r_1^5}\left(J_1+2\,J_2 \;-\; \frac{5}{r_1^2}\,J_2\,y^2\right)\right)y=0. \tag{7}
$$

These points will evaluate from Equations (6) and (7) in two subsections.

## 4.1. Positions of collinear stationary points:

Collinear points are the solutions of Equations (6) and (7) by taking $x \neq 0,\, y=0$. That is, these points lie on the line joining the heterogeneous primary and the radiating finite straight segment secondary. Hence from Equation (6), let

$$
\begin{aligned}
f(x,0)=n^2\,\phi_2\,x \;-\; \frac{q\,\mu\,(x+\mu-1)}{|x+\mu-1|^3}\left(1+\frac{s_1^2}{|x+\mu-1|^2}\right) \\
-\frac{(1-\mu)(x+\mu)}{|x\,+\,\mu|^3} \;-\; \frac{3\,(x+\mu)\,J_1}{2\,|x\,+\,\mu|^5}=0.
\end{aligned}
\tag{8}
$$

To perform the positions of collinear points, we divide $x-$axis into 3 parts: $x<-\mu$, $-\mu<x<1-\mu$ and $1-\mu<x$. These parts will discuss in 3 cases.

**Case I**

That is, $x<-\mu \Rightarrow x\,\in\,(-\infty,-\mu)$. Then Equation (8) can be written as:

$$
f(x)=n^2\,\phi_2\,x \;+\; \frac{q\,\mu}{(x+\mu-1)^2}\,(1+\frac{s_1^2}{(x+\mu-1)^2}) \;+\; \frac{(1-\mu)}{(x\,+\,\mu)^2} \;+\; \frac{3\,J_1}{2\,(x\,+\,\mu)^4}. \tag{9}
$$

And its derivative with respect to $x$ will be

$$
f'(x)=n^2\,\phi_2 \;+\; \frac{2\,q\,\mu}{|x+\mu-1|^3}\left(1+\frac{2\,s_1^2}{|x+\mu-1|^2}\right) \;+\; \frac{2\,(1-\mu)}{|x\,+\,\mu|^3} \;+\; \frac{6\,J_1}{|x\,+\,\mu|^5}. \tag{10}
$$

This implies that $f'(x)\;>\;0$ for $x\;\in\;(-\infty,-\mu)$. Now, $\lim_{x\to-\infty} f(x)\;=\;-\infty$ and $\lim_{x\to-\mu^-} f(x)\;=\;\infty$. Therefore $f(x)$ is monotonically increasing function. And also $\lim_{x\to-\infty} f(x)<0$ and $\lim_{x\to-\mu^-} f(x)>0$, it shows that only a single point $x\,\in\,(-\infty,-\mu)$ exists for which $f(x)=0$. This point is one of the stationary points which is denoted as $L_1$.

**Case II**

That is, $-\mu < x < 1-\mu \Rightarrow x \in (-\mu, 1-\mu)$. Then Equation (8) will be

$$f(x,0) = n^2\,\phi_2\,x + \frac{q\,\mu}{(x+\mu-1)^2}\left(1+\frac{s_1^2}{(x+\mu-1)^2}\right) - \frac{(1-\mu)}{(x+\mu)^2} - \frac{3\,J_1}{2\,(x+\mu)^4}. \tag{11}$$

And its derivative with respect to $x$ will be

$$f'(x) = n^2\,\phi_2 + \frac{2\,q\,\mu}{|x+\mu-1|^3}\left(1+\frac{2\,s_1^2}{|x+\mu-1|^2}\right) + \frac{2\,(1-\mu)}{|x+\mu|^3} + \frac{6\,J_1}{|x+\mu|^5}. \tag{12}$$

This implies that $f'(x) > 0$ for $x \in (-\mu, 1-\mu)$. Now, $\lim_{x\to -\mu^+} f(x) = -\infty$ and $\lim_{x\to(1-\mu)^+} f(x) = \infty$. Therefore, $f(x)$ is a monotonically increasing function and also $\lim_{x\to-\mu^+} f(x) < 0$ and $\lim_{x\to(1-\mu)^+} f(x) > 0$. It shows that the single point $x \in (-\mu, 1-\mu)$ exists for which $f(x) = 0$. This point is also one of the stationary points which is denoted as $L_2$.

**Case III**

That is, $1-\mu < x \Rightarrow x \in (1-\mu, \infty)$. Then Equation (8) can be written as:

$$f(x,0) = n^2\,\phi_2\,x - \frac{q\,\mu}{(x+\mu-1)^2}\left(1+\frac{s_1^2}{(x+\mu-1)^2}\right) - \frac{(1-\mu)}{(x+\mu)^2} - \frac{3\,J_1}{2\,(x+\mu)^4}. \tag{13}$$

And its derivative with respect to $x$ will be

$$f'(x) = n^2\,\phi_2 + \frac{2\,q\,\mu}{|x+\mu-1|^3}\left(1+\frac{2\,s_1^2}{|x+\mu-1|^2}\right) + \frac{2\,(1-\mu)}{|x+\mu|^3} + \frac{6\,J_1}{|x+\mu|^5}. \tag{14}$$

This implies that $f'(x) > 0$ for $x \in (1-\mu, \infty)$. Now, $\lim_{x\to(1-\mu)^-} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$. Therefore, $f(x)$ is a monotonically increasing function. Also, $\lim_{x\to(1-\mu)^-} f(x) < 0$ and $\lim_{x\to\infty} f(x) > 0$. It shows that the single point $x \in (1-\mu, \infty)$ exists for which $f(x) = 0$. This point is also one of the stationary points which is denoted as $L_3$.

### 4.2. Locations of triangular stationary points:

From Equations (6) and (7) one can obtained the non-collinear stationary points when $x \neq 0$ as well as $y \neq 0$. Let us consider that $r_1 = 1+\psi_1$ & $r_2 = 1+\psi_2$, $\psi_1 << 1, \psi_2 << 1$. Hence, from Equation (5), we obtain

$$x = \frac{1}{2} - \mu + \psi_1 - \psi_2, \qquad y = \pm\frac{\sqrt{3}}{2}\left(1+\frac{2}{3}(\psi_1+\psi_2)\right). \tag{15}$$

Utilizing the values of $r_1$, $r_2$, $x$ and $y$ in Equations (6) and (7) as well as neglecting the higher powers of $\psi_i$ (i = 1, 2), we get

$$\psi_1 = -\left(\frac{\psi_{10} + q\,\psi_{11} + J_1\,\psi_{12} + J_2\,\psi_{13}}{\psi_{14} + q\,\psi_{15} + J_1\,\psi_{16} + J_2\,\psi_{17}}\right), \tag{16}$$

$$\psi_2 = -\left(\frac{\psi_{20} + q\,\psi_{21} + J_1\,\psi_{22} + J_2\,\psi_{23}}{\psi_{24} + q\,\psi_{25} + J_1\,\psi_{26} + J_2\,\psi_{27}}\right), \tag{17}$$

where

$$\psi_{10} = (1-\mu)\{1-\mu-2\,\phi_2\,(1+s_1^2)\},$$

$$\psi_{11} = \mu\{(-1+\phi_2+\mu-3\phi_2\,\mu) + s_1^2(-3+4\phi_2+3\mu-8\,\phi_2\,\mu) + s_1^4(3-5\mu)\phi_2\},$$

$$\psi_{12} = 3\,(1-\mu-2\,\phi_2-\phi_2\,s_1^2), \qquad \psi_{13} = \tfrac{33}{4}\{-1+\mu+\phi_2\,(1+s_1^2)(1-\tfrac{10}{11}\mu)\},$$

$$\psi_{14} = 3(1-\mu)\{-1+\mu+\phi_2\,(1+s_1^2)\},$$

$$\psi_{15} = \mu\{3(1+\phi_2-\mu) + s_1^2(7+8\,\phi_2-7\,\mu) + 5\,\phi_2\,s_1^4\},$$

$$\psi_{16} = 12(-1+\phi_2+\mu+\tfrac{5}{8}\phi_2\,s_1^2), \qquad \psi_{17} = \tfrac{15}{8}\{14(1-\mu)-5\phi_2\,(1+s_1^2)\},$$

$$\psi_{20} = 64\{(-1+2\phi_2+2\phi_2\,s_1^2) + \mu(2-5\phi_2-5\phi_2\,s_1^2) + \mu^2(-1+3\phi_2+3\,\phi_2\,s_1^2)\},$$

$$\psi_{21} = 64\mu\{1+2\,\phi_2-\mu+s_1^2\,(1+4\phi_2-\mu)2\phi_2\,s_1^4\},$$

$$\psi_{22} = 96\{-2+2\,\phi_2+2\,\phi_2\,s_1^2+\mu(2-5\phi_2-5\phi_2\,s_1^2) + \tfrac{\phi_2}{1-\mu}(2-5\mu+3\mu^2)\},$$

$$\psi_{23} = 24\{\phi_2\,(-2+45\mu)(1+s_1^2) + 14(1-\mu)\},$$

$$\psi_{24} = 192\{1-\phi_2-\phi_2\,s_1^2+\mu\,(-2+\phi_2+\phi_2\,s_1^2) + \mu^2\},$$

$$\psi_{25} = -64\,\mu\{3+\phi_2-3\mu+s_1^2(7+8\phi_2-7\mu) + 5\phi_2\,s_1^4\},$$

$$\psi_{26} = 768(1-\phi_2-\mu-\tfrac{5}{8}\phi_2\,s_1^2), \psi_{27} = 120\{14(-1+\mu) + 5\phi_2\,(1+s_1^2)\}.$$

Equation (15) is denoting the locations of triangular stationary points in which positive and negative signs of $y$ represent $L_4$ and $L_5$, respectively. The numerical illustration will confirm the locations and existence of the five stationary points in the next subsection.

### 4.3. Numerical illustration for the positions of stationary points

For showing the locations and existence of the five stationary points graphically, we have considered the following numerical values as $s_1$ = 0.01, $J_1$ = 0.0015, $J_2$ = 0.0002, $\mu$ = 0.024, $\phi_2$ = 1.1, q = 0.95, and given in Figure 1 and Figure 2. Figure 1 shows the locations of stationary points at the variation of the solar radiation parameter q. We observed from here that as the value of $q$ reduces, there is no change in the location of the stationary point $L_1$ while the stationary points $L_2$ and $L_3$ are moving away and towards the origin, respectively. Also, the stationary points $L_{4,5}$ both are

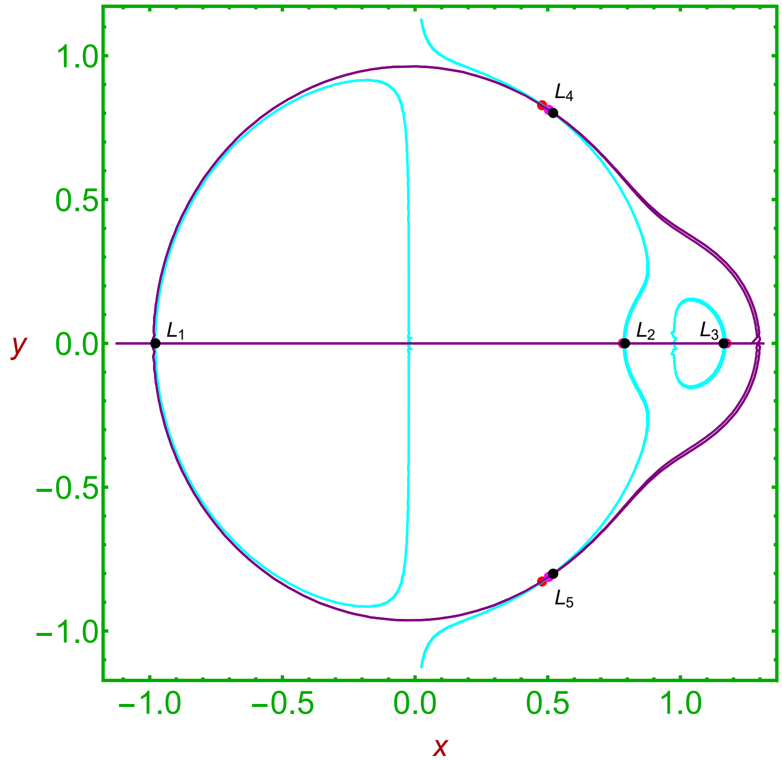


**Figure 1.** Locations of stationary points for three different values of q = 0.95 (Red), 0.90 (Magenta), 0.85 (Black)

moving towards the abscissa. Similarly, Figure 2 represents the variation of the coriolis parameter $\phi_2$. We observed from here that on increase the value of $\phi_2$, the stationary points $L_{1,2,3}$ are moving towards the origin while the stationary points $L_{4,5}$ are moving towards the abscissa. In this way the parameters considered in this problem have excellent impact on the motion of the third smallest body.

## 5. Stability

The variational equation for the system (3) can be obtained by putting $x = x_0 + \alpha$, $\alpha << 1$ and $y = y_0 + \beta$, $\beta << 1$, as

$$\ddot{\alpha} - 2\,n\,\phi_1\,\dot{\beta} = \Omega^0_{xx}\,\alpha + \Omega^0_{xy}\,\beta, \quad \ddot{\beta} + 2\,n\,\phi_1\,\dot{\alpha} = \Omega^0_{xy}\,\alpha + \Omega^0_{yy}\,\beta, \tag{18}$$

where the second derivatives of the potential function $\Omega$ can be evaluated corresponding to the stationary points and denoted by the superscript $0$.

Further we will have a non-trivial solution if

$$\begin{vmatrix} \lambda^2 - \Omega^0_{xx} & -2\,n\,\phi_1\,\lambda - \Omega^0_{xy} \\ 2\,n\,\phi_1\,\lambda - \Omega^0_{xy} & \lambda^2 - \Omega^0_{yy} \end{vmatrix} = 0.$$

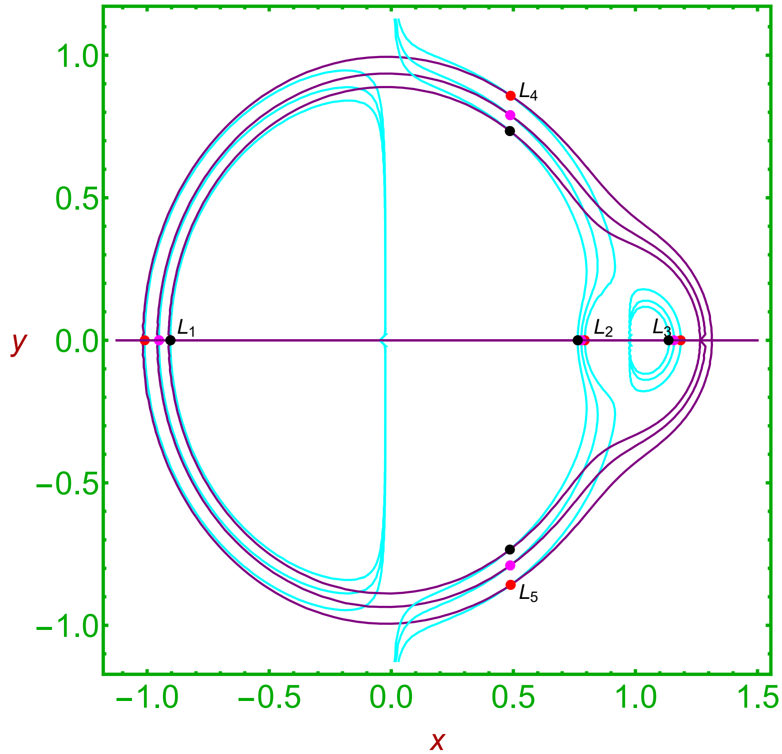


**Figure 2.** Locations of stationary points for three different values of $\phi_2$ = 1 (Red), 1.2 (Magenta), 1.4 (Black)

From the above determinant, we have

$$g(\lambda) = \lambda^4 + B_1\,\lambda^2 + B_0, \tag{19}$$

with

$$B_1 = 4\,n^2\,\phi_1^2 - \Omega^0_{xx} - \Omega^0_{yy}, \quad B_0 = \Omega^0_{xx}\,\Omega^0_{yy} - (\Omega^0_{xy})^2, \tag{20}$$

$$\begin{aligned} \Omega_{x\,x} =& n^2\,\phi_2 + \frac{4\,q\,s_1^2\,\mu\,(x+\mu-1)^2}{r_2^7} - \frac{2\,q\,s_1^2\,\mu}{3r_2^5} - \frac{30\,J_2\,y^2\,(x+\mu)^2}{r_1^9} + \frac{3\,J_2\,y^2}{r_1^7} \\ & - \frac{1-\mu}{r_1^3} + \frac{3\,(1-\mu)(x+\mu)^2}{r_1^5} - \frac{q\,\mu}{r_2^3}\left(1+\frac{s_1^2}{3r_2^2}\right) - \frac{3}{2\,r_1^5}\left(J_1 - \frac{3\,J_2\,y^2}{r_1^2}\right) \\ & + \frac{3\,q\,\mu\,(x+\mu-1)^2}{r_2^5}(1+\frac{s_1^2}{3r_2^2}) + \frac{15\,(x+\mu)^2}{2\,r_1^7}\left(J_1 - \frac{3\,J_2\,y^2}{r_1^2}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} \Omega_{y\,y} =& n^2\,\phi_2 + \frac{4\,q\,s_1^2\,y^2\,\mu}{r_2^7} - \frac{2\,q\,s_1^2\,\mu}{3\,r_2^5} + \frac{3\,y^2\,(1-\mu)}{r_1^5} + \frac{3\,q\,y^2\,\mu}{r_2^5}\left(1+\frac{s_1^2}{3\,r_2^2}\right) \\ & - \frac{1-\mu}{r_1^3} - \frac{q\,\mu}{r_2^3}\left(1+\frac{s_1^2}{3r_2^2}\right) - \frac{3\,J_2}{r_1^5}\left(1-\frac{5\,y^2}{r_1^2}\right) + \frac{4\,y^4}{r_1^4} \\ & + \frac{18\,J_2\,y}{r_1^7}\left(1-\frac{y^2}{r_1^2}\right) - \frac{3}{2\,r_1^5}\left(1-\frac{5\,y^2}{r_1^2}\right)\left(J_1 - \frac{3\,y^2\,J_2}{r_1^2}\right), \end{aligned} \tag{22}$$

$$\Omega_{x\,y} = \frac{4\,q\,s_1^2\,y\,\mu\,(x+\mu-1)}{r_2^7} - \frac{21\,J_2\,y^3\,(x+\mu)}{r_1^9} + \frac{6\,J_2\,y\,(x+\mu)}{r_1^7} + \frac{3\,y\,(1-\mu)(x+\mu)}{r_1^5} + \frac{3\,q\,y\,\mu\,(x+\mu-1)}{r_2^5}\left(1+\frac{s_1^2}{3\,r_2^2}\right) + \frac{9\,(x+\mu)\,J_2\,y}{r_1^7}\left(1-\frac{y^2}{r_1^2}\right) + \frac{15\,y\,(x+\mu)}{2\,r_1^7}\left(J_1-\frac{3\,J_2\,y^2}{r_1^2}\right). \tag{23}$$

It is well known that the classical cr3bp has 5 stationary points out of which 3 are collinear and 2 are non-collinear stationary points where the 3 collinear points are always unstable while triangular points are stable for some values of mass ratio $\mu$. In our case if $\lambda \to \infty$ then $g(\lambda) \to \infty$ and $g(0) = B_0$. Here, the value of $B_0$ will decide the stability, i.e., if $B_0 < 0$, then at least one positive root will exists, so these points will be unstable.

## 6. Conclusion

The present study investigated the motion properties of the third body which is orbiting under the gravitational forces of the heterogeneous primary having $N$-layers with different densities $\rho_i$ of each layers and the radiating finite straight segment of secondary body. The evaluated equations of motion is varied from the classical case by the perturbation parameters $\phi_1$, $\phi_2$, q, $J_1$, $J_2$ and $s_1$. Further, we have determined the locations of five stationary points out of which three are collinear and two are triangular stationary points. These stationary points are depending on the above said perturbation parameters. Afterward, we have performed the stability of these stationary points and observed that the stability is depending on the values of the derivatives of the potential function $\Omega$.

## REFERENCES

Abouelmagd, E.I. (2013). The effect of photogravitational force and oblateness in the perturbed restricted three-body problem, Astrophys. Space Sci., Vol. 346, No. 1, pp. 51–69.

Abouelmagd, E.I. and Ansari, A.A. (2019). The motion properties of the infinitesimal body in the framework of bicircular Sun-perturbed Earth-Moon system, New Astronomy, Vol. 73, no. 101282.

Ansari, A.A. (2017). Effect of Albedo on the motion of the infinitesimal body in circular restricted three-body problem with variable masses, Italian J. of Pure and Applied Math., Vol. 38, pp. 581–600.

Ansari, A.A. (2018). The circular restricted four- body problem with triaxial primaries and variable infinitesimal mass, Applications and Applied Mathematics: An International Journal, Vol. 13, No. 2, pp. 818-838.

Ansari, A.A. (2020). Kind of Robe's restricted problem with heterogeneous irregular primary of N-layers when outer most layer has viscous fluid, New Astronomy, Vol. 83. doi:10.1016/j.newast.2020.101496

Ansari, A.A. and Abouelmagd, E.I. (2020). Gravitational potential formulae between two bodies with finite dimensions, Astronomical Notes. doi: 10.1002/asna.202013726

Ansari, A.A., Kellil, R. and Alhussain, Z. (2019). Behavior of an infinitesimal-variable-mass body in CR3BP; The primaries are finite straight segments, Punjab Univ. J. of Math., Vol. 51 , No. 5, pp. 107–120.

Arribas, M. and Elipe, A. (2001). Non-integrability of the motion of a particle around a massive straight segment, Physics Letters A, Vol. 281, No. 2-3, pp. 142–148.

Bouaziz, F. and Ansari, A.A. (2021). Perturbed Hill's problem with variable mass, Astron. Nachr. doi: 10.1002/asna.202113870

Chernikov, Yu.A. (1970). The photo-gravitational restricted three-body problem, Soviet Astronomy, Vol. 14, pp. 176.

Douskos, C.N. and Markellos, V.V. (2006). Out-of-plane equilibrium points in the restricted three-body problem with oblateness, Astronomy and Astrophysics, Vol. 446, No. 1, pp. 357–360.

Jain, R. and Sinha, D. (2014). Stability and regions of motion in the restricted three-body problem when both the primaries are finite straight segments, Astrophys. Space Sci., Vol. 351, No. 1, pp. 87–100.

Kushvah, B.S. (2008). Linear stability of equilibrium points in the generalized photogravitational Chermnykh's problem, Astrophys. Space Sci., Vol. 318, No. 1, pp. 41–50.

Poynting, J.H. (1903). Radiation in the solar system: Its effect on temperature and its pressure on small bodies, Proceedings of the Royal Society of London Series I, Vol. 72, pp. 265–266.

Riaguas, A., Elipe, A. and López-Moratalla, T. (2001). Non-linear stability of the equilibria in the gravity field of a finite straight segment, Celest. Mech. Dyn. Astro., Vol. 81, No. 3, pp. 235–248.

Sahdev, S.K. and Ansari, A.A. (2021a). Generalized Robe's problem having oblate heterogeneous primary containing viscous fluid inside the outer most layer and radiating spherical secondary with modified Newtonian potential, Sci. Int. Lahore, Vol. 33, No. 2, pp. 147-151.

Sahdev, S.K. and Ansari, A.A. (2021b). Motion of test particle in the outer layer of heterogeneous body, Gedrag and Organisatie Review, Vol. 34, No. 1, pp. 1-12.

Sharma, R. K. and SubbaRao, P.V. (1975). Collinear equilibria and their characteristic exponents in the restricted three-body problem when the primaries are oblate spheroids, Celest. Mech., Vol. 12, No. 2, pp. 189–201.

Simmons, J.F.L., McDonald, A.J.C. and Brown, J.C. (1985). The restricted 3-body problem with radiation pressure, Celest. Mech., Vol. 35, No. 2, pp. 145–187.

Singh, J. and Haruna, S. (2020). Equilibrium points and stability in R3BP with heterogeneous oblate spheroid, Int. Astronomy and Astrophysics Research Journal, Vol. 29, pp. 16–29.