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Süleyman Şenyurt
Ordu University

Kemal Eren
Sakarya University

Kebire Hilal Ayvacı
Ordu University

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A Study on Inextensible Flows of Polynomial Curves with Flc Frame

[1]**Süleyman Şenyurt**, [2*]**Kemal Eren, and** [3]**Kebire Hilal Ayvacı**

[1,3]Department of Mathematics
Faculty of Arts and Science
Ordu University
52200, Ordu, Turkey
[2]Department of Mathematics
Faculty of Arts and Sciences
Sakarya University
54100, Sakarya, Turkey
[1]senyurtsuleyman52@gmail.com; [2]kemal.eren1@ogr.sakarya.edu.tr;
[3] kebirehilalayvaci@odu.edu.tr

*Corresponding Author

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Abstract

In this paper, we investigate the inextensible flows of polynomial space curves in $\mathbb{R}^3$. We calculate that the necessary and sufficient conditions for an inextensible curve flow are represented as a partial differential equation involving the curvatures. Also, we expressed the time evolution of the Frenet like curve (Flc) frame. Finally, an example of the evolution of the polynomial curve with Flc frame is given and graphed.

Keywords: Flc frame; Polynomial curves; Inextensible curve; Flow; Evolution

MSC 2010: 53C44, 53A04, 53A05

1. Introduction

Most nonlinear problems in physics, chemistry and biology can be explained with the help of the parameters of curves and surfaces. In addition, the evolution equations of curves and surfaces have important application areas in computer image processing. Inelastic curve moving is a motion in which the is maintained at the initial and final positions of the curve throughout the moving process. In other words, if the arc-length change of the curve with respect to time is zero, this curve is called an inelastic curve. The flows of the inelastic curve produce moving that do not contain strain energy. For example, the swinging motion of a fixed-length string produces an inelastic curve motion. Such examples are frequently encountered in physical applications. The moving of the inelastic curve appears in computing imaging, computer animations, and even structural mechanics. All these applications involve the evolution of curves over time. Gage and Hamilton found new methods (Gage (1984); Gage and Hamilton (1986)) to study the curvature vector fields of curves, that is, their variation with time along acceleration vectors, and Grayson (1987) proved the conversion of closed planar curves into a circle using the heat equation. In addition, Gage (1985) examined the plane curve evolution that preserves the area, and Kwon et al. investigate the moving of the inelastic curves in Euclidean space (Kwon and Park (1999); Kwon et al. (2005)). Many studies have been done on the flow of curves for different frames and different spaces (Baş and Körpınar (2013); Yıldız et al. (2013); Yıldız et al. (2014); Yıdız and Tosun (2017); Gürbüz (2018); Eren and Kösal (2020); Kelleci and Eren (2020); Hussien and Mohamed (2016); Körpınar and Baş (2019); Latifi and Ravazi (2008); Mohamed (2017); Turhan and Ayyıldız (2015); Öğrenmiş and Yeneroğlu (2010); Solouma (2020); Solouma and Al-Dayel (2020)).

Our aim in this study is to investigate the inextensible flow of the polynomial curves. For this, firstly, the concepts related to Flc frame defined along polynomial curves are given. Then, the conditions of inextensible of the polynomial curve flow are expressed and the graph of this curve is drawn using the Matlab program by giving an example.

2. Inextensible Flows of Polynomial Curves

In this section, let's give the Flc frame expressed by Dede to be used throughout the article. Let $\alpha = \alpha(u)$ be a differentiable polynomial curve with arc-length parameter u. The Flc frame formula of the polynomial curve α is given as

$$T' = v\left(d_1 D_2 + d_2 D_1\right),\ D_2{}' = v\left(-d_1 T + d_3 D_1\right),\ D_1{}' = v\left(-d_2 T - d_3 D_2\right), \tag{1}$$

where $\|\alpha'\| = v$, the vectors T, D_2 and D_1 are the tangent, the normal-like and binormal-like vector field and also, d_1, d_2 and d_3 are the curvatures of the polynomial curve α (see for more details (Dede (2019); Güven (2020))).

From now on, the main results for inextensible flows of polynomial space curves are discussed. We suppose that

$$\begin{aligned} \alpha : [0, l] \times [0, w] &\to \mathbb{R}^3, \\ (u, t) &\to \alpha\left(u, t\right), \end{aligned}$$

is a one parameter family of polynomial curves, where l is the arc-length of the initial polynomial curve. Let u be the polynomial curve parameterization variable, $0 \leq u \leq l$. The speed of the curve

α is defined by $v = \left\| \frac{\partial \alpha}{\partial u} \right\|$ and also the arc-length of the curve α is represented by

$$s(u) = \int_0^u \left\| \frac{\partial \alpha}{\partial u} \right\| du = \int_0^u v du,$$

where $\frac{\partial}{\partial s} = \frac{1}{v} \frac{\partial}{\partial u}$ or $ds = v du$.

Definition 2.1.

Let α be a differentiable polynomial curve with Flc frame $\{T, D_2, D_1\}$ in $\mathbb{R}^3$. Any flow of the curve α can be presented in the following form:

$$\frac{\partial \alpha(u,t)}{\partial t} = fT + gD_2 + hD_1,$$

where f, g and h are scalar speeds of the curve α.

We suppose that $s(u,t) = \int_0^u v du$ is be the arc-length variation. In that case, the curve α cannot be any elongation or compression, if one can be expressed by the condition

$$\frac{\partial s(u,t)}{\partial t} = \int_0^u \frac{\partial v}{\partial t} du = 0,$$

here $u \in [0, l]$.

Definition 2.2.

A polynomial curve evolution $\alpha(u,t)$ and its flow $\frac{\alpha(u,t)}{\partial t}$ in $\mathbb{R}^3$ are said to be inextensible if

$$\frac{\partial}{\partial t} \left\| \frac{\partial \alpha(u,t)}{\partial u} \right\| = 0.$$

Theorem 2.1.

Let α be a differentiable polynomial curve with respective to Flc frame $\{T, D_2, D_1\}$ in $\mathbb{R}^3$. If $\frac{\partial \alpha}{\partial t} = fT + gD_2 + hD_1$ is a flow of α, then there exists the following equation:

$$\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u} - v(gd_1 + hd_2),$$

where f, g and h are scalar speeds of the curve α.

Proof:

Taking the differential of the equation $v^2 = \left\langle \frac{\partial\alpha}{\partial u}, \frac{\partial\alpha}{\partial u} \right\rangle$ with respect to t, one is easily found that

$$2v\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}\left\langle \frac{\partial\alpha}{\partial u}, \frac{\partial\alpha}{\partial u} \right\rangle = 2\left\langle \frac{\partial\alpha}{\partial u}, \frac{\partial}{\partial t}\frac{\partial\alpha}{\partial u} \right\rangle.$$

Since $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial u}$ commute and also using the Flc formula and the equation $\frac{\partial\alpha}{\partial t} = fT + gD_2 + hD_1$, we have

$$\begin{aligned}
2v\tfrac{\partial v}{\partial t} &= 2\left\langle \tfrac{\partial\alpha}{\partial u}, \tfrac{\partial}{\partial u}\left(fT + gD_2 + hD_1\right) \right\rangle \\
&= 2\left\langle vT, \left(\tfrac{\partial f}{\partial u} - vgd_1 - vhd_2\right)T + \left(\tfrac{\partial g}{\partial u} + vfd_1 - vhd_3\right)D_2 + \left(\tfrac{\partial h}{\partial u} + vfd_2 + vgd_3\right)D_1 \right\rangle \\
&= 2v\left(\tfrac{\partial f}{\partial u} - vgd_1 - vhd_2\right).
\end{aligned}$$

So, we get

$$\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u} - v\left(gd_1 + hd_2\right). \tag{2}$$

■

Theorem 2.2.

Let $\frac{\partial\alpha}{\partial t} = fT + gD_2 + hD_1$ be a flow of with Flc frame $\{T, D_2, D_1\}$ in $\mathbb{R}^3$. Then the flow is inextensible if and only if

$$\frac{\partial f}{\partial s} = gd_1 + hd_2.$$

Proof:

Assume that the flow of α is inextensible. Considering the equations $\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u}$ and (2) together, we have

$$\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u} - v\left(gd_1 + hd_2\right) = \frac{\partial f}{\partial s}\frac{\partial s}{\partial u} - v\left(gd_1 + hd_2\right) = \left(\frac{\partial f}{\partial s} - gd_1 - hd_2\right)v.$$

Since $v \neq 0$, it follows that

$$\frac{\partial f}{\partial s} = gd_1 + hd_2. \tag{3}$$

Conversely, the proof is easily completed by following a similar way to the above. ■

Theorem 2.3.

Let $\frac{\partial \alpha}{\partial t} = fT + gD_2 + hD_1$ be a flow of α with the Flc frame $\{T, D_2, D_1\}$ in $\mathbb{R}^3$. If the flow of α is inextensible, then the differentiations of $\{T, D_2, D_1\}$ with respect to t is

$$\frac{\partial T}{\partial t} = \left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right) D_2 + \left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right) D_1,$$

$$\frac{\partial D_2}{\partial t} = -\left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right) T + \varphi D_1,$$

$$\frac{\partial D_1}{\partial t} = -\left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right) T - \varphi D_2,$$

where $\varphi = \left\langle \frac{\partial D_2}{\partial t}, D_1 \right\rangle$.

Proof:

Suppose that the flow $\frac{\partial \alpha}{\partial t} = fT + gD_2 + hD_1$ of α with the Flc frame $\{T, D_2, D_1\}$ is inextensible. It seems that

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial s}\right) = \frac{\partial}{\partial s}\left(\frac{\partial \alpha}{\partial t}\right) = \frac{\partial}{\partial s}\left(fT + gD_2 + hD_1\right)$$

$$= \frac{\partial f}{\partial s}T + f\frac{\partial T}{\partial s} + \frac{\partial g}{\partial s}D_2 + g\frac{\partial D_2}{\partial s} + \frac{\partial h}{\partial s}D_1 + h\frac{\partial D_1}{\partial s}.$$

Using the Flc formula given by (2.1) and Equation (3), we get

$$\frac{\partial T}{\partial t} = \left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right) D_2 + \left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right) D_1.$$

Differentiating the Flc frame with respect to t as follows;

$$0 = \frac{\partial}{\partial t}\langle T, D_2\rangle = \left\langle \frac{\partial T}{\partial t}, D_2 \right\rangle + \left\langle T, \frac{\partial D_2}{\partial t} \right\rangle = \frac{\partial g}{\partial s} + fd_1 - hd_3 + \left\langle T, \frac{\partial D_2}{\partial t} \right\rangle,$$

$$0 = \frac{\partial}{\partial t}\langle T, D_1\rangle = \left\langle \frac{\partial T}{\partial t}, D_1 \right\rangle + \left\langle T, \frac{\partial D_1}{\partial t} \right\rangle = \frac{\partial h}{\partial s} + fd_2 + gd_3 + \left\langle T, \frac{\partial D_1}{\partial t} \right\rangle,$$

$$0 = \frac{\partial}{\partial t}\langle D_2, D_1\rangle = \left\langle \frac{\partial D_2}{\partial t}, D_1 \right\rangle + \left\langle D_2, \frac{\partial D_1}{\partial t} \right\rangle = \varphi + \left\langle D_2, \frac{\partial D_1}{\partial t} \right\rangle,$$

$$0 = \frac{\partial}{\partial t}\langle D_2, D_2\rangle = \left\langle \frac{\partial D_2}{\partial t}, D_2 \right\rangle + \left\langle D_2, \frac{\partial D_2}{\partial t} \right\rangle = 2\left\langle \frac{\partial D_2}{\partial t}, D_2 \right\rangle,$$

$$0 = \frac{\partial}{\partial t}\langle D_1, D_1\rangle = \left\langle \frac{\partial D_1}{\partial t}, D_1 \right\rangle + \left\langle D_1, \frac{\partial D_1}{\partial t} \right\rangle = 2\left\langle \frac{\partial D_1}{\partial t}, D_1 \right\rangle.$$

Thus, we obtain

$$\frac{\partial D_2}{\partial t} = -\left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right)T + \varphi D_1,$$

$$\frac{\partial D_1}{\partial t} = -\left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right)T - \varphi D_2,$$

where $\varphi = \left\langle \frac{\partial D_2}{\partial t}, D_1 \right\rangle$. ■

Theorem 2.4.

Suppose that the curve flow $\frac{\partial \alpha}{\partial t} = fT + gD_2 + hD_1$ is inextensible. Then the partial differential equations of the curvatures d_1, d_2 and d_3 of the curve α with respect to t satisfy

$$\frac{\partial d_1}{\partial t} = \frac{\partial}{\partial s}\left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right) - d_3\left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right) + d_2\varphi,$$

$$\frac{\partial d_2}{\partial t} = \frac{\partial}{\partial s}\left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right) + d_3\left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right) - d_1\varphi,$$

$$\frac{\partial d_3}{\partial t} = d_1\left(\frac{\partial h}{\partial s} + fd_2 + gd_3\right) - d_2\left(\frac{\partial g}{\partial s} + fd_1 - hd_3\right) + \frac{\partial \varphi}{\partial s},$$

where $\varphi = \left\langle \frac{\partial D_2}{\partial t}, D_1 \right\rangle$.

Proof:

Since the curve α is inextensible curve, it implies that $\frac{\partial}{\partial s}$ and $\frac{\partial}{\partial t}$ commutative, that is,

$$\frac{\partial}{\partial s}\left(\frac{\partial T}{\partial t}\right)=\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial s}\right),\ \frac{\partial}{\partial s}\left(\frac{\partial D_2}{\partial t}\right)=\frac{\partial}{\partial t}\left(\frac{\partial D_2}{\partial s}\right) \text{ and } \frac{\partial}{\partial s}\left(\frac{\partial D_1}{\partial t}\right)=\frac{\partial}{\partial t}\left(\frac{\partial D_1}{\partial s}\right).$$

From here,

$$\frac{\partial}{\partial s}\left(\frac{\partial T}{\partial t}\right)=\left(-d_1\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-d_2\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)\right)T+$$

$$\left(\frac{\partial}{\partial s}\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-d_3\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)\right)D_2+$$

$$\left(d_3\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)+\frac{\partial}{\partial s}\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)\right)D_1$$

and

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial s}\right)=\left(-d_1\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-d_2\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)\right)T+$$

$$\left(\frac{\partial d_1}{\partial t}-d_2\varphi\right)D_2+\left(\frac{\partial d_2}{\partial t}+d_1\varphi\right)D_1.$$

Comparing the last two equations, we obtain:

$$\frac{\partial d_1}{\partial t}=\frac{\partial}{\partial s}\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-d_3\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)+d_2\varphi,$$

$$\frac{\partial d_2}{\partial t}=\frac{\partial}{\partial s}\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)+d_3\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-d_1\varphi.$$

Similarly, by using $\frac{\partial}{\partial s}\left(\frac{\partial D_1}{\partial t}\right)=\frac{\partial}{\partial t}\left(\frac{\partial D_1}{\partial s}\right)$, it is seen that

$$\frac{\partial}{\partial s}\left(\frac{\partial D_1}{\partial t}\right)=\left(-\frac{\partial}{\partial s}\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)+d_1\varphi\right)T+$$

$$\left(-d_1\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)-\frac{\partial\varphi}{\partial s}\right)D_2+$$

$$\left(-d_2\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)-\varphi d_3\right)D_1$$

and

$$\frac{\partial}{\partial t}\left(\frac{\partial D_1}{\partial s}\right)=\left(d_3\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-\frac{\partial d_2}{\partial t}\right)T+$$

$$\left(-d_2\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)-\frac{\partial d_3}{\partial t}\right)D_2+$$

$$\left(-d_2\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)-d_3\varphi\right)D_1.$$

Thus, we find

$$\frac{\partial d_3}{\partial t}=d_1\left(\frac{\partial h}{\partial s}+fd_2+gd_3\right)-d_2\left(\frac{\partial g}{\partial s}+fd_1-hd_3\right)+\frac{\partial \varphi}{\partial s}.$$

■

Example 2.1.

Let $\alpha(u,t)$ be a polynomial curve evolution expressed by

$$\alpha(u,t)=\left(u+t,\frac{u^2+t^2}{2},\frac{u^3+t^3}{6}\right).$$

For the flow $\frac{\partial\alpha(u,t)}{\partial t}=fT+gD_2+hD_1$ of α , it is found that

$$\frac{\partial\alpha(u,t)}{\partial u}=\left(1,u,\frac{u^2}{2}\right).$$

Also, the speed of the curve is calculated as $v=\left\|\frac{\partial\alpha}{\partial u}\right\|=\frac{(2+u^2)}{2}$. So, since $\frac{\partial v}{\partial t}=0$, the flow $\frac{\partial\alpha(u,t)}{\partial t}$ is inextensible. Moreover, the tangent, principal normal like, binormal like vector field and the curvatures of the polynomial curve are as follows:

$$T=\left(\frac{2}{2+u^2},\frac{2u}{2+u^2},\frac{u^2}{2+u^2}\right),$$

$$D_2=\left(-\frac{u^2}{\sqrt{1+u^2}\,(2+u^2)},-\frac{u^3}{\sqrt{1+u^2}\,(2+u^2)},\frac{2\sqrt{1+u^2}}{2+u^2}\right),$$

$$D_1=\left(\frac{u}{\sqrt{1+u^2}},-\frac{1}{\sqrt{1+u^2}},0\right)$$

and

$$d_1 = \frac{4u}{\sqrt{1+u^2}(2+u^2)^2}, \quad d_2 = -\frac{4}{\sqrt{1+u^2}(2+u^2)^2}, \quad d_3 = \frac{2u^2}{(1+u^2)\,(2+u^2)^2}.$$

For the curve flow, from the equation $\frac{\partial\alpha(u,t)}{\partial t} = \left(1, t, \frac{t^2}{2}\right) = fT + gD_2 + hD_1$, we get

$$f = \left\langle \frac{\partial\alpha\,(u,t)}{\partial t}, T \right\rangle = \frac{(2+ut)^2}{2\,(2+u^2)},$$

$$g = \left\langle \frac{\partial\alpha\,(u,t)}{\partial t}, D_2 \right\rangle = -\frac{(u-t)\,(u+t+u^2t)}{\sqrt{1+u^2}\,(2+u^2)},$$

$$h = \left\langle \frac{\partial\alpha\,(u,t)}{\partial t}, D_1 \right\rangle = \frac{u-t}{\sqrt{1+u^2}}.$$

From here, differentiation of f with respect to u, we have

$$\frac{\partial f}{\partial u} = -\frac{2\,(u-t)\,(2+ut)}{(2+u^2)^2}.$$

On the other hand, the following given equation can be calculated:

$$v\,(gd_1 + hd_2) = -\frac{2\,(u-t)\,(2+ut)}{(2+u^2)^2}.$$

It is easily seen from these last equations that $\frac{\partial f}{\partial u} = v\,(gd_1 + hd_2)$. So, we can say that the curve flow is inextensible.

3. Conclusion

The inextensible flows of any space curve are considered from a different point of view, but at points where the n-th order derivative is zero, the inextensible flows of the curves are not examined so far. With the help of the Flc frame defined for polynomial curves at points where the n-th order derivative is zero, we investigate the inextensible flows of polynomial space curves in $\mathbb{R}^3$. For this purpose, we have calculated that the necessary and sufficient conditions for an inextensible curve flow are represented as a partial differential equation containing the curvatures. We also expressed the time evolution of the Flc frame. Thus, a new perspective has been given to the flows of curves and become a new source for further research.

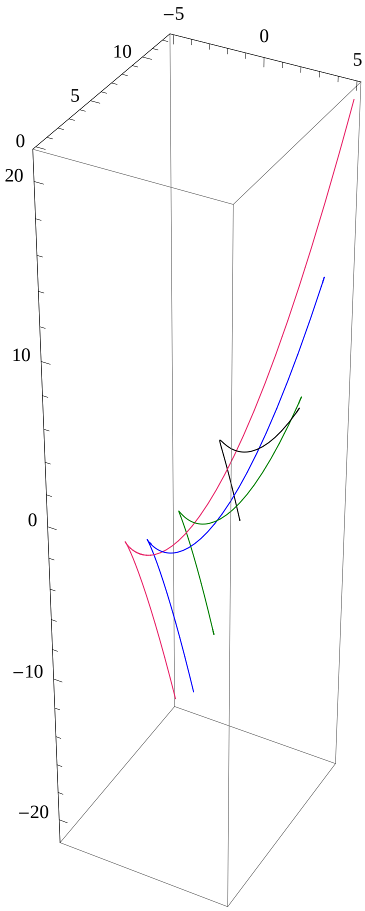

Figure 1. The evolution of the red, blue, green, and black colored the polynomial curve for $t = 0, 1, 2, 3$, respectively, and $u \in (-5, 5)$

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