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On the Asymptotic Stability a System with Nonlinear Perturbations and Constant Delay

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Abstract

In this paper, we consider a nonlinear perturbed system of neutral delay integro-differential equations (NDIDEs). We prove two new theorems, Theorems 1 and 2, such that these theorems include sufficient conditions and are related to asymptotical stability of zero solution of the perturbed system of NDIDEs. The technique of the proofs depends upon the definitions of two new and more suitable Lyapunov-Krasovskiĭ functionals (LKFs). When we compared the results of this paper with those found in the literature, our results improve and extend some classical results, and contribute new contents to the topic of NDIDEs and to the profession.

Keywords: Asymptotic stability; Neutral delay integro-differential equations; Lyapunov functional; System; Linear matrix inequality (LMI)

MSC (2010): 34K2, 34K40, 34K45, 93D30

1. Introduction

From the relevant literature, it can be seen that the stability problems of neutral delay-differential systems and some other kind of various differential equations have received the considerable attention of researchers over the last decades. Some of the papers and books that can be found are given by references to this paper. In most of these papers and books, the techniques of Lyapunov functions (LFs) or Lyapunov-Krasovskiĭ functionals (LKFs) with basic inequality techniques and linear matrix inequalities have been used to prove qualitative results related to the considered

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equations. Indeed, most of the stability criteria of NDIDEs in the mentioned sources are expressed in terms of matrix norm or matrix measure of the system matrices depending on the derivative of LFs and LKFs used in works. Unfortunately, the choice of some kind of the LFs or LKFs usually affects this.

In fact, the motivation of this paper has been inspired by the results of Akbulut and Tunç (2019), Benhadri (2021), Boyd et al. (1994), Brayton and Willoughby (1967), Chen et al. (2006), Cong and Yang (2000), Hale and Verduyn Lunel (1993), Gözen and Tunc (2017), Gözen and Tunç (2018), Gözen and Tunç (2019), Gözen and Tunç (2021), Guo and Yan (2004), Hu (1996), Khusainov and Yun'kova (1988), Kolmanovskiĭ and Myshkis (1992), Kuang et al. (2011), Kuang et al. (1994), Kwon et al. (2008), Li (1988), Li et al. (2014), Liu (2006), Liu and Xu (2006), Mesmouli et al. (2016), Mesmouli et al. (2018), Park and Won (1999), Park and Won (2000a), Park (2002), Park and Kwon (2005), Sun (2011), Tunç (2014), Tunç (2015), Tunç and Tunç (2021), Tunç et al. (2021), Tunç at al. (2021), Yang and Liu (2002) and, in particular, from Park and Won (2000b) (Theorem 1, Theorem 2).

2. Preliminaries

As for this paper, we should mention the work of Park and Won (2000b). Indeed, Park and Won (2000b) considered the following nonlinear perturbed system of NDDEs:

$$\dot{x}(t) = Ax(t) + Bx(t-h) + C\dot{x}(t-h) + Q(x(t), x(t-h), \dot{x}(t-h)). \tag{1}$$

In their work, two new theorems, which include sufficient conditons on the asymptotic stability of the perturbed system of NDDEs (Park and Won (2000b), Theorem 1, Theorem 2), are proved via LKF method and linear matrix inequalities (LMIs). In Park and Won (2000b), two different LKFs are defined and the proofs of (Park and Won (2000b),Theorem 1, Theorem 2) are done by using those LKFs.

Here, instead of the nonlinear perturbed system of NDDEs (1), we consider the following more general nonlinear perturbed system of NDIDEs :

$$\begin{aligned}\dot{x}(t) = Ax(t) + Bx(t-h) + C\dot{x}(t-h) + \int_{t-h}^{t} K(t,s)x(s)ds \\ + Q(x(t), x(t-h), \dot{x}(t-h)),\end{aligned} \tag{2}$$

where $x(t)\in R^n$ is the state vector, $\phi(t)\in C([-h,0],R^n)$ is initial function, $x(t)=\phi(t)$ on $[-h,0]$, $A\in C([0,\infty),R^{n\times n})$, $B,\ C\in R^{n\times n}$, $K(t,s)\in C([-h,\infty)\times[-h,\infty),R^{n\times n})$, h is a positive constant, fixed constant delay, $Q\in C[R^n\times R^n\times R^n,R^n]$ represents the nonlinear perturbation, and $Q(0,x(t-h),\dot{x}(t-h))=0.$

The aim of this paper is to extend and improve the results of Park and Won (2000b) (Theorem 1, Theorem 2) and obtain them under less conservative and more optimal conditions, and do suitable contributions to the relevant literature. As we know, it is worth to mention that the choice of proper LFs and LKFs for stability problems under investigation can lead to stronger or weaker conditions. In this paper, we chose very simple and more suitable two LKFs to extend and obtain the results of Park and Won (2000b) (Theorem 1, Theorem 2) under less conservative, i.e., weaker conditions. These cases can be easily seen when we compare equation (1) and equation (2) and the conditions of Park and Won (2000b) (Theorem 1, Theorem 2) and the conditions of our theorems, Theorems 1 and 2, to be given at the following. These are the novelty, originality and contributions of the results of this paper to the relevant literature.

3. Main results

The following information are necessary to develop the new stability criteria of this paper. Let D and E be real matrices of appropriate dimensions. Then, for any scalar $\varepsilon>0$,

$$DE+E^TD^T\le\varepsilon DD^T+\varepsilon^{-1}E^TE \quad \text{(see, Park and Won (2000b)).}$$

For the symmetric matrices Y and $R,$ the linear matrix inequality

$$\begin{bmatrix} Y & S \\ S^T & R \end{bmatrix}>0,$$

is equivalent to

$$R>0 \text{ and } Y-SR^{-1}S^T>0 \quad \text{(see, Park and Won (2000b)).}$$

Our first main result is the following theorem, Theorem 1.

Theorem 1.

We assume that the following conditions are satisfied:

$(A1)$ There exist $n\times n-$ symmetric positive definite matrices $P,\ R$ and positive scalars $\varepsilon_1,\varepsilon_2,\varepsilon_3$ and ε_4 such that the following LMI holds:

$$M=\begin{bmatrix} A^T(t)P+PA(t)+\varepsilon_4^{-1}PP+3\beta_0^2\varepsilon_4+R & PB & 0 & P \\ B^TP & 3\beta_0^2\varepsilon_4-R & PC & 0 \\ 0 & C^TP & 3\beta_2^2\varepsilon_4 & 0 \\ P & 0 & 0 & 0 \end{bmatrix}<0.$$

$(A2)$ There exist positive scalars β_0, β_1 and β_2 such that

$$\|Q(x(t),x(t-h),\dot{x}(t-h))\| \le \beta_0\|x(t)\|+\beta_1\|x(t-h)\|+\beta_2\|\dot{x}(t-h)\|.$$

Then, the zero solution of system (2) is asymptotical stable for all $h>0$.

Proof:

Define a Lyapunov functional $V=V(t,x_t)$ by

$$V=x^T(t)Px(t)+\int_{-h}^{0}x^T(t+s)Rx(t+s)ds$$

for simplicity, we define

$$W_2=\int_{-h}^{0}x^T(t+s)Rx(t+s)ds=\int_{t-h}^{t}x^T(s)Rx(s)ds.$$

Then, we can write

$$V=x^T(t)Px(t)+W_2.$$

By the derivative of the functional V along the system (2), we have

$$\begin{aligned}\dot{V}&=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)+\dot{W}_2\\ &=x^T[A^T(t)P+PA(t)]x+2x^TPBx_h+2x_h{}^TPC\dot{x}_h\\ &\quad+2x^TPQ+2x^TP\int_{t-h}^{t}K(t,s)x(s)ds+\dot{W}_2,\end{aligned}$$

where

$$x=x(t), x_h=x(t-h), \quad \dot{x}_h=\dot{x}(t-h)$$

and

$$\dot{W}_2=x^T(t)Rx(t)-x^T(t-h)Rx(t-h)=x^TRx-x_h^TRx_h.$$

By some elementary calculations and Lemma 1, it is clear that

$$Q^TQ=\|Q\|^2\le\beta_0^2\|x\|^2+\beta_1^2\|x_h\|^2+\beta_2^2\|\dot{x}_h\|^2+2\beta_0\beta_1\|x\|\ \|x_h\|$$

$$
\begin{aligned}
&+2\beta_1\beta_2\|x_h\|\ \|\dot{x}_h\| + 2\beta_0\beta_2\|x\|\ \|\dot{x}_h\| \\
&\le \beta_0^2\|x\|^2 + \beta_1^2\|x_h\|^2 + \beta_2^2\|\dot{x}_h\|^2 + \beta_0^2\|x\|^2 + \beta_1^2\|x_h\|^2 \\
&\quad + \beta_1^2\|x_h\|^2 + \beta_2^2\|\dot{x}_h\|^2 + \beta_0^2\|x\|^2 + \beta_2^2\|\dot{x}_h\|^2 \\
&= 3\beta_0^2\|x\|^2 + 3\beta_1^2\|x_h\|^2 + 3\beta_2^2\|\dot{x}_h\|^2 \qquad (3)
\end{aligned}
$$

and

$$
\begin{aligned}
2x^T PQ &= Q^T Px + x^T PQ \le \varepsilon_4 Q^T Q + \varepsilon_4^{-1} x^T PPx \\
&\le \varepsilon_4^{-1} x^T PPx + \varepsilon_4(3\beta_0 x^T x + 3\beta_1^2 x_h^T x_h + 3\beta_2^2 \dot{x}_h^T \dot{x}_h). \qquad (4)
\end{aligned}
$$

Using the inequalities (3) and (4), we derive that

$$
\begin{aligned}
\dot{V} &\le x^T[A^T(t)P + PA(t)]x + 2x^T PBx_h + 2x_h{}^T PC\dot{x}_h + \varepsilon_4^{-1}x^T PPx \\
&\quad + \varepsilon_4(3\beta_0^2 x^T x + 3\beta_1^2 x_h^T x_h + 3\beta_2^2 \dot{x}_h^T \dot{x}_h) \\
&\quad + 2x^T P\int_{t-h}^{t} K(t,s)x(s)ds + x^T Rx - x_h^T Rx_h. \\
&= x^T[A^T(t)P + PA(t) + \varepsilon_4^{-1}PP + 3\beta_0^2\varepsilon_4 + R]x \\
&\quad + 2x^T PBx_h + 2x^T P\int_{t-h}^{t} K(t,s)x(s)ds \\
&\quad + x_h^T[3\beta_1^2\varepsilon_4 - R]x_h + 2x_h^T PC\dot{x}_h + 3\beta_2^2\dot{x}_h^T\dot{x}_h.
\end{aligned}
$$

We can now arrange the last inequality in the matrix form as the following:

$$
\dot{V} \le \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix}^T
\times \begin{bmatrix} A^T(t)P + PA(t) + \varepsilon_4^{-1}PP + 3\beta_0^2\varepsilon_4 + R & PB & 0 & P \\ B^T P & 3\beta_0^2\varepsilon_4 - R & PC & 0 \\ 0 & C^T P & 3\beta_2^2\varepsilon_4 & 0 \\ P & 0 & 0 & 0 \end{bmatrix}
\times \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix}
$$

$$= \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix}^T M \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix}.$$

Since the matrix M is negative definite, hence the derivative $\dot{V}$ is negative definite. This result completes the proof of Theorem 1.

We now give the second main result of this paper.

Theorem 2.
We assume that the following condition is satisfied:

($A3$) There exist $n \times n$ symmetric positive definite matrices P, R and positive scalars β_0, β_1, β_2, $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_4 such the following LMI holds:

$$M = \begin{bmatrix} \Sigma & 0 & 0 & P \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_2^{-1}C^T C + 3\beta_2^2(\varepsilon_5^{-1}+1) & 0 \\ P & 0 & 0 & 1 \end{bmatrix} < 0,$$

where

$$\Sigma = A_0^T P + PA_0 + hA^T(t)A(t) + 4hPBB^T P + (\varepsilon_2 + \varepsilon_5)PP + 2\varepsilon_1 PBB^T P$$
$$+ 3\beta_0^2(1+\varepsilon_5^{-1}) + 3\beta_1^2(1+\varepsilon_5^{-1}) + hB^T B + 2\varepsilon_1^{-1}C^T C.$$

Then, the zero solution of system (2) is asymptotical stable for any constant time-delay $h > 0$.

Proof:

Without loss of generality, it is assumed that $x(t)$ is continuously differentiable on the interval $[-2h, -h]$. Let $A_0 = A(t) + B$. Then, system (2) can be arranged as following:

$$\dot{x}(t) = (A(t)+B)x(t) - B\int_{t-h}^{t} \dot{x}(s)ds + C\dot{x}(t-h) + Q((x(t), x(t-h), \dot{x}(t-h)) + \int_{t-h}^{t} K(t,s)x(s)ds$$
$$= A_0 x(t) - B\int_{t-h}^{t} \{A(s)\,x(s) + Bx(s-h) + C\dot{x}(s-h) + Q((x(t), x(t-h), \dot{x}(t-h))$$
$$+ \int_{s-h}^{s} K(s,v)x(v)dv\} ds + C\dot{x}(t-h) + Q((x(t), x(t-h), \dot{x}(t-h)) + \int_{t-h}^{t} K(t,s)x(s)ds.$$

Let

$$\eta_1 = \int_{t-h}^{t} A(s)x(s)ds,\ \eta_2 = \int_{t-h}^{t} Bx(s-h)ds,$$

$$\eta_3 = \int_{t-h}^{t} Q(x(s),x(s-h),\dot{x}(s-h))ds,\ \eta_4 = \int_{t-h}^{t}\int_{s-h}^{s} K(s,v))x(v)dvds.$$

Hence, we have

$$\dot{x}(t) = A_0x(t) - B(\eta_1+\eta_2+\eta_3+\eta_4) - BC[x(t-h)-x(t-2h)] + C\dot{x}(t-h)$$
$$+Q(x(t),x(t-h),\dot{x}(t-h)) + \int_{t-h}^{t} K(t,s)x(s)ds. \tag{5}$$

Consider the following candidate Lyapunov functional $V_1 = V_1(t,x_t)$ for the system (5):

$$V_1 = x^T(t)Px(t) + \int_{-h}^{0} x^T(t+s)R_1x(t+s)ds + \int_{-2h}^{0} x^T(t+s)R_2x(t+s)ds$$
$$+\int_{-h}^{0}\{\int_{t+\theta}^{t}\|A(s)x(s)\|^2 ds + \int_{t+\theta}^{t}\|Bx(s-h)\|^2 ds + \int_{t+\theta}^{t}\|Q(x(s),x(s-h),\dot{x}(s-h)\|^2 ds$$
$$+\int_{t+\theta}^{t}\int_{s-h}^{s}\|K(s,v)x(v)dv\|^2 d\,s\}\,d\theta, \tag{6}$$

where R_1 and R_2 are the positive semi-definite symmetric matrices.

For simplicity, define

$$Z_1 = \int_{-h}^{0} x^T(t+s)R_1x(t+s)ds, \tag{7}$$

$$Z_2 = \int_{-2h}^{0} x^T(t+s)R_2x(t+s)ds, \tag{8}$$

$$Z_3 = \int_{-h}^{0}\{\int_{t+\theta}^{t}\|A(s)x(s)\|^2 ds + \int_{t+\theta}^{t}\|Bx(s-h)\|^2 ds$$
$$+\int_{t+\theta}^{t}\|Q(x(s),x(s-h),\dot{x}(s-h)\|^2 ds \quad + \int_{t+\theta}^{t}\int_{s-h}^{s}\|K(s,v)x(v)dv\|^2 dsd\theta\}. \tag{9}$$

In view of (6), (7), (8) and (9), it follows that

$$V_1 = x^T(t)Px(t) + Z_1 + Z_2 + Z_3$$

and

$$\dot{V}_1 = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \dot{Z}_1 + \dot{Z}_2 + \dot{Z}_3$$
$$= [x^T A_0^T - (\eta_1+\eta_2+\eta_3+\eta_4)^T B^T - x_h^T C^T B^T + x_{2h}^T C^T B^T + \dot{x}_h^T C^T + Q^T$$

$$+\left(\int_{t-h}^{t}K(t,s)x(s)ds\right)^{T}]Px+x^{T}P[A_0x-B(\eta_1+\eta_2+\eta_3+\eta_4)-BCx_h+BCx_{2h}+C\dot{x}_h$$
$$+Q+\int_{t-h}^{t}K(t,s)x(s)ds]+\dot{Z}_2+\dot{Z}_3+\dot{Z}_4$$
$$=x^{T}(A_0^{T}P+PA_0)x-2x^{T}PB(\eta_1+\eta_2+\eta_3+\eta_4)-2x^{T}PBCx_h+2x^{T}PBCx_{2h}$$
$$+2x^{T}PC\dot{x}_h+2x^{T}PQ+2x^{T}P\int_{t-h}^{t}K(t,s)x(s)ds+\dot{Z}_1+\dot{Z}_2+\dot{Z}_3. \tag{10}$$

We now calculate $\dot{Z}_1, \dot{Z}_2$ and $\dot{Z}_3$. Hence, as for the next step, we derive that

$$\dot{Z}_1=x^{T}R_1x-x_h^{T}R_1x_h,$$
$$\dot{Z}_2=x^{T}R_2x-x_{2h}^{T}R_2x_{2h},$$
$$\dot{Z}_3=\int_{-h}^{0}\left(\|A(t)x(t)\|^2-\|A(t+\theta)x(t+\theta)\|^2\right)d\theta+\int_{-h}^{0}\left(\|Bx(t-h)\|^2-\|Bx(t+\theta-h)\|^2\right)d\theta$$
$$+\int_{-h}^{0}\left(\|Q(x(t),x(t-h),\dot{x}(t-h)\|^2-\|Q(x(t+\theta)x(t+\theta-h)\dot{x}(t+\theta-h)\|^2\right)d\theta$$
$$+\int_{-h}^{0}[\int_{t-h}^{t}\|K(t,v)x(v)dv\|^2d\theta-\int_{t+\theta-h}^{t+\theta}\|K(t+\theta,v)x(v)dv\|^2]d\theta$$
$$=h\left\{\|A(t)x(t)\|^2+\|Bx(t-h)\|^2+\|Q(x(t),x(t-h),\dot{x}(t-h)\|^2+\int_{t-h}^{t}\|K(t,s)x(s)\|^2ds\right\}$$
$$-\int_{t-h}^{t}\|A(s)x(s)\|^2ds-\int_{t-h}^{t}\|Bx(s-h)\|^2ds-\int_{t-h}^{t}\|Q(x(s),x(s-h),\dot{x}(s-h)\|^2ds$$
$$-\int_{t-h}^{t}\int_{\theta-h}^{\theta}\|K(\theta,s)x(s)\|^2dsd\theta.$$

By Lemma 1, the terms on the right-hand side of (10) satisfy the following inequalities:

$$-2x^{T}PB\eta_1\le hx^{T}PBB^{T}Px+h^{-1}\eta_1^{T}\eta_1,$$
$$-2x^{T}PB\eta_2\le hx^{T}PBB^{T}Px+h^{-1}\eta_2^{T}\eta_2,$$
$$-2x^{T}PB\eta_3\le hx^{T}PBB^{T}Px+h^{-1}\eta_3^{T}\eta_3,$$
$$-2x^{T}PB\eta_4\le hx^{T}PBB^{T}Px+h^{-1}\eta_4^{T}\eta_4,$$
$$-2x^{T}PBCx_h\le\varepsilon_1x^{T}PBB^{T}Px+\varepsilon_1^{-1}x_h^{T}C^{T}Cx_h,$$
$$2x^{T}PBCx_{2h}\le\varepsilon_1x^{T}PBB^{T}Px+\varepsilon_1^{-1}x_{2h}^{T}C^{T}Cx_{2h},$$
$$2x^{T}PC\dot{x}_h\le\varepsilon_2x^{T}PPx+\varepsilon_2^{-1}\dot{x}_h^{T}C^{T}C\dot{x}_h,$$
$$2x^{T}PQ\le\varepsilon_5x^{T}PPx+\varepsilon_5^{-1}Q^{T}Q=\varepsilon_5x^{T}PPx+\varepsilon_5^{-1}(3\beta_0^2x^{T}x+3\beta_1^2x_h^{T}x_h+3\beta_2^2\dot{x}_h^{T}\dot{x}_h),$$

$$h\left\|Q(x(t),x(t-h),\dot{x}(t-h))\right\|^2 \le h(3\beta_0^2 x^T x + 3\beta_1^2 x_h^T x_h + 3\beta_2^2 \dot{x}_h^T \dot{x}_h).$$

Additionally, we also have the following inequalities:

$$\eta_1^T\eta_1 = \left\|\int_{t-h}^{t} A(s)x(s)ds\right\|^2 \le \left(\int_{t-h}^{t}\left\|A(s)x(s)\right\|ds\right)^2 \le h\int_{t-h}^{t}\left\|A(s)x(s)\right\|^2 ds,$$

$$\eta_2^T\eta_2 = \left\|\int_{t-h}^{t} Bx(s-h)ds\right\|^2 \le \left(\int_{t-h}^{t}\left\|Bx(s-h)\right\|ds\right)^2 \le h\int_{t-h}^{t}\left\|Bx(s-h)\right\|^2 ds,$$

$$\eta_3^T\eta_3 = \left\|\int_{t-h}^{t} Q(x(s),x(s-h)\dot{x}(s-h)ds\right\|^2 \le \left(\int_{t-h}^{t}\left\|Q(x(s),x(s-h),\dot{x}(s-h))\right\|ds\right)^2$$
$$\le h\int_{t-h}^{t}\left\|Q(x(s),x(s-h),\dot{x}(s-h))\right\|^2 ds,$$

$$\eta_4^T\eta_4 = \left\|\int_{t-h}^{t}\int_{s-h}^{s} K(s,v)x(v)dvds\right\|^2 \le \left(\int_{t-h}^{t}\left\|\int_{s-h}^{s} K(s,v)x(v)dv\right\|ds\right)^2$$
$$\le h\int_{t-h}^{t}\left\|\int_{s-h}^{s} K(s,v)x(v)dv\right\|^2 ds$$

$$-2x^T PB(\eta_1+\eta_2+\eta_3+\eta_4) \le hx^T PBB^T Px + h^{-1}\eta_1^T\eta_1 + hx^T PBB^T Px + h^{-1}\eta_2^T\eta_2$$
$$+ hx^T PBB^T Px + h^{-1}\eta_3^T\eta_3 + hx^T PBB^T Px + h^{-1}\eta_4^T\eta_4$$
$$\le 4hx^T PBB^T Px + \int_{t-h}^{t}\left\|A(s)x(s)\right\|^2 ds + \int_{t-h}^{t}\left\|Bx(s-h)\right\|^2 ds$$
$$+ \int_{t-h}^{t}\left\|Q(x(s),x(s-h),\dot{x}(s-h))\right\|^2 ds + \int_{t-h}^{t}\left\|\int_{s-h}^{s} K(s,v)x(v)dv\right\|^2 ds.$$

Next, from $\dot{Z}_3$, it follows that

$$\dot{Z}_3 \le hx^T A^T(t)A(t)x + hx_h^T B^T Bx_h + 3\beta_0^2 x^T x + 3\beta_1^2 x_h^T x_h + 3\beta_2^2 \dot{x}_h^T \dot{x}_h$$
$$+\left(\int_{t-h}^{t} K(t,s)x(s)ds\right)^T \int_{t-h}^{t} K(t,s)x(s)ds - \int_{t-h}^{t}\left\|A(s)x(s)\right\|^2 ds$$
$$-\int_{t-h}^{t}\left\|Bx(s-h)\right\|^2 ds - \int_{t-h}^{t}\left\|Q(x(s),x(s-h),\dot{x}(s-h))\right\|^2 ds$$
$$-\int_{t-h}^{t}\left\|\int_{\theta-h}^{\theta} K(\theta,s)x(s)ds\right\|^2 d\theta.$$

When the above inequalities are combined, it is derived from (10) that

$$\begin{aligned}
\dot{V}_1 \le\ & x^T(A_0^T P + PA_0)x + 4hx^T PBB^T Px + \int_{t-h}^{t}\|A(s)x(s)\|^2 ds + \int_{t-h}^{t}\|Bx(s-h)\|^2 ds \\
& + \int_{t-h}^{t}\|Q(x(s),x(s-h),\dot{x}(s-h))\|^2 ds + \int_{t-h}^{t}\left\|\int_{s-h}^{s}K(s,v)x(v)dv\right\|^2 ds \\
& + \varepsilon_1 x^T PBB^T Px + \varepsilon_1^{-1}x_h^T C^T Cx_h + \varepsilon_1 x^T PBB^T Px + \varepsilon_1^{-1}x_{2h}^T C^T Cx_{2h} \\
& + \varepsilon_2 x^T P^T Px + \varepsilon_2^{-1}\dot{x}_h^T C^T C\dot{x}_h + \varepsilon_5 x^T PPx + \varepsilon_5^{-1}(3\beta_0^2 x^T x + 3\beta_1^2 x_h^T x_h + 3\beta_2^2 \dot{x}_h^T \dot{x}_h) \\
& + 2x^T P\int_{t-h}^{t}K(t,s)x(s)ds + x^T R_1 x - x_h^T R_1 x_h + x^T R_2 x - x_{2h}^T R_2 x_{2h} \\
& + hx^T A^T(t)A(t)x + hx_h^T B^T Bx_h + 3\beta_0 x^T x + 3\beta_1^2 x_h^T x_h + 3\beta_2^2 \dot{x}_h^T \dot{x}_h \\
& + \left(\int_{t-h}^{t}K(t,s)x(s)ds\right)^T \int_{t-h}^{t}K(t,s)x(s)ds - \int_{t-h}^{t}\|A(s)x(s)\|^2 ds \\
& - \int_{t-h}^{t}\|Bx(s-h)\|^2 ds - \int_{t-h}^{t}\|Q(x(s),x(s-h),\dot{x}(s-h))\|^2 ds - \int_{t-h}^{t}\left\|\int_{\theta-h}^{\theta}K(\theta,s)x(s)ds\right\|^2 d\theta \\
\le\ & x^T[A_0^T P + PA_0 + R_1 + R_2 + hA^T(t)A(t) + 4hPBB^T P \\
& + (\varepsilon_2+\varepsilon_5)PP + 2\varepsilon_1 PBB^T P + 3\beta_0^2(1+\varepsilon_5^{-1})]x \\
& + \dot{x}_h^T[\varepsilon_2^{-1}C^T C + 3\beta_2^2(\varepsilon_5^{-1}+1)]\dot{x}_h \\
& + x_h^T[hB^T B + \varepsilon_1^{-1}C^T C + 3\beta_1^2(\varepsilon_5^{-1}+1) - R_1]x_h \\
& + x_{2h}^T[\varepsilon_1^{-1}C^T C - R_2]x_{2h} + 2x^T P\int_{t-h}^{t}K(t,s)x(s)ds \\
& + \left(\int_{t-h}^{t}K(t,s)x(s)ds\right)^T \int_{t-h}^{t}K(t,s)x(s)ds.
\end{aligned}$$

Let

$$R_1 = hB^T B + \varepsilon_1^{-1}C^T C + 3\beta_1^2(\varepsilon_5^{-1}+1)$$

and

$$R_2 = \varepsilon_1^{-1}C^T C.$$

Then, we obtain

$$\begin{aligned}
\dot{V}_1 \le\ & x^T[A_0^T P + PA_0 + hA^T(t)A(t) + 4hPBB^T P + (\varepsilon_2+\varepsilon_5)PP + 2\varepsilon_1 PBB^T P \\
& + 3\beta_0^2(1+\varepsilon_5^{-1}) + 3\beta_1^2(1+\varepsilon_5^{-1}) + hB^T B + 2\varepsilon_1^{-1}C^T C]x \\
& + \dot{x}_h^T[\varepsilon_2^{-1}C^T C + 3\beta_2^2(\varepsilon_5^{-1}+1)]\dot{x}_h + 2x^T P\int_{t-h}^{t}K(t,s)x(s)ds \\
& + \left(\int_{t-h}^{t}K(t,s)x(s)ds\right)^T \int_{t-h}^{t}K(t,s)x(s)ds.
\end{aligned}$$

Hence, it follows that

$$\begin{aligned}\dot{V} &\le x^T[A_0^T P + PA_0 + hA^T(t)A(t) + 4hPBB^T P + (\varepsilon_2+\varepsilon_5)PP + 2\varepsilon_1 PBB^T P \\ &\quad + 3\beta_0^2(1+\varepsilon_5^{-1}) + 3\beta_1^2(1+\varepsilon_5^{-1}) + hB^T B + 2\varepsilon_1^{-1}C^T C]x \\ &\quad + x^T P\int_{t-h}^{t} K(t,s)x(s)ds + \dot{x}_h^T[\varepsilon_2^{-1}C^T C + 3\beta_2^2(\varepsilon_5^{-1}+1)]\dot{x}_h \\ &\quad + \left(\int_{t-h}^{t} K(t,s)x(s)ds\right)^T Px \\ &\quad + \left(\int_{t-h}^{t} K(t,s)x(s)ds\right)^T \int_{t-h}^{t} K(t,s)x(s)ds. \\ &\le \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix}^T \begin{bmatrix} \Sigma & 0 & 0 & P \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_2^{-1}C^T C + 3\beta_2^2(\varepsilon_5^{-1}+1) & 0 \\ P & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix} \\ &= \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix}^T M \begin{bmatrix} x \\ x_h \\ \dot{x}_h \\ \int_{t-h}^{t} K(t,s)x(s)ds \end{bmatrix},\end{aligned}$$

where

$$\begin{aligned}\Sigma &= A_0^T P + PA_0 + hA^T(t)A(t) + 4hPBB^T P + (\varepsilon_2+\varepsilon_5)PP + 2\varepsilon_1 PBB^T P \\ &\quad + 3\beta_0^2(1+\varepsilon_5^{-1}) + 3\beta_1^2(1+\varepsilon_5^{-1}) + hB^T B + 2\varepsilon_1^{-1}C^T C.\end{aligned}$$

Since the matrix M is negative definite, then it follows that $\dot{V}$ is negative definite. This result completes the proof of Theorem 2.

4. Conclusion

A nonlinear perturbed system of NDIDEs is considered. New and sufficient conditions for the asymptotical stability of zero solution of the perturbed system of NDIDEs are constructed by Theorems 1-2. The technique used in the proofs of Theorems 1-2, depends on two new LKFs. By the results of this paper, we did new contributions to theory of NDIDEs and obtained some former results under weaker conditions.

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