Applications and Applied Mathematics: An International Journal (AAM)

Volume 17 | Issue 1 | Article 15

6-2022

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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 17, Issue 1 (June 2022), pp. 227 – 242

Further Results on the Admissibility of Singular Systems with Delays

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Received: December 6, 2021; Accepted: March 17, 2022

Abstract

In this article, admissibility problem for a kind of singular systems with delays is studied. Firstly, the given singular system with delays is transformed into a neutral system with delays. Secondly, new sufficient criteria are obtained on the stability of the new neutral system by aid of Wirtinger-based integral inequality, linear matrix inequality (LMI) and suitable Lyapunov-Krasovskii functionals (LKFs). The obtained criteria are valid for both of the systems. At the end, two numerical examples are given to illustrate the applicability of the obtained results using MATLAB-Simulink software. By this article, we extend and improve some results of the past literature.

Keywords: Admissibility; Impulse-free; Lyapunov-Krasovskii functional (LKF); Neutral system; Regular; Singular system

MSC 2020 No.: 34A08, 34K40

1. Introduction

In the last few decades, because of their extensive applications in the electrical and mechanical models; singular systems, which are also called descriptor systems, semi-state systems, implicit systems, have been one of the major research field of control theory. Many books and articles related to singular systems have been discussed and many results have been obtained regarding the stability and admissibility of these systems. Thus, the problem of stability and admissibility analysis for singular systems with delays is very important both theoretically and practically. It is noted that the study of singular systems is much more complicated than that for regular time delay systems because of the existence of algebraic constraints. Lyapunov-Krasovskii functional method is very important technique for stability of time delay systems (for instance, Liu (2017), Seuret and Gouaisbaut (2013), Wu et al. (2013), Xu and Lam (2006), Yiğit and Tunç (2022) and references therein).

There are very important reference books, which include various qualitative conditions about singular systems, such as Dai (1989), Xu and Lam (2006), and Yang et al. (2013). In recent years, numerous important and interesting results on the qualitative properties for various linear systems of first order have been obtained by applying linear matrix inequality, the second Lyapunov method, the Lyapunov-Krasovskii method, perturbation approach, and so on. Cong (2014) created a way to prove the stability by using a perturbation approach and Lyapunov functional approach. Liu and Hou (2014) proved some sufficient conditions for first order linear differential equation systems. Liu et al. (2014) obtained new stability criteria for linear singular time-delay systems. Tunç and Yiğit (2020) obtained certain sufficient conditions for the solutions of nonlinear delay differential equations, which include two variable delays. Yiğit and Tunç (2020) obtained new conditions for singular time-delay systems by using some well-known inequalities and LKFs. For some recent interesting and related papers on various qualitative properties of solutions non-singular integro-differential equations and some others, we referee the readers to the papers of Khan et al. (2020a, 2020b), Sohail (2018), Yiğit and Tunç (2022a, 2022b), Tunç (2020), Tunç and Tunç (2018, 2021), Tunç et al. (2021) and the references of these papers.

2. Preliminaries

The motivation of this paper has been inspired by the results of Liu et al. (2014) regarding singular system with time- delay. In that paper, they obtained new stability criteria for this linear singular time-delay systems by using an LKF and the Wirtinger-based integral inequality method. Thus, we consider the following linear singular integro-system with three delays:

$$\mathrm{E}\dot{x}(t) = \mathrm{A}x(t) + \mathrm{A}_d x(t-\tau).$$

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The motivation of this paper has been inspired by the results of Liu et al. (2014) and the formers related works in the literature.

We consider the following linear singular integro-system with three delays:

$$\mathrm{E}\dot{x}(t) = \mathrm{A}x(t) + \sum_{i=1}^{2} \mathrm{A}_{d_i} x(t-\tau_i(t)) + \mathrm{A}_\tau \int_{t-\tau}^{t} x(s)ds, \tag{1}$$

$$x(t) = \varphi(t),\ t \in [-\tau, 0],\ \tau > 0.$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\phi(t)$ is a continuous initial function defined on $[-\tau, 0]$. $\mathrm{A} \in \mathbb{R}^{n\times n}$ is a negative definite real constant matrix and $\mathrm{E}, \mathrm{A}_\tau, \mathrm{A}_{d_i} \in \mathbb{R}^{n\times n}$ are real constant matrices, the matrix $\mathrm{E} \in \mathbb{R}^{n\times n}$ is singular, and it is assumed that $rank\ \mathrm{E} = r \le n, n \ge 1$. The time-varying delays $\tau_i(t)$ are continuous differentiable and satisfying

$$0 < \tau_i(t) \le \tau_i,\ \sigma_i \le \dot{\tau}_i(t) \le h_i \le 1, (i = 1,2),\ \tau = \max\{\tau_1, \tau_2\}, \tau > 0, \tag{2}$$

where τ, τ_i, σ_i and h_i are some known constant delays.

We now give some information, which are needed in advance. The pair (E, A) is said to be regular if $\det(s\mathrm{E} - \mathrm{A}) \ne 0$. The pair (E, A) is said to be impulse-free if $\deg\ (\det\ (s\mathrm{E} - \mathrm{A})) = rank\ (\mathrm{E})$ (Dai (1989)). The singular delay system (1) is said to be regular and impulse-free if the pair (E, A) is regular and impulse-free. The singular delay system (1) is said to be admissible if it is regular, impulse-free and stable (Xu and Lam (2006)).

If the pair (E, A) is regular and impulse-free, then Dai (1989) shows that there exist two non-singular matrices $M,\ N \in \mathbb{R}^{n\times n}$ with their respective matrices such that

$$MEN = \begin{bmatrix} \mathrm{I}_r & 0 \\ 0 & 0 \end{bmatrix}, \quad M\mathrm{A}N = \begin{bmatrix} \mathrm{A}_1 & 0 \\ 0 & \mathrm{I}_{n-r} \end{bmatrix}.$$

Let $R \in \mathbb{R}^{n\times n}$ be any constant symmetric matrix and $x : [a,b] \to \mathbb{R}^n$ be a continuously differentiable function. Then, the following inequality holds:

$$\int_a^b \dot{x}^{\mathrm{T}}(s) R \dot{x}(s) ds \ge \frac{1}{b-a}[x(b) - x(a)]^T R[x(b) - x(a)] + \frac{3}{b-a}\Omega^{\mathrm{T}} R \Omega,$$

where

$$\Omega = x(a) + x(b) - \left(\frac{2}{b-a}\right)\int_a^b x(s)ds \text{ (Seuret and Gouaisbaut (2013)).}$$

Firstly, let us transform the linear singular system (1) to a neutral system. By above information, there exist two regular matrices $D,\ K \in \mathbb{R}^{n\times n}$ such that

$$DEK = \begin{bmatrix} \mathrm{I}_r & 0 \\ 0 & 0 \end{bmatrix} = \overline{\mathrm{E}}, \quad DAK = \begin{bmatrix} \mathrm{A}_1 & 0 \\ 0 & \mathrm{I}_{n-r} \end{bmatrix} = \overline{\mathrm{A}},$$

$$D\mathrm{A}_{d_i}K = \begin{bmatrix} \mathrm{A}_{d_i 1} & \mathrm{A}_{d_i 2} \\ \mathrm{A}_{d_i 3} & \mathrm{A}_{d_i 4} \end{bmatrix} = \overline{\mathrm{A}_{d_i}},\ (i = 1, 2),$$

$$D\mathrm{A}_\tau K = \begin{bmatrix} \mathrm{A}_{\tau 1} & \mathrm{A}_{\tau 2} \\ \mathrm{A}_{\tau 3} & \mathrm{A}_{\tau 4} \end{bmatrix} = \overline{\mathrm{A}_\tau}, \quad K^{-1}x(t) = \mu(t) = \begin{bmatrix} \mu_1(t) \\ \mu_2(t) \end{bmatrix}.$$

Then, we write the system (1) as the following system:

$$\overline{\mathrm{E}}\dot{\mu}(t) = \overline{\mathrm{A}}\mu(t) + \sum_{i=1}^{2}\overline{\mathrm{A}_{d_i}}\mu(t-\tau_i(t)) + \overline{\mathrm{A}_\tau}\int_{t-\tau}^{t}\mu(s)ds,$$

which can be decomposed into the following system:

$$\dot{\mu}_1(t) = \mathrm{A}_1\mu_1(t) + \sum_{i=1}^{2}\mathrm{A}_{d_i1}\mu_1(t-\tau_i(t)) + \sum_{i=1}^{2}\mathrm{A}_{d_i2}\mu_2(t-\tau_i(t))$$

$$+\mathrm{A}_{\tau1}\int_{t-\tau}^{t}\mu_1(s)ds + \mathrm{A}_{\tau2}\int_{t-\tau}^{t}\mu_2(s)ds, \tag{3}$$

$$0 = \mu_2(t) + \sum_{i=1}^{2}\mathrm{A}_{d_i3}\mu_1(t-\tau_i(t)) + \sum_{i=1}^{2}\mathrm{A}_{d_i4}\mu_2(t-\tau_i(t))$$

$$+\mathrm{A}_{\tau3}\int_{t-\tau}^{t}\mu_1(s)ds + \mathrm{A}_{\tau4}\int_{t-\tau}^{t}\mu_2(s)ds. \tag{4}$$

Taking the time derivative of the equation (4), we have

$$0 = \dot{\mu}_2(t) + \sum_{i=1}^{2}\mathrm{A}_{d_i3}(1-\dot{\tau}_i(t))\dot{\mu}_1(t-\tau_i(t))$$

$$+\sum_{i=1}^{2}\mathrm{A}_{d_i4}(1-\dot{\tau}_i(t))\dot{\mu}_2(t-\tau_i(t))$$

$$+\mathrm{A}_{\tau3}[\mu_1(t)-\mu_1(t-\tau)] + \mathrm{A}_{\tau4}[\mu_2(t)-\mu_2(t-\tau)]. \tag{5}$$

Combining the equation (4) and the equation (5), we obtain

$$\dot{\mu}_2(t) = -\mu_2(t) - \sum_{i=1}^{2}\mathrm{A}_{d_i3}\mu_1(t-\tau_i(t)) - \sum_{i=1}^{2}\mathrm{A}_{d_i4}\mu_2(t-\tau_i(t))$$

$$-\mathrm{A}_{\tau3}\int_{t-\tau}^{t}\mu_1(s)ds - \mathrm{A}_{\tau4}\int_{t-\tau}^{t}\mu_2(s)ds - \sum_{i=1}^{2}\mathrm{A}_{d_i3}(1-\dot{\tau}_i(t))\dot{\mu}_1(t-\tau_i(t))$$

$$-\sum_{i=1}^{2}\mathrm{A}_{d_i4}(1-\dot{\tau}_i(t))\dot{\mu}_2(t-\tau_i(t)) - \mathrm{A}_{\tau3}[\mu_1(t)-\mu_1(t-\tau)]$$

$$-\mathrm{A}_{\tau4}[\mu_2(t)-\mu_2(t-\tau)]. \tag{6}$$

In view of equations (3) and (6), it follows that

$$\begin{bmatrix} \dot{\mu}_1(t) \\ \dot{\mu}_2(t) \end{bmatrix} = \begin{bmatrix} A_1\mu_1(t) \\ -\mu_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -A_{\tau 3}\mu_1(t) - A_{\tau 4}\mu_2(t) \end{bmatrix}$$

$$+\begin{bmatrix} A_{d_1 1}\mu_1(t-\tau_1(t)) + A_{d_1 2}\mu_2(t-\tau_1(t)) \\ -A_{d_1 3}\mu_1(t-\tau_1(t)) - A_{d_1 4}\mu_2(t-\tau_1(t)) \end{bmatrix}$$

$$+\begin{bmatrix} A_{d_2 1}\mu_1(t-\tau_2(t)) + A_{d_2 2}\mu_2(t-\tau_2(t)) \\ -A_{d_2 3}\mu_1(t-\tau_2(t)) - A_{d_2 4}\mu_2(t-\tau_2(t)) \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ A_{\tau 3}\mu_1(t-\tau) + A_{\tau 4}\mu_2(t-\tau) \end{bmatrix}$$

$$+\begin{bmatrix} A_{\tau 1}\int_{t-\tau}^{t}\mu_1(s)ds + A_{\tau 2}\int_{t-\tau}^{t}\mu_2(s)ds \\ -A_{\tau 3}\int_{t-\tau}^{t}\mu_1(s)ds - A_{\tau 4}\int_{t-\tau}^{t}\mu_2(s)ds \end{bmatrix}$$

$$+(1-\dot{\tau}_1(t))\begin{bmatrix} 0 & 0 \\ -A_{d_1 3} & -A_{d_1 4} \end{bmatrix}\begin{bmatrix} \dot{\mu}_1(t-\tau_1(t)) \\ \dot{\mu}_2(t-\tau_1(t)) \end{bmatrix}$$

$$+(1-\dot{\tau}_2(t))\begin{bmatrix} 0 & 0 \\ -A_{d_2 3} & -A_{d_2 4} \end{bmatrix}\begin{bmatrix} \dot{\mu}_1(t-\tau_2(t)) \\ \dot{\mu}_2(t-\tau_2(t)) \end{bmatrix},$$

which is equivalent to the following neutral system with three delays:

$$\dot{\mu}(t) - \sum_{i=1}^{2}\hat{C}_i(t)\dot{\mu}(t-\tau_i(t)) = \left(\hat{A} + \hat{B}\right)\mu(t) + \sum_{i=1}^{2}\hat{A}_{d_i}\,\mu(t-\tau_i(t))$$

$$-\hat{B}\,\mu(t-\tau) + \hat{A}_{\tau}\int_{t-\tau}^{t}\mu(s)ds, \tag{7}$$

$$\mu(t) = \phi(t),\ t\in[-\tau,0],\ \tau = \max\{\tau_1,\tau_2\},\ \tau>0,$$

where

$$\hat{A} = \begin{bmatrix} A_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix},\ \hat{A}_{d_1} = \begin{bmatrix} A_{d_1 1} & A_{d_1 2} \\ -A_{d_1 3} & -A_{d_1 4} \end{bmatrix},\ \hat{A}_{d_2} = \begin{bmatrix} A_{d_2 1} & A_{d_2 2} \\ -A_{d_2 3} & -A_{d_2 4} \end{bmatrix},$$

$$\hat{A}_{\tau} = \begin{bmatrix} A_{\tau 1} & A_{\tau 2} \\ -A_{\tau 3} & -A_{\tau 4} \end{bmatrix},\ \hat{B} = \begin{bmatrix} 0 & 0 \\ -A_{\tau 3} & -A_{\tau 4} \end{bmatrix},\ C_1 = \begin{bmatrix} 0 & 0 \\ -A_{d_1 3} & -A_{d_1 4} \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 \\ -A_{d_2 3} & -A_{d_2 4} \end{bmatrix}, \hat{C}_1(t) = (1-\dot{\tau}_1(t))C_1, \hat{C}_2(t) = (1-\dot{\tau}_2(t))C_2. \quad (8)$$

It should be noted that the systems (1) and (7) are not equivalent, however the asymptotic stability of the system (7) guarantees the asymptotic stability of system (1), and vice versa (see, Liu et al. (2014)). In the light of the above information, we can conclude that the system (1) is asymptotically admissible.

3. Main Results and Numerical Applications

A. Assumptions

Throughout this work, we suppose the following condition holds.

(A1) We suppose that the pair (E, A) is regular and impulse-free and the eigenvalues of

$\sum_{i=1}^{2} \hat{C}_i(t)$ are inside the unit circle,

i.e., $\rho(\hat{C}_i(t)) = \max\{|(1-\sigma_i)\rho(C_i)|, |(1-h_i)\rho(C_i)|\} < 1$,

where the symbol ρ denotes the spectral radius of the matrix.

(A2) There are symmetric positive definite $Q_i \in \mathbb{R}^{n\times n}$, $R_i \in \mathbb{R}^{n\times n}$, $S_i \in \mathbb{R}^{n\times n}$, $W_i \in \mathbb{R}^{n\times n}$, $(i=1,2)$ and $P \in \mathbb{R}^{6n\times 6n}$ matrices such that the following LMIs hold:

$$\Psi = \Psi_0 - F_2^T \overline{\overline{S_1}} F_2 - F_3^T \overline{\overline{S_2}} F_3 \langle 0,$$

where

$$\Psi_0 = Sym\{F_1^T P F_0\} + \overline{Q_1} + \overline{Q_2} + \overline{R_1} + \overline{R_2} + \overline{W_1} + \overline{W_2} + F_0^T \overline{\overline{R_1}} F_0 + F_0^T \overline{\overline{R_2}} F_0 + \tau_1^2 F_0^T \overline{S_1} F_0 + \tau_2^2 F_0^T \overline{S_2} F_0,$$

$$\overline{Q_1} = diag(Q_1, -(1-\dot{\tau}_1(t))Q_1, 0_{7n}), \overline{Q_2} = diag(Q_1, 0_n, -(1-\dot{\tau}_2(t))Q_2, 0_{6n}),$$

$$\overline{W_1} = diag(W_1, 0_{2n}, -W_1, 0_{5n}),\ \overline{W_2} = diag(\tau W_2, 0_{7n}, -\tau W_2),$$

$$\overline{R_1} = diag(0_{4n}, -(1-\dot{\tau}_1(t))R_1, 0_{4n}),\ \overline{R_2} = diag(0_{5n}, -(1-\dot{\tau}_2(t))R_2, 0_{3n}),$$

$$\overline{\overline{R_1}} = diag(R_1, 0_{5n}), \overline{\overline{R_2}} = diag(R_2, 0_{5n}), \overline{S_1} = diag(S_1, 0_{5n}),$$

$$\overline{S_2} = diag(S_2, 0_{5n}),\ \overline{\overline{S_1}} = diag(S_1, 3S_1), \overline{\overline{S_2}} = diag(S_2, 3S_2),$$

$$F_0=\begin{bmatrix} \hat{A}+\hat{B} & \hat{A}_{d_1} & \hat{A}_{d_2} & -\hat{B} & (1-\dot{\tau}_1(t))C_1 & (1-\dot{\tau}_2(t))C_2 & 0 & 0 & \tau A_\tau \\ 0 & 0 & 0 & 0 & (1-\dot{\tau}_1(t))I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-\dot{\tau}_2(t))I & 0 & 0 & 0 \\ I & 0 & 0 & 0 & -(1-\dot{\tau}_1(t))I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -(1-\dot{\tau}_2(t))I & 0 & 0 & 0 \\ I & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$F_1=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_1(t)I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_2(t)I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau I \end{bmatrix}, \quad F_2=\begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 & -2I & 0 & 0 \end{bmatrix}, \quad F_3=\begin{bmatrix} I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & -2I & 0 \end{bmatrix}$$

and $\hat{A}$, $\hat{A}_{d_i}$, $\hat{B}$, $\hat{A}_\tau$, $\hat{C}_i(t)$ and $C_i, (i=1,2)$ are defined by (8).

Theorem 3.1.

If the conditions (2), (A1) and (A2) hold, then the system (7) is asymptotically stable and thus the system (1) is asymptotically admissible.

Proof:

Firstly, we define

$$\xi(t)=[\mu^T(t)\ \mu^T(t-\tau_1(t))\ \mu^T(t-\tau_2(t))\ \int_{t-\tau_1(t)}^{t}\mu^T(s)ds\ \int_{t-\tau_2(t)}^{t}\mu^T(s)ds\ \int_{t-\tau}^{t}\mu^T(s)ds\ ]^T,$$

$$\eta(t)=[\mu^T(t)\ \mu^T(t-\tau_1(t))\ \mu^T(t-\tau_2(t))\ \mu^T(t-\tau)\ \dot{\mu}^T(t-\tau_1(t))\ \dot{\mu}^T(t-\tau_2(t))$$

$$\frac{1}{\tau_1(t)}\int_{t-\tau_1(t)}^{t}\mu^T(s)ds\ \ \frac{1}{\tau_2(t)}\int_{t-\tau_2(t)}^{t}\mu^T(s)ds\ \ \frac{1}{\tau}\int_{t-\tau}^{t}\mu^T(s)ds\ ]^T$$

and note that

$$\xi(t)=F_1\eta(t),\ \dot{\xi}(t)=F_0\eta(t).$$

We now prove the asymptotic stability of the system (7). For this, we define a new LKF as follows:

$$V(t)=\sum_{i=1}^{5}V_i(t), \tag{9}$$

where

$$V_1(t)=\xi^T(t)\mathrm{P}\xi(t),$$

$$V_2(t)=\int_{t-\tau_1(t)}^{t}\mu^T(s)Q_1\mu(s)ds+\int_{t-\tau_2(t)}^{t}\mu^T(s)Q_2\mu(s)ds$$

$$+\int_{t-\tau}^{t}\mu^{\mathrm{T}}(s)W_1\mu(s)ds,$$

$$V_3(t)=\int_{t-\tau_1(t)}^{t}\dot{\mu}^T(s)R_1\dot{\mu}(s)ds+\int_{t-\tau_2(t)}^{t}\dot{\mu}^T(s)R_2\dot{\mu}(s)ds,$$

$$V_4(t)=\tau_1(t)\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{\mu}^T(s)S_1\dot{\mu}(s)dsd\theta$$

$$+\tau_2(t)\int_{-\tau_2(t)}^{0}\int_{t+\theta}^{t}\dot{\mu}^{\mathrm{T}}(s)S_2\dot{\mu}(s)dsd\theta,$$

$$V_5(t)=\int_{-\tau}^{0}\int_{t+\theta}^{t}\mu^T(s)W_2\mu(s)dsd\theta\,.$$

It is clear that the LKF (9) is positive definite. By the time derivative of the LKF $V(t)$ of (9) along the system (7), we obtain

$$\dot{V}(t)=\sum_{i=1}^{5}\dot{V}_i(t), \tag{10}$$

where

$$\dot{V}_1(t)=\dot{\xi}^T(t)\mathrm{P}\xi(t)+\xi^T(t)P\dot{\xi}(t)$$

$$=\begin{bmatrix}\dot{\mu}(t)\\(1-\dot{\tau}_1(t))\dot{\mu}(t-\tau_1(t))\\(1-\dot{\tau}_2(t))\dot{\mu}(t-\tau_2(t))\\\mu(t)-(1-\dot{\tau}_1(t))\mu(t-\tau_1(t))\\\mu(t)-(1-\dot{\tau}_2(t))\mu(t-\tau_2(t))\\\mu(t)-\mu(t-\tau)\end{bmatrix}^{\mathrm{T}} P\begin{bmatrix}\mu(t)\\\mu(t-\tau_1(t))\\\mu(t-\tau_2(t))\\\int_{t-\tau_1(t)}^{t}\mu(s)ds\\\int_{t-\tau_2(t)}^{t}\mu(s)ds\\\int_{t-\tau}^{t}\mu(s)ds\end{bmatrix}+\begin{bmatrix}\mu(t)\\\mu(t-\tau_1(t))\\\mu(t-\tau_2(t))\\\int_{t-\tau_1(t)}^{t}\mu(s)ds\\\int_{t-\tau_2(t)}^{t}\mu(s)ds\\\int_{t-\tau}^{t}\mu(s)ds\end{bmatrix}^{\mathrm{T}} P\begin{bmatrix}\dot{\mu}(t)\\(1-\dot{\tau}_1(t))\dot{\mu}(t-\tau_1(t))\\(1-\dot{\tau}_2(t))\dot{\mu}(t-\tau_2(t))\\\mu(t)-(1-\dot{\tau}_1(t))\mu(t-\tau_1(t))\\\mu(t)-(1-\dot{\tau}_2(t))\mu(t-\tau_2(t))\\\mu(t)-\mu(t-\tau)\end{bmatrix}$$

$$= \eta^T(t)\{F_0^T \mathrm{P} F_1\}\eta(t) + \eta^T(t)\{F_1^T \mathrm{P} F_0\}\eta(t)$$

$$= \eta^{\mathrm{T}}(t) Sym\{F_1^{\mathrm{T}} P F_0\}\eta(t), \tag{11}$$

$$\dot{V}_2(t) = \mu^T(t) Q_1 \mu(t) - (1-\dot{\tau}_1(t))\mu^T(t-\tau_1(t)) Q_1 \mu(t-\tau_1(t))$$
$$+ \mu^{\mathrm{T}}(t) Q_2 \mu(t) - (1-\dot{\tau}_2(t))\mu^{\mathrm{T}}(t-\tau_2(t)) Q_2 \mu(t-\tau_2(t))$$
$$+ \mu^{\mathrm{T}}(t) W_1 \mu(t) - \mu^{\mathrm{T}}(t-\tau) W_1 \mu(t-\tau)$$
$$= \eta^{\mathrm{T}}(t)\overline{Q_1}\eta(t) + \eta^{\mathrm{T}}(t)\overline{Q_2}\eta(t) + \eta^{\mathrm{T}}(t)\overline{W_1}\eta(t), \tag{12}$$

$$\dot{V}_3(t) = \dot{\mu}^T(t) R_1 \dot{\mu}(t) - (1-\dot{\tau}_1(t))\dot{\mu}^T(t-\tau_1(t)) R_1 \dot{\mu}(t-\tau_1(t))$$
$$+ \dot{\mu}^{\mathrm{T}}(t) R_2 \dot{\mu}(t) - (1-\dot{\tau}_2(t))\dot{\mu}^{\mathrm{T}}(t-\tau_2(t)) R_2 \dot{\mu}(t-\tau_2(t))$$
$$= [(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t) + \sum_{i=1}^{2} \hat{\mathrm{A}}_{d_i}\, \mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau))$$
$$+ \sum_{i=1}^{2} \hat{C}_i(t)\dot{\mu}(t-\tau_i(t)) + \hat{\mathrm{A}}_\tau \int_{t-\tau}^{t} \mu(s)ds]^{\mathrm{T}} R_1 [(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t)$$
$$+ \sum_{i=1}^{2} \hat{\mathrm{A}}_{d_i}\, \mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau)) + \sum_{i=1}^{2} \hat{C}_i(t) + \hat{\mathrm{A}}_\tau \int_{t-\tau}^{t} \mu(s)ds]$$
$$+ [(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t) + \sum_{i=1}^{2} \hat{\mathrm{A}}_{d_i}\, \mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau))$$
$$+ \sum_{i=1}^{2} \hat{C}_i(t)\dot{\mu}(t-\tau_i(t)) + \hat{\mathrm{A}}_\tau \int_{t-\tau}^{t} \mu(s)ds]^{\mathrm{T}} \times R_2 [(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t)$$
$$+ \sum_{i=1}^{2} \hat{\mathrm{A}}_{d_i}\, \mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau)) + \sum_{i=1}^{2} \hat{C}_i(t)\dot{\mu}(t-\tau_i(t))$$
$$+ \hat{\mathrm{A}}_\tau \int_{t-\tau}^{t} \mu(s)ds] - (1-\dot{\tau}_1(t))\dot{\mu}^{\mathrm{T}}(t-\tau_1(t)) R_1 \dot{\mu}(t-\tau_1(t))$$
$$- (1-\dot{\tau}_2(t))\dot{\mu}^{\mathrm{T}}(t-\tau_2(t)) R_2 \dot{\mu}(t-\tau_2(t))$$
$$= \eta^{\mathrm{T}}(t)[\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2} \quad -\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1 \quad (1-\dot{\tau}_2(t))C_2 \; 0 \quad 0 \quad \tau\hat{\mathrm{A}}_\tau]^{\mathrm{T}}$$
$$\times R_1 [\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2} \quad -\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1$$
$$(1-\dot{\tau}_2(t))C_2 \quad 0 \quad 0 \quad \tau\hat{\mathrm{A}}_\tau]\eta(t) + \eta^{\mathrm{T}}(t)[\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1}$$
$$\hat{\mathrm{A}}_{d_2} \quad -\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1 \quad (1-\dot{\tau}_2(t))C_2 \quad 0 \quad 0 \quad \tau\hat{\mathrm{A}}_\tau]^{\mathrm{T}} R_2$$
$$\times [\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2} \quad -\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1 \quad (1-\dot{\tau}_2(t))C_2 \quad 0$$

$$0 \quad \tau \hat{\mathrm{A}}_{\tau}]\eta(t) - \eta^{\mathrm{T}}(t)\overline{R_1}\eta(t) - \eta^{\mathrm{T}}(t)\overline{R_2}\eta(t)$$

$$= \eta^{\mathrm{T}}(t)F_0^{\mathrm{T}}\overline{\overline{R_1}}F_0\eta(t) + \eta^{\mathrm{T}}(t)F_0^{\mathrm{T}}\overline{\overline{R_2}}F_0\eta(t)$$

$$-\eta^{\mathrm{T}}(t)\overline{R_1}\eta(t) - \eta^{\mathrm{T}}(t)\overline{R_2}\eta(t), \tag{13}$$

$$\dot{V}_4(t) = \tau_1^2(t)[(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t) + \sum_{i=1}^{2}\hat{\mathrm{A}}_{d_i}\,\mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau)$$

$$+ \sum_{i=1}^{2}\hat{C}_i(t)\dot{\mu}(t-\tau_i(t)) + \hat{\mathrm{A}}_{\tau}\int_{t-\tau}^{t}\mu(s)ds]^{\mathrm{T}} S_1[(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t)$$

$$+ \sum_{i=1}^{2}\hat{\mathrm{A}}_{d_i}\,\mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau) + \sum_{i=1}^{2}\hat{C}_i(t)\dot{\mu}(t-\tau_i(t))$$

$$+ \hat{\mathrm{A}}_{\tau}\int_{t-\tau}^{t}\mu(s)ds] + \tau_2^2(t)[(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t) + \sum_{i=1}^{2}\hat{\mathrm{A}}_{d_i}\,\mu(t-\tau_i(t))$$

$$- \hat{\mathrm{B}}\,\mu(t-\tau)) + \sum_{i=1}^{2}\hat{C}_i(t)\dot{\mu}(t-\tau_i(t)) + \mathrm{A}_{\tau}\int_{t-\tau}^{t}\mu(s)ds]^{\mathrm{T}} S_2$$

$$\times[(\hat{\mathrm{A}} + \hat{\mathrm{B}})\mu(t) + \sum_{i=1}^{2}\hat{\mathrm{A}}_{d_i}\,\mu(t-\tau_i(t)) - \hat{\mathrm{B}}\,\mu(t-\tau))$$

$$+ \sum_{i=1}^{2}\hat{C}_i(t)\dot{\mu}(t-\tau_i(t)) + \hat{\mathrm{A}}_{\tau}\int_{t-\tau}^{t}\mu(s)ds]$$

$$-\tau_1(t)\int_{t-\tau_1(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)S_1\dot{\mu}(s)ds - \tau_2(t)\int_{t-\tau_2(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)S_2\dot{\mu}(s)ds$$

$$= \tau_1^2(t)\eta^{\mathrm{T}}(t)[\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2} \quad -\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1$$

$$(1-\dot{\tau}_2(t))C_2 \quad 0 \quad 0 \quad \tau\hat{\mathrm{A}}_{\tau}]^{\mathrm{T}} S_1[\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2}$$

$$-\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1 \quad (1-\dot{\tau}_2(t))C_2 \quad 0 \quad 0 \quad \tau\hat{\mathrm{A}}_{\tau}]\eta(t)$$

$$+ \tau_2^2(t)\eta^{\mathrm{T}}(t)[\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2} \quad -\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1$$

$$(1-\dot{\tau}_2(t))C_2 \quad 0 \quad 0 \quad \tau\hat{\mathrm{A}}_{\tau}]^{\mathrm{T}} S_2[\hat{\mathrm{A}} + \hat{\mathrm{B}} \quad \hat{\mathrm{A}}_{d_1} \quad \hat{\mathrm{A}}_{d_2}$$

$$-\hat{\mathrm{B}} \quad (1-\dot{\tau}_1(t))C_1 \quad (1-\dot{\tau}_2(t))C_2 \quad 0 \quad 0 \quad \tau\hat{\mathrm{A}}_{\tau}]\eta(t)$$

$$-\tau_1(t)\int_{t-\tau_1(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)S_1\dot{\mu}(s)ds - \tau_2(t)\int_{t-\tau_2(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)S_2\dot{\mu}(s)ds\,. \tag{14}$$

Applying the Wirtinger inequality, for the last two terms in (14), we obtain the following inequalities, respectively:

$$
\begin{aligned}
-\tau_1(t)\int_{t-\tau_1(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)S_1\dot{\mu}(s)ds &\leq -[\mu(t)-\mu(t-\tau_1(t))]^{\mathrm{T}}S_1[\mu(t)-\mu(t-\tau_1(t))] \\
&\quad -[\mu(t)+\mu(t-\tau_1(t))-\frac{2}{\tau_1(t)}\int_{t-\tau_1(t)}^{t}\mu(s)ds]^{\mathrm{T}}3S_1 \\
&\quad \times[\mu(t)+\mu(t-\tau_1(t))-\frac{2}{\tau_1(t)}\int_{t-\tau_1(t)}^{t}\mu(s)ds] \\
&\leq -\eta^{\mathrm{T}}(t)\begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 & -2I & 0 & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} S_1 & 0 \\ 0 & 3S_1 \end{bmatrix} \\
&\quad \times\begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 & -2I & 0 & 0 \end{bmatrix}\eta(t)
\end{aligned} \tag{15}
$$

and

$$
\begin{aligned}
-\tau_2(t)\int_{t-\tau_2(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)S_2\dot{\mu}(s)ds &\leq -[\mu(t)-\mu(t-\tau_2(t))]^{\mathrm{T}}S_2[\mu(t)-\mu(t-\tau_2(t))] \\
&\quad -[\mu(t)+\mu(t-\tau_2(t))-\frac{2}{\tau_2(t)}\int_{t-\tau_2(t)}^{t}\mu(s)ds]^{\mathrm{T}}3S_2 \\
&\quad \times[\mu(t)+\mu(t-\tau_2(t))-\frac{2}{\tau_2(t)}\int_{t-\tau_2(t)}^{t}\mu(s)ds] \\
&\leq -\eta^{\mathrm{T}}(t)\begin{bmatrix} I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & -2I & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} S_2 & 0 \\ 0 & 3S_2 \end{bmatrix} \\
&\quad \times\begin{bmatrix} I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & -2I & 0 \end{bmatrix}\eta(t).
\end{aligned} \tag{16}
$$

Hence, we get

$$
\begin{aligned}
\dot{V}_4(t) &\leq \tau_1^2\eta^{\mathrm{T}}(t)F_0^T\overline{S_1}F_0\eta(t)+\tau_2^2\eta^{\mathrm{T}}(t)F_0^T\overline{S_2}F_0\eta(t) \\
&\quad -\eta^{\mathrm{T}}(t)F_2^{\mathrm{T}}\overline{\overline{S_1}}F_2\eta(t)-\eta^{\mathrm{T}}(t)F_3^{\mathrm{T}}\overline{\overline{S_2}}F_3\eta(t).
\end{aligned} \tag{17}
$$

Additionally, calculating the derivative of $V_5(t)$ and using Jensen inequality (see Tunç and Yiğit (2020), Lemma 1.2), we obtain that

$$
\dot{V}_5(t)=\tau\mu^{\mathrm{T}}(t)W_2\mu(t)-\int_{t-\tau}^{t}\mu^{\mathrm{T}}(s)W_2\mu(s)ds
$$

$$\leq \tau\mu^{\mathrm{T}}(t)W_2\mu(t)-\left(\frac{1}{\tau}\int_{t-\tau}^{t}\mu^{\mathrm{T}}(s)ds\right)\tau W_2\left(\frac{1}{\tau}\int_{t-\tau}^{t}\mu(s)ds\right)$$

$$\leq \eta^{\mathrm{T}}(t)\overline{W_2}\eta(t)\,. \tag{18}$$

Combining (10)-(18), we have

$$\dot{V}(t)\leq \eta^{\mathrm{T}}(t)\Psi\,\eta(t).$$

Since $\Psi\langle 0$, we arrive at $\dot{V}(t)<0$. Thus, we conclude that the neutral system (7) is asymptotically stable. Consequently, since the singular system (1) is regular, impulse free and asymptotically stable, it is also asymptotically admissible. This result completes the proof of Theorem 3.1.

Example 3.2.

For the particular case of the system (1), we consider the following linear singular system with three delays:

$$\frac{d}{dt}\left(\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\right)=\begin{bmatrix}-8 & 0\\ -2 & -0.5\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}-4 & 0\\ 0.38 & 0.1\end{bmatrix}\begin{bmatrix}x_1(t-(0.05+0.05\sin^2 t))\\ x_2(t-(0.05+0.05\sin^2 t))\end{bmatrix}$$

$$+\begin{bmatrix}2 & 0\\ 0.16 & 0.05\end{bmatrix}\begin{bmatrix}x_1(t-(0.025+0.025\sin^2 t))\\ x_2(t-(0.025+0.025\sin^2 t))\end{bmatrix}$$

$$+\begin{bmatrix}-0.4 & -0.1\\ -1.4 & -0.5\end{bmatrix}\begin{bmatrix}\int_{t-0.1}^{t}x_1(s)ds\\ \int_{t-0.1}^{t}x_2(s)ds\end{bmatrix},$$

where

$$E=\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix},\ A=\begin{bmatrix}-8 & 0\\ -2 & -0.5\end{bmatrix},\ A_{\mathrm{d}_1}=\begin{bmatrix}-4 & 0\\ 0.38 & 0.1\end{bmatrix},$$

$$A_{\mathrm{d}_2}=\begin{bmatrix}2 & 0\\ 0.16 & 0.05\end{bmatrix},\ A_{\tau}=\begin{bmatrix}-0.4 & -0.1\\ -1.4 & -0.5\end{bmatrix}$$

and

$$0<\tau_1(t)=0.05+0.05\sin^2 t\leq 0.1=\tau_1,$$

$$-0.05\leq\dot{\tau}_1(t)=0.05\sin 2t\leq 0.05=h_1,$$

$$0<\tau_2(t)=0.025+0.025\sin^2 t\leq 0.05=\tau_2,$$

$$-0.025 \le \dot{\tau}_2(t) = 0.025 \sin 2t \le 0.05 = h_2, \ \tau = 0.1.$$

It is clear that the pair (E, A) is regular and impulse free. Thus, there exist two regular matrices

$$D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad K = \begin{bmatrix} 0.25 & 0 \\ -1 & -1 \end{bmatrix}$$

such that

$$DEK = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, DAK = \begin{bmatrix} -4 & 0 \\ 0 & 1 \end{bmatrix}, DA_{d_1}K = \begin{bmatrix} -2 & 0 \\ -0.01 & -0.2 \end{bmatrix},$$

$$DA_{d_2}K = \begin{bmatrix} 1 & 0 \\ -0.02 & -0.1 \end{bmatrix}, DA_{\tau}K = \begin{bmatrix} 0 & 0.2 \\ 0.3 & 1 \end{bmatrix}.$$

Hence, we have

$$\hat{A} = \begin{bmatrix} -4 & 0 \\ 0 & -1 \end{bmatrix}, \hat{A}_{d_1} = \begin{bmatrix} -2 & 0 \\ 0.01 & 0.2 \end{bmatrix}, \hat{A}_{d_2} = \begin{bmatrix} 1 & 0 \\ 0.02 & 0.1 \end{bmatrix},$$

$$\hat{A}_{\tau} = \begin{bmatrix} 0 & 0.2 \\ -0.3 & -1 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 & 0 \\ -0.3 & -1 \end{bmatrix}, \hat{C}_1 = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.2 \end{bmatrix},$$

$$\hat{C}_2 = \begin{bmatrix} 0 & 0 \\ 0.02 & 0.1 \end{bmatrix}.$$

Let

$$P = \mathrm{diag}(5,2,0.01,0.01,0.3,0.2,0.1,0.2,3,0.2,0.3,0.1),$$

$$Q_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0.1 \end{bmatrix}, Q_2 = \begin{bmatrix} 3 & 1 \\ 1 & 1.2 \end{bmatrix}, R_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, R_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 8 & 0 \\ 0 & 3 \end{bmatrix}, S_2 = \begin{bmatrix} 8 & 0 \\ 0 & 9 \end{bmatrix}. W_1 = \begin{bmatrix} 8 & 1 \\ 1 & 2 \end{bmatrix}, W_2 = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 2 \end{bmatrix}.$$

Under the above conditions, all eigenvalues of the LMI defined by Ψ satisfy $\lambda_{\max}(\Psi) \le -0.0583$. Consequently, it is clear that all conditions of Theorem 3.1 can be satisfied. Thus, the system (1) is asymptotically admissible, which is asymptotically stable, regular and impulse-free.

Trajectories of the solutions of the above system is given in Figure 1. The given system is solved by MATLAB-Simulink software.

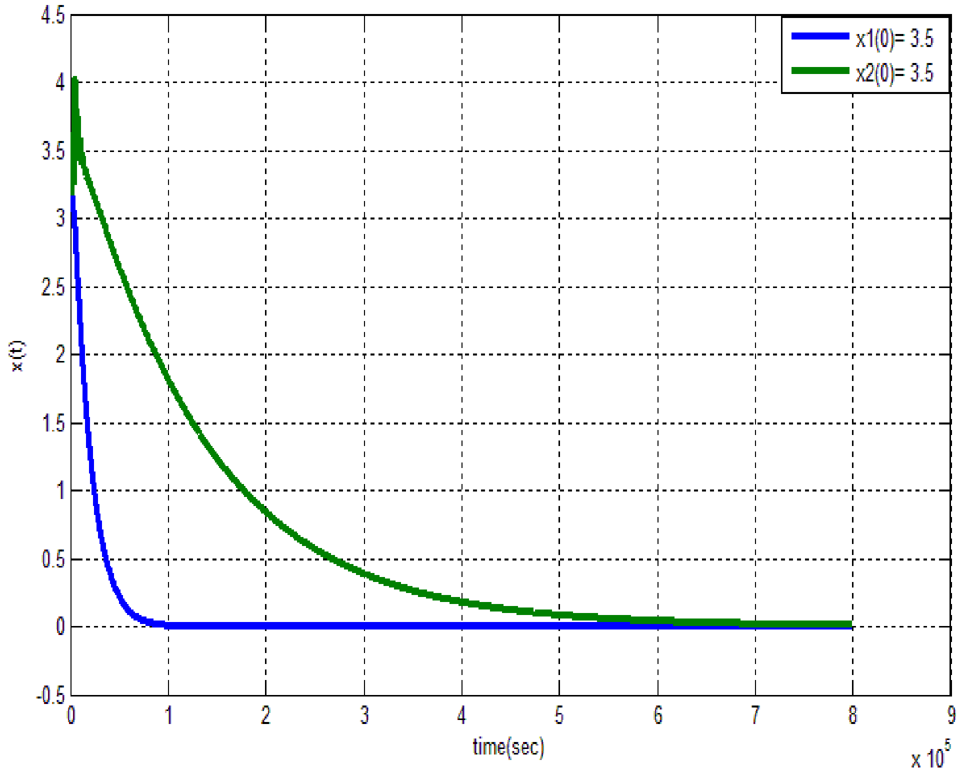

Figure 1. Trajectories of the solution $x(t)$ of the system in Example 3.2, when $\tau = 0,1$

Example 3.3.

Consider the singular system (1) with

$$E = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 4 \\ 0.5 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} -2 & 0 & 4 \\ 0 & -5 & -20 \\ -1 & 0 & 3 \end{bmatrix}, A_{d_1} = \begin{bmatrix} -2 & 0 & 4 \\ 0 & 1 & 4 \\ -1 & 0 & 1.77 \end{bmatrix},$$

$$A_{d_2} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0.49 & 0 & -1.52 \end{bmatrix}, A_{\tau} = \begin{bmatrix} 0 & 0 & 0.02 \\ 0 & 0 & 0.01 \\ 0.015 & 0.02 & 0.06 \end{bmatrix}$$

and

$$0 < \tau_1(t) = 0.025 + 0.025\sin^2 t \le 0.05 = \tau_1,$$

$$-0.025 \le \dot{\tau}_1(t) = 0.025\sin 2t \le 0.025 = h_1,$$

$$0 < \tau_2(t) = 0.0125 + 0.0125\sin^2 t \le 0.025 = \tau_2,$$

$$-0.0125 \le \dot{\tau}_2(t) = 0.0125\sin 2t \le 0.0125 = h_2,\ \ \tau = 0.05.$$

Trajectories of the solutions of the above system is given in Figure 2. The given system is solved by MATLAB-Simulink software.

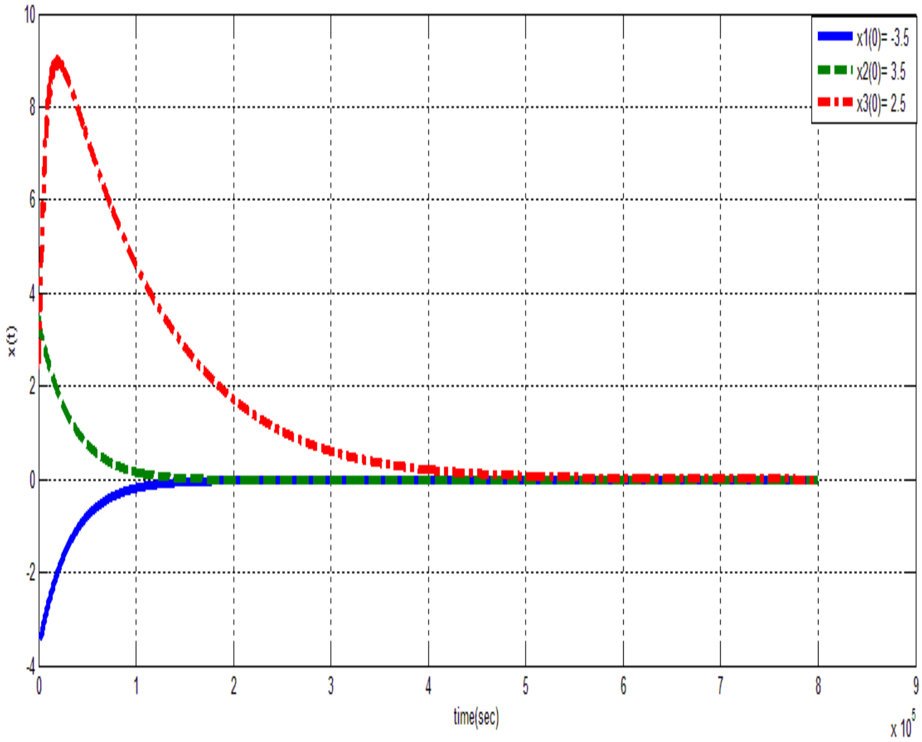

Figure 2. Trajectories of the solution $x(t)$ of system in Example 3.3, when $\tau = 0.05$

3. Conclusion

In this paper, we consider a class of linear singular systems with mixed delays. By a suitable transform, we reduce the considered system to a non-singular neutral system with mixed delays. Then, using a new LKF, LMI and Wirtinger-based integral inequality, we investigate asymptotic admissibility. Finally, two numerical examples are also given with their simulations to demonstrate the applications of the main results. The obtained results include and generalize some recent results in the literature.

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