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Poonam Gupta
Hindu Girls College

Rajni Gupta
Hindu College

Sangeeta Malik
Baba Mastnath University

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Impatient Customers in Queueing System with Optional Vacation Policies and Power Saving Mode

[1]Poonam Gupta, [2,3,*]Rajni Gupta and [3]Sangeeta Malik

[1] Department of Mathematics
Hindu Girls College
Sonipat, India
poonammittal2207@gmail.com

[2] Department of Mathematics
Hindu College
Sonipat, India
goelrajni0101@gmail.com

[3] Department of Mathematics
Baba Mastnath University
Asthal Bohar, Rohtak, India
sangeetastat@gmail.com

*Corresponding author

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Abstract

In this manuscript, a queueing system with two optional vacation policies, power-saving mode under reneging and retention of reneged customers in both vacations is analyzed. If the server is free, it chooses either of the vacations, classical vacation or working vacation. During vacations, the customers may get impatient due to delays and may leave the system, but they are retained in the system with some convincing mechanisms. On vacation completion, if the system is empty, the server is turned off to facilitate better utilization of the resources. Some of the operating system characteristics are derived using the probability generating functions technique. The numerical results are graphically represented by using MATLAB software.

Keywords: Queueing; Vacations; Classical vacation; Working vacation; Bernoulli schedule; Setup time; Retention; Reneging

MSC 2010 No.: 60K25, 68M20, 90B22

1. Introduction

Queueing systems with server vacations have been extensively studied by many researchers in the last few decades due to their practical applications in many real-life systems such as telecommunication networks, inventory systems, etc. Vacations in the queueing system refer to the duration in which the server is either not available at all to serve the customers or serve them at a comparatively slower rate. The server takes vacations due to many reasons. It may be due to server breakdowns or some auxiliary works like maintenance, testing, etc., assigned to the server. Levy and Yechiali (1975) performed pioneer work on vacation queueing models. Doshi (1986) presented a good survey on queues with vacations. Depending on the situation, two types of vacations are broadly discussed in the literature. Classical vacations in which server does not provide service to customers at all and working vacation in which server continues to serve the customers but at a lower rate. Servi and Finn (2002) first analyzed queueing systems with working vacation policy. Later on, many authors contributed to the field. For such models, we refer the reader to Tian and Zhang (2006), Banik et al. (2007), Gupta and Kumar (2021a), and references therein. Vacation in a queueing system is a special case of a queue with delayed service. Some research works concerning the delay in service may be referred to in Haghighi et al. (2008, 2011, 2016) and references therein. Some authors Ibe and Isijola (2014), Zhang and Zhou (2017), Unni and Mary (2019) studied queues with differentiated vacation policies. Later on Gupta and Kumar (2021b) extended the differentiated vacation policy to retrial queueing systems.

The impatience of customers is an important feature, which needs to be included in the study of queueing systems to model practical situations more closely. Analyzing the need, the queueing systems with impatient behavior of customers have been studied by many researchers. The impatience of customers may lead to reneging or balking of customers, which results in potential loss hence adversely affect the organizations. The main reason for the loss of customers in queueing systems is generally the unavailability of servers either due to vacations or server failures. To compensate for the loss, organizations adopt some convincing methods to retain the customers in the system. Such models are also discussed in the literature. We may refer to Haghighi et al. (1986), Yechiali (2004), Altman and Yechiali (2006), Yue et al. (2012), Ammar (2015, 2017), Bouchentouf et al.(2020a, 2020b, 2021), Kadi et al. (2020), Gupta and Kumar (2021c) for the related works.

Power saving is another growing issue in today's scenario. An idle server consumes significant power so it is inevitable to turn off the server when it is not in use. However, a trade-off arises as the arriving customer has to wait till the server is reactivated and a setup cost is also incurred in such models. Some authors worked on queueing systems with setup times in the recent past to find a solution to the issue. Some of the related works can be found in Xu et al. (2009), Azhagappan and Deepa (2019), Manoharan and Jeeva (2020).

Despite a rich literature on queueing theory, there is no work available in the literature on optional vacation policy (Bernoulli vacation policy) with customers' impatience and power-saving mode. We have extended the work of Manoharan and Ashok (2018) on optional vacations by incorporating the impatience of customers and power-saving mode. The present paper deals with a single server queueing system under Bernoulli's vacation policy (two optional vacations), setup times and retention of customers on vacations. The model provides an option to the server to choose one of the vacations either classical or working of different durations with probabilities p or $\bar{p}$ depending on the requirement, every time it becomes free from the normal state. The model reduces to a queueing system with working vacation, reneging and setup times on taking $p = 0$. Further, if working vacation time also equals zero, it changes to queue with setup times. If p is taken as 1, it boils down to queue with classical vacation, reneging and retention of customers under setup times. The rest of the paper is organized as follows. Section 2 describes real-life application of the model. The mathematical description of the model is presented in Section 3. Differential –difference equations for the proposed model are discussed in Section 4. Balance equations and steady-state probabilities are obtained in Section 5. Some operating characteristics of the system are obtained in section 6. The graphical behavior of operating characteristics of the system is shown in Section 7. Section 8 concludes the paper.

2. Real-life application of the model

Consider a flour mill or spice mill with an operator. As long as customers arrive for service, they are provided service in order of their arrivals. When the operator gets free after serving all the customers, he may get engaged in some secondary tasks. During this period, the arriving customers may get service depending on the availability of the assistant. Of course, the assistant will serve the customers at a comparatively low rate than the operator. In absence of the operator, due to slow or no service, customers may lose patience and leave without being served. To save power, the machine is turned off when not in use. If any of the customers are not satisfied with the service then he/she may rejoin to get satisfactory service. In queueing terminology, operator, availability of assistant, non-availability of assistant, power off, rejoining of customers corresponds to sever, working vacation, classical vacation, closed down state, feedback respectively.

3. Mathematical description of the model

The model is assumed to satisfy the following assumptions:

1. The customers arrive according to the Poisson process with a mean arrival rate of $1/\lambda$.
2. The server serves the customers on an FCFS basis with exponentially distributed service time. The mean service rate is taken as $1/\mu$ in the normal/active state of the server.
3. After serving all the customers, the server may opt to go on classical or working vacations with probabilities p or $\bar{p}$ respectively from the active state.
4. In classical vacation, the server stays idle whereas, in working vacation, the customers are served but at a slower rate $\mu_v < \mu$. The duration of both the vacations is assumed to be exponentially distributed with different parameters θ_1 and θ_2 respectively in working and classical vacations.
5. The customers may get impatient due to long wait and activate impatient timers T_1 and T_2 in working and classical vacations respectively. These impatience timers are assumed to be exponentially distributed with rates ϕ_1 and ϕ_2 respectively. The reneging customers may be retained in the system by some convincing mechanisms with probabilities $\overline{q_1}$ and $\overline{q_2}$ respectively.
6. If there is no waiting customer in the system, the server is turned off on vacation completion instant, i.e., enters in closed downstate.

7. The customer arriving during the off state of the server; activates the server. The setup time is assumed to follow an exponential distribution with parameter η. The customers arriving during the setup process will have to wait in the queue for their turn.
8. The inter-arrival times, service times, reneging times and setup times are all assumed to be identically and independently distributed.

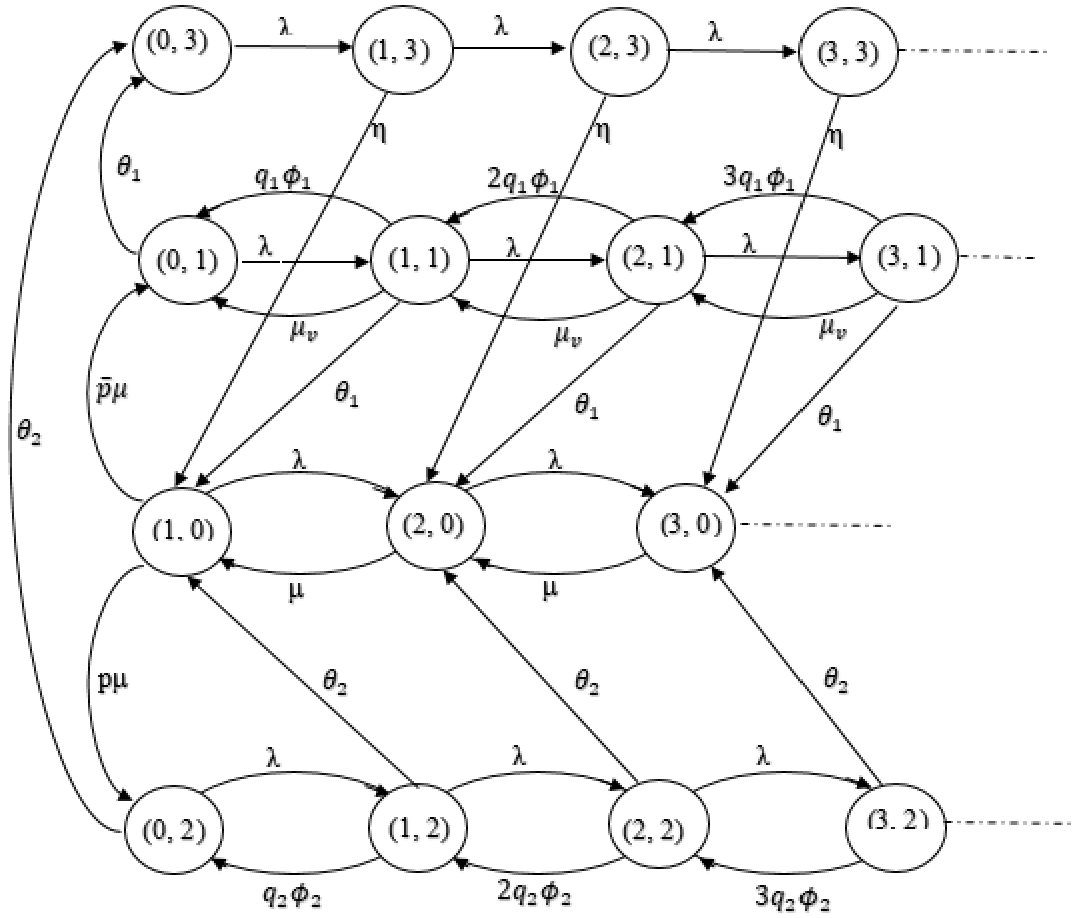

Figure 1. Transition state diagram of the system

4. Differential difference equations

Denoting the number of customers in the system and the state of the server at time t by N(t) and S(t) respectively, we observe that {N(t), S(t)} is a continuous Markov chain. The different possible states of the server are given as follows:

$$S(t) = \begin{cases} 0, & server\ in\ normal/active\ state, \\ 1, & server\ in\ working\ vacation\ state, \\ 2, & server\ in\ classical\ vacation\ state, \\ 3, & server\ in\ closed\ down/setup\ state. \end{cases}$$

Taking $p_{n\,i}(t)$ as the probability of the system being in i^{th} server state with n customers at time and using the Markov process, the differential-difference equations for the proposed quasi birth-death model are

$$\frac{d}{dt}p_{1\,0}(t) = \mu p_{2\,0}(t) + \theta_2 p_{1\,2}(t) + \theta_1 p_{1\,1}(t) + \eta p_{1\,3}(t) - (\lambda+\mu)p_{1\,0}(t)\,, \tag{1}$$

$$\frac{d}{dt}p_{n\,0}(t) = \mu p_{n+1\,0}(t) + \theta_2 p_{n\,2}(t) + \theta_1 p_{n\,1}(t) + \eta p_{n\,3}(t) + \lambda p_{n-1\,0}(t) - (\lambda+\mu)p_{n\,0}(t)\,, \qquad n \geq 2, \tag{2}$$

$$\frac{d}{dt}p_{0\,1}(t) = (\mu_v + q_1\phi_1)p_{1\,1}(t) + \bar{p}\mu p_{1\,0}(t) - (\lambda+\theta_1)p_{0\,1}(t), \tag{3}$$

$$\frac{d}{dt}p_{n\,1}(t) = \lambda p_{n-1\,1}(t) + \mu_v p_{n+1\,1}(t) + (n+1)q_1\phi_1 p_{n+1\,1}(t) - (\lambda+\theta_1+\mu_v+nq_1\phi_1)p_{n\,1}(t)\,, \qquad n \geq 1, \tag{4}$$

$$\frac{d}{dt}p_{0\,2}(t) = q_2\phi_2 p_{1\,2}(t) + p\mu p_{1\,0}(t) - (\lambda+\theta_2)p_{0\,2}(t), \tag{5}$$

$$\frac{d}{dt}p_{n\,2}(t) = \lambda p_{n-1\,2}(t) + (n+1)q_2\phi_2 p_{n+1\,2}(t) - (\lambda+\theta_2+nq_2\phi_2)p_{n\,2}(t), \qquad n \geq 1, \tag{6}$$

$$\frac{d}{dt}p_{0\,3}(t) = \theta_1 p_{0\,1}(t) + \theta_2 p_{0\,2}(t) - \lambda p_{0\,3}(t)\,, \tag{7}$$

$$\frac{d}{dt}p_{n\,3}(t) = \lambda p_{n-1\,3}(t) - (\lambda+\eta)p_{n\,3}(t). \qquad n \geq 1, \tag{8}$$

5. Balance equations and stationary probabilities

Under stability condition $\lambda < \mu$, taking limit $t \to \infty$, we have

$$\left.\begin{array}{l} \lim_{t\to\infty} p_{n\,i}(\mathrm{t}) = p_{n\,i} \\ \frac{d}{dt}p_{n\,i}(t) = 0 \end{array}\right\}$$

Thus, the balance equations are as follows

$$(\lambda+\mu)p_{1\,0} = \mu p_{2\,0} + \theta_2 p_{1\,2} + \theta_1 p_{1\,1} + \eta p_{1\,3,} \tag{9}$$

$$(\lambda+\mu)p_{n\,0} = \mu p_{n+1\,0} + \theta_2 p_{n\,2} + \theta_1 p_{n\,1} + \eta p_{n\,3} + \lambda p_{n-1\,0}, \quad n \geq 2, \tag{10}$$

$$(\lambda+\theta_1)p_{0\,1} = (\mu_v + q_1\phi_1)p_{1\,1} + \bar{p}\mu p_{1\,0}, \tag{11}$$

$$(\lambda+\theta_1+\mu_v+nq_1\phi_1)p_{n\,1} = \lambda p_{n-1\,1} + \mu_v p_{n+1\,1} + (n+1)q_1\phi_1 p_{n+1\,1}, \; n \geq 1, \tag{12}$$

$$(\lambda+\theta_2)p_{0\,2} = q_2\phi_2 p_{1\,2} + p\mu p_{1\,0}, \tag{13}$$

$$(\lambda+\theta_2+nq_2\phi_2)p_{n\,2} = \lambda p_{n-1\,2} + (n+1)q_2\phi_2 p_{n+1\,2}, \; n \geq 1, \tag{14}$$

$$\lambda p_{0\,3} = \theta_1 p_{0\,1} + \theta_2 p_{0\,2}\,, \tag{15}$$

$$(\lambda+\eta)p_{n\,3} = \lambda p_{n-1\,3}, \qquad n \geq 1. \tag{16}$$

Taking probability generating functions of the number of customers in the system as

$$H_i(z) = \sum_{n=0}^{\infty} p_{n\,i}\, z^n , \qquad i = 1, 2, 3 , \tag{17}$$

$$H_0(z) = \sum_{n=1}^{\infty} p_{n\,0} z^n . \tag{18}$$

Multiplying equations (9) and (10) with appropriate powers of z, summing over all possible values of n and using equation (18), we get

$$\left(\lambda + \mu - \lambda z - \frac{\mu}{z}\right) H_0(z) - \theta_1 H_1(z) - \theta_2 H_2(z) - \eta H_3(z) \\ = -\mu p_{1\,0} - \theta_1 p_{0\,1} - \theta_2 p_{0\,2} - \eta p_{0\,3} . \tag{19}$$

Similarly, equations (11), (12) yields

$$q_1\phi_1(1-z)H_1'(z) + \left(-\lambda(1-z) + \mu_v\left(\frac{1-z}{z}\right) - \theta_1\right) H_1(z) \\ = -\bar{p}\mu p_{1\,0} + \mu_v\left(\frac{1-z}{z}\right) p_{0\,1}. \tag{20}$$

Multiplying equations (13) and (14) with appropriate power of z and summing over all values of n and using equation (17) simultaneously, we get

$$(1-z)q_2\phi_2 H_2'(z) + (\lambda z - \lambda - \theta_2)H_2(z) = -p\mu p_{1\,0}. \tag{21}$$

Similarly multiplying equation (15) and (16) by z^n and adding over all possible values of n and using probability generating functions

$$(\lambda + \eta - \lambda z)H_3(z) = (\lambda + \eta)p_{0\,3.} \tag{22}$$

Equation (20) can be re-written as

$$H_1'(z) + \left(-\frac{\lambda}{q_1\phi_1} + \frac{\mu_v}{q_1\phi_1 z} - \frac{\theta_1}{q_1\phi_1(1-z)}\right) H_1(z) = -\frac{A_1}{1-z} + \frac{A_2}{z}, \tag{23}$$

where

$$A_1 = \frac{p\mu}{q_1\phi_1} p_{1\,0} , \tag{24}$$

$$A_2 = \frac{\mu_v}{q_1\phi_1} p_{0\,1}. \tag{25}$$

Solving this differential equation, we obtain

$$H_1(z) = e^{\frac{\lambda z}{q_1\phi_1}} z^{-\frac{\mu_v}{q_1\phi_1}} (1-z)^{-\frac{\theta_1}{q_1\phi_1}} \left(-A_1 B_1(z) + A_2 B_2(z)\right), \tag{26}$$

where,

$$B_1(z) = \int_0^z e^{\frac{-\lambda z}{q_1\phi_1}} z^{\frac{\mu_v}{q_1\phi_1}} (1-z)^{\frac{\theta_1}{q_1\phi_1}-1} dz , \tag{27}$$

$$B_2(z) = \int_0^z e^{\frac{-\lambda z}{q_1\phi_1}} z^{\frac{\mu_v}{q_1\phi_1}-1}(1-z)^{\frac{\theta_1}{q_1\phi_1}} dz\,. \tag{28}$$

Solving the differential equation (21), we obtain

$$H_2(z) = \frac{-e^{\frac{\lambda z}{q_2\phi_2}}(1-z)^{\frac{-\theta 2}{q_2\phi_2}}}{q_2\phi_2}\int_0^z e^{\frac{-\lambda z}{q_2\phi_2}}(1-z)^{\frac{\theta 2}{q_2\phi_2}-1} p\mu p_{1\,0} dz\,. \tag{29}$$

Taking limit $z \to 1$ in equation (19), we obtain

$$\mu p_{1\,0} + \theta_1 p_{0\,1} + \theta_2 p_{0\,2} + \eta p_{0\,3} = \theta_1 H_1(1) + \theta_2 H_2(1) + \eta H_3(1)\,. \tag{30}$$

Using equations (19) and (30) together, we get

$$H_0(z) = \frac{\theta_1 H_1(z)+\theta_2 H_2(z) + \eta H_3(z) - (\theta_1 H_1(1) + \theta_2 H_2(1) + \eta H_3(1))}{(1-z)(\lambda - \frac{\mu}{z})}\,. \tag{31}$$

Taking limit $z \to 1$ in equation (22), we get

$$H_3(1) = \frac{\lambda + \eta}{\eta} p_{0\,3}\,. \tag{32}$$

We see that on taking limit $z \to 1$ in equation (26), the denominator tends to zero so, the numerator must also tend to zero, i.e., we must have

$$-A_1 B_1(1) + A_2 B_2(1) = 0. \tag{33}$$

Equation (33) gives,

$$p_{0\,1} = \frac{p\mu B_1(1)}{\mu_v B_2(1)} p_{1\,0}. \tag{34}$$

Further, adding equations (9), (10), (12), (14), (16) and using recursion, we obtain

$$\begin{aligned}\lambda p_{n\,0} + \lambda p_{n\,1} + \lambda p_{n\,2} + \lambda p_{n\,3} - \mu p_{n+1\,0} - \mu_v p_{n+1\,1} - (n+1) q_2\phi_2 p_{n+1\,2} \\ - (n+1) q_1\phi_1 p_{n+1\,1} \\ = -\mu p_{1\,0} + \lambda p_{0\,2} - q_2\phi_2 p_{1\,2} + \lambda p_{0\,1} - (\mu_v + q_1\phi_1) p_{1\,1} + \lambda p_{0\,3}\end{aligned} \tag{35}$$

Using equations (11), (13) and (15) in the above equation, we get

$$\begin{aligned}\lambda p_{n\,0} + \lambda p_{n\,1} + \lambda p_{n\,2} + \lambda p_{n\,3} - \mu p_{n+1\,0} - \mu_v p_{n+1\,1} - (n+1) q_2\phi_2 p_{n+1\,2} \\ - (n+1) q_1\phi_1 p_{n+1\,1} = 0\,.\end{aligned} \tag{36}$$

Summing over all possible values of n and using normalization condition, it changes to

$$\lambda - \mu H_0(1) - \mu_v H_1(1) - q_2\phi_2 H_2'(1) - q_1\phi_1 H_1'(1) + \mu_v p_{0\,1} = 0\,. \tag{37}$$

Taking limit $z \to 1$ in equation (21), we get

$$H_2(1) = \frac{p\mu}{\theta_2} p_{1\,0}. \tag{38}$$

From equation (30), we have

$$\mu p_{1\,0} = \theta_1 H_1(1) + \theta_2 H_2(1), \tag{39}$$

$$H_1(1) = \frac{\mu p_{1\,0} - \theta_2 H_2(1)}{\theta_1}. \tag{40}$$

Now, taking limit $z \to 1$ in equation (31) and using L- Hospital rule, we get

$$H_0(1) = \frac{\theta_1 H_1'(1) + \theta_2 H_2'(1) + \eta H_3'(1)}{\mu - \lambda}. \tag{41}$$

To obtain the values of $H_1'(1)$ *and* $H_2'(1)$, differentiate equations (20) and (21) and taking limit $z \to 1$,

$$H_1'(1) = \frac{\mu_v p_{0\,1} + (\lambda - \mu_v) H_1(1)}{\theta_1 + q_1 \phi_1}, \tag{42}$$

$$H_2'(1) = \frac{\lambda}{\theta_2 + q_2 \phi_2} H_2(1). \tag{43}$$

Similarly, differentiating equation (22) and taking a limit, we get

$$H_3'(1) = \frac{\lambda(\lambda + \eta)}{\eta^2} p_{0\,3}. \tag{44}$$

On similar steps, differentiating equation (31), we get

$$H_0'(1) = \frac{(\theta_1 H_1''(1) + \theta_2 H_2''(1) + \eta H_3''(1))(\mu - \lambda) + 2\mu(\theta_1 H_1'(1) + \theta_2 H_2'(1) + \eta H_3'(1))}{2(\mu - \lambda)^2}, \tag{45}$$

where $H_2''(1)$ is obtained by differentiating equation (21) twice and taking limit $z \to 1$,

$$H_2''(1) = \frac{2\lambda\, H_2'(1)}{\theta_2 + 2 q_2 \phi_2}. \tag{46}$$

Similarly, differentiating equation (20) twice and taking a limit, we obtain

$$H_1''(1) = \frac{-2\mu_v p_{0\,1} + 2(\lambda - \mu_v) H_1'(1) + 2\mu_v H_1(1)}{\theta_1 + 2 q_1 \phi_1}. \tag{47}$$

As we see from the above equations, all the probability generating functions are expressed in terms of $p_{0\,1}, p_{0\,3}$ and $p_{1\,0}$.

To find the values of $p_{0\,1}, p_{0\,3}$ and $p_{1\,0}$,

Simple mathematical calculations in equation (37) and using eq. (34) gives the following relation between $p_{0\,3}$ and $p_{1\,0}$

$$p_{0\,3} = \frac{\eta(\lambda + D p_{1\,0})}{\lambda\mu(\lambda + \eta)}, \tag{48}$$

where,

$$D = \frac{(1 - C_1)p\mu B_1(1)}{B_2(1)} - \frac{\bar{p}\mu C_1(\lambda - \mu_v)}{\theta_1} - \frac{C_2 p\mu}{\theta_2} - \frac{\mu\mu_v}{\theta_1}, \tag{49}$$

$$C_1 = \frac{q_1\phi_1 + \frac{\mu\theta_1}{\mu - \lambda}}{\theta_1 + q_1\phi_1}, \tag{50}$$

$$C_2 = \frac{\lambda q_2\phi_2 + \frac{\mu\lambda\theta_2}{\mu - \lambda}}{\theta_2 + q_2\phi_2}. \tag{51}$$

Now, $p_{0\,3}$ and $p_{0\,1}$ are both explicitly expressed in terms of $p_{1\,0}$, which can be obtained by using normalization condition

$$\sum_{i=0}^{3} H_i(1) = 1. \tag{52}$$

6. System operating characteristics

Expected system length $EL_S = EL_0 + EL_1 + EL_2 + EL_3$, where, EL_i, $i = 0, 1, 2, 3$ denotes mean system length in i^{th} state of the server.

$$EL_S = \sum_{i=0}^{3}\sum_{n=1}^{\infty} np_{n\,i}$$

$$= \sum_{i=0}^{3} H_i'(1), \tag{53}$$

$H_0'(1)$, $H_1'(1)$, $H_2'(1)$, $H_3'(1)$ are obtained from equations (42) to (45).

Expected Sojourn time W_S = Mean waiting time experienced by customers in the system

$$= \frac{EL_S}{\lambda}. \tag{54}$$

Probability of server in the active/normal state $P_N = H_0(1)$. (55)

Probability of server in setup/closed down state $P_{CD} = H_3(1)$. (56)

Probability of server in working vacation state $P_{WV} = H_1(1)$. (57)

Probability of server in the classical vacation state $P_{CV} = H_2(1)$. (58)

Rate of abandonment in working vacation $R_{WV} = q_1\phi_1 H_1'(1)$. (59)

Rate of abandonment in classical vacation $R_{CV} = q_2\phi_2 H_2'(1)$. (60)

7. Numerical and graphical illustration

In this section, we study the graphical behavior of various system characteristics for different values of system parameters. The values of various parameters are taken as as $\lambda = 1.5$, $\mu = 4$,

$\mu_v = 3,\ p = 0.4, \theta_1 = 3,\ \theta_2 = 3.2,\ \phi_1 = 0.6,\ \phi_2 = 0.9,\ q_1 = 0.6,\ q_2 = 0.8,\ \eta = 4$ unless varied as shown in graphs.

(a) Sensitivity analysis

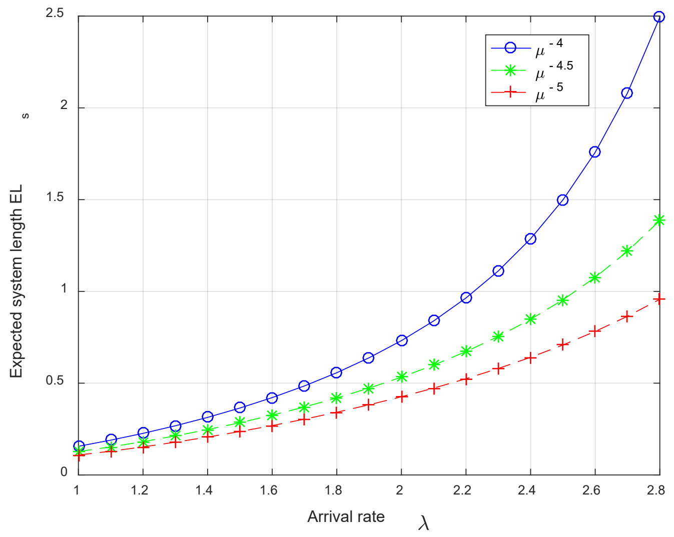

Figure 2. Expected system length EL_S versus λ for different values of μ

Figure 2 shows that with the increase in arrival rate λ, the mean system length increases, for a fixed value of service rate μ in the normal service state. It is due to a decrease in the mean inter-arrival time of customers. For fixed λ, as μ increases, the mean service time decreases resulting in a reduction in expected system length.

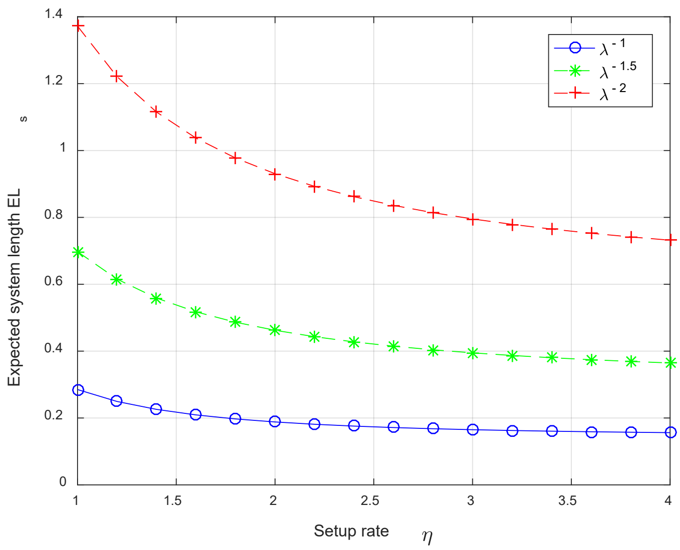

Figure 3. Expected system length EL_S versus η for different values of λ

We observe from Figure 3 the behavior of the expected system length with setup rate η for different values of arrival rate λ. We see that the expected system length decreases with an increase in setup rate η, for a fixed value of arrival rate λ. This decrease in mean system length with an increase in setup rate is theoretically expected due to the corresponding decrease in the mean setup time of the server. The figure depicts that with the increase in arrival rate, for a fixed setup rate, the mean system length goes on increasing.

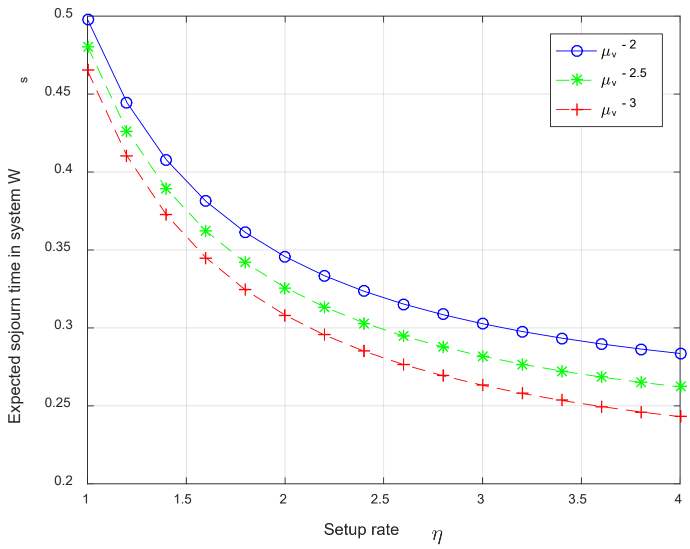

Figure 4. Expected sojourn time W_S versus η for different values of μ_v

From Figure 4, we see the variation in mean sojourn time experienced by customers in the system with setup rate η. For a fixed value of service rate in working vacation μ_v, expected sojourn time decreases with an increase in setup rate. Actually, with the increase in setup rate, the mean setup time decreases which leads to early return to the active state of the server thereby reducing the mean sojourn time of customers in the system. As the service rate in working vacation increases, again due to fast service, the mean sojourn time further decreases for a fixed setup rate.

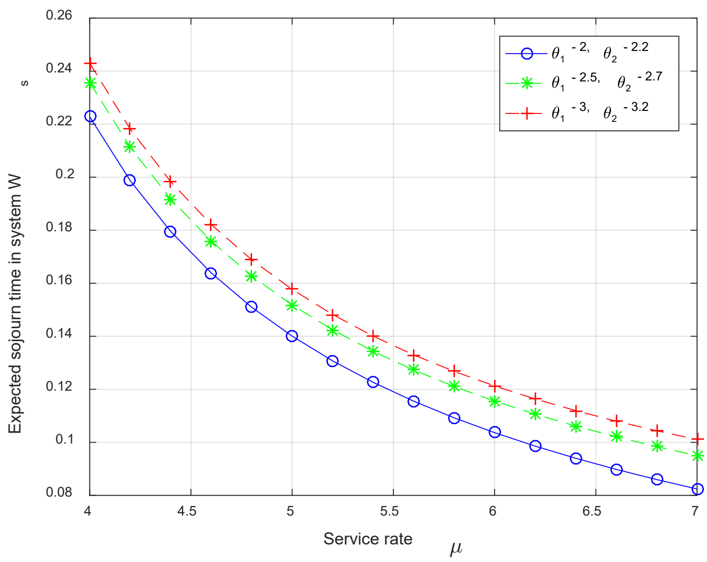

Figure 5. Effect of service rate μ on W_S for different θ_1 and θ_2

Figure 5 reveals the variation in mean waiting/sojourn time observed by customers in the system with service rate μ. As the service rate increases, the expected sojourn time decreases for fixed values of classical and working vacation rates θ_1 and θ_2. This decrease is due to a reduction in mean service time with an increase in mean service rate μ. For fixed service rate, the mean sojourn time of customers in the system increases with an increase in vacation rates θ_1 and θ_2.

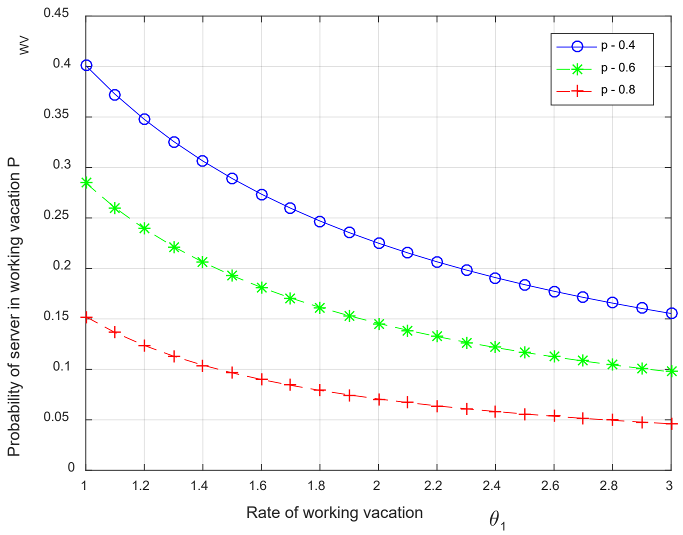

Figure 6. Effect of θ_1 on P_{WV} for different values of p

We observe from Figure 6 the variation in the probability of the server being in a working vacation state with the rate of working vacation θ_1. For a fixed value of probability p, as θ_1 increases, the mean duration of working vacation decreases resulting in a decrease in the probability of the server staying in a working vacation state.

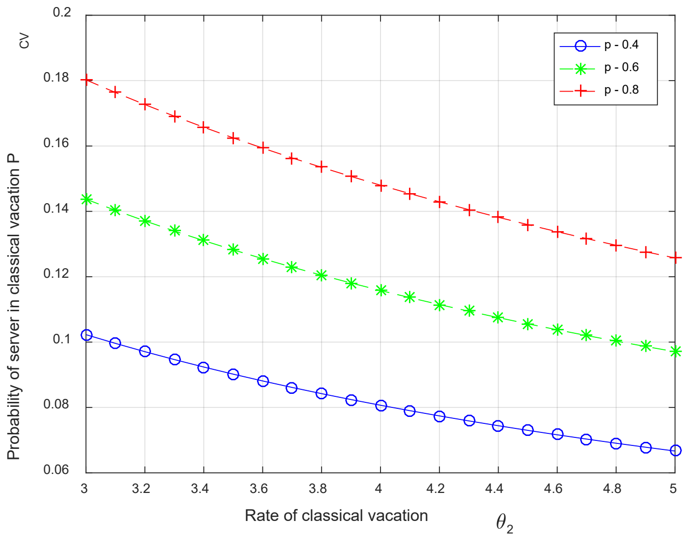

Figure 7. Effect of θ_2 on P_{CV} for different values of p

Figure 7 represents the variation in the probability of the server being on classical vacation with a rate of classical vacation θ_2. For a fixed value of p, as θ_2 increases, the classical vacation gets of shorter duration hence the probability of the server being in classical vacation reduces. For a particular value of θ_2, the probability of the server being in classical vacation increases with an increase in p.

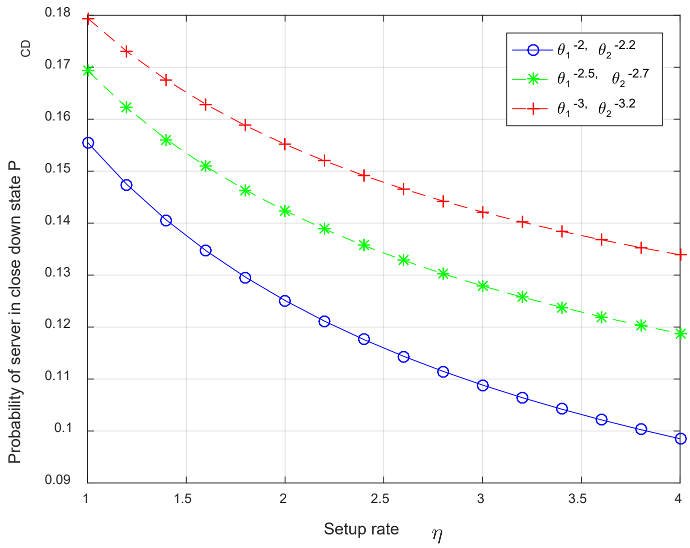

Figure 8. Variation in P_{CD} versus η for different θ_1 and θ_2

We observe from the above figure that for particular values of θ_1 and θ_2, the probability of the server being in a closed-down state reduces with an increase in the setup rate η. This is because as the setup rate increases, the time taken in the setup of the server reduces hence the probability of the server being in a closed-down state reduces. Further probability of the server being in a closed-down state increases with vacation rates θ_1 and θ_2. This is because as vacation rates increase, the duration of both types of vacations decreases hence the chances of the server going to the closed down state increases.

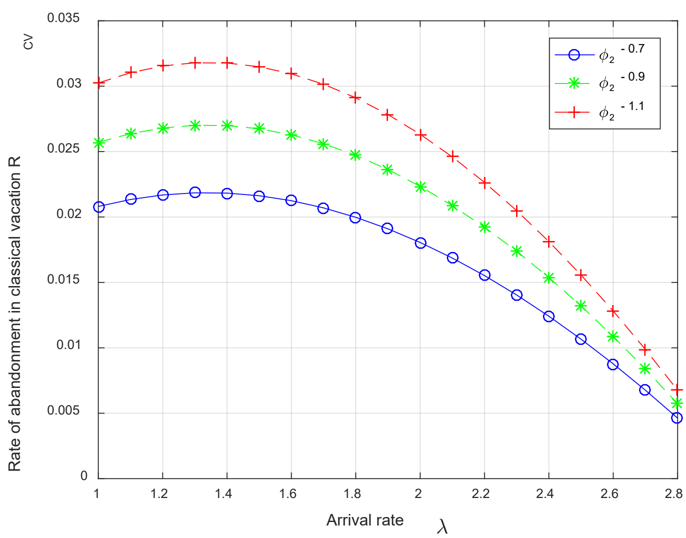

Figure 9. Effect of λ on R_{CV} for different values of ϕ_2

Figure 9 depicts the variation in abandonment rate during the classical vacation with arrival rate for different values of reneging rate in classical vacation. For a fixed value of arrival rate, the abandonment rate in classical vacation increases with an increase in reneging rate ϕ_2. As ϕ_2 increases, the mean impatience time T_2 in classical vacation decreases hence customers get impatient more quickly thereby increasing the corresponding rate of abandonment.

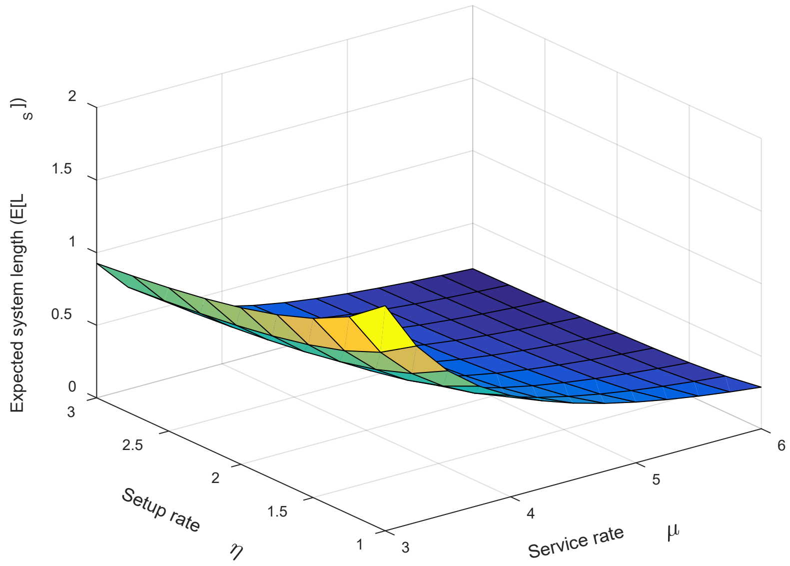

Figure 10. Effect of μ and η on mean system length EL_S

Figure 10 represents the variation in mean system length with setup rate and service rate in normal state simultaneously.

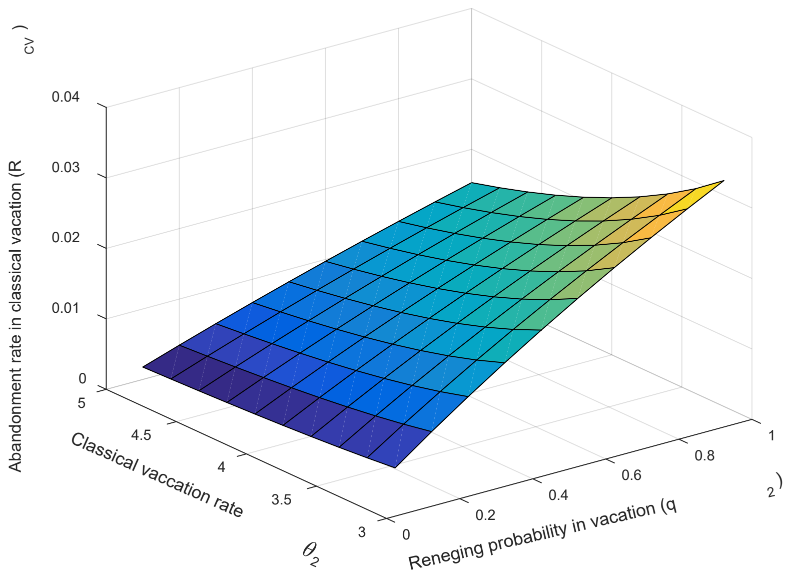

Figure 11. Effect of q_2 and θ_2 on abandonment rate R_{CV}

Figure 11 represents the variation in the rate of abandonment in classical vacation R_{CV} with the rate of classical vacation θ_2 and probability of reneging in classical vacation q_2 together.

(b) Cost optimization

The optimal cost concerning service rate μ is obtained via the quadratic fit approach. The different cost elements are fixed as $C_{L_S} = 25$, $C_\mu = 35$, $C_{\mu_v} = 32$, $C_{\theta_1} = 30$, $C_{\theta_2} = 20$, $C_\eta = 10$, for the purpose and operating cost function per unit time is taken as

$$F(\mu) = EL_S\, C_{L_S} + \mu C_\mu + \mu_v C_{\mu_v} + \theta_1 C_{\theta_1} + \theta_2 C_{\theta_2} + \eta C_\eta.$$

The optimal operating cost = 310.030501 is obtained corresponding to μ = 2.852213 with the permissible error of 10^{-5} as shown in Table 1. Figure 12 verifies the observed results.

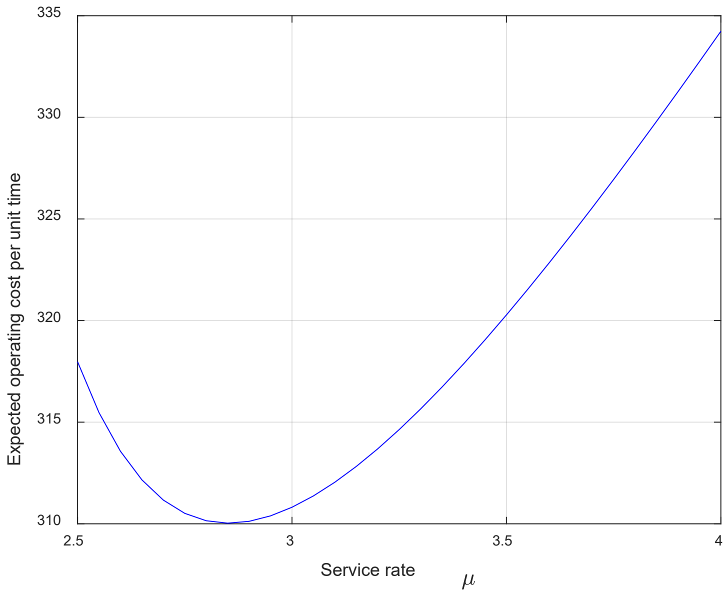

Figure 12. Expected operating cost per unit time versus μ

Table 1. Cost optimization via quadratic fit approach

x_1	x_2	x_3	$F(x_1)$	$F(x_2)$	$F(x_3)$	x_L
2.00000	2.20000	3.00000	452.364967	357.437682	310.813424	2.669989
2.20000	2.669989	3.00000	357.437682	311.719180	310.813424	2.846608
2.669989	2.846608	3.00000	311.719180	310.031809	310.813424	2.865909
2.846608	2.865909	3.00000	310.031809	310.038157	310.813424	2.851633
2.846608	2.851633	2.865909	310.031809	310.030515	310.038157	2.852255
2.851633	2.852255	2.865909	310.030515	310.030501	310.038157	2.852217
2.851633	2.852217	2.852255	310.030515	310.030501	310.030501	2.852213

8. Conclusion

A queueing system with Bernoulli vacation policies, retention of customers during vacations and setup times is studied. The probability generating functions of the number of customers in the system are derived in the active state, setup state, classical and working vacation states of the server. The results obtained in this study are of great importance for many real-life systems like manufacturing, inventory and other related ones. The effect of various parameters on some operating characteristics of the system like rate of abandonment, mean size of the system, mean sojourn times and different state probabilities of the server is numerically and graphically analyzed via MATLAB software. For future work, it will be interesting to analyze the model with multi-server and general service times.

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