Revised Variational Iteration Method for Solving Systems of Ordinary Differential Equations

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Abstract

A modification of the variational iteration method applied to systems of linear/non-linear ordinary differential equations, which yields a series solution with accelerated convergence, has been presented. Illustrative examples have been given.

Keywords: System of Ordinary Differential Equations, Revised Variational Iteration Method

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110

Applications and Applied Mathematics: An International Journal (AAM), Vol. 5 [2010], Iss. 3, Art. 10

AAM: Intern. J., Special Issue No. 1 (August 2010)   111

1. Introduction

In 1999, the variational iteration method (VIM) was proposed by He in He (2006). This method is now widely used by many researchers to study linear and non-linear partial differential equations. Numerous problems in Physics, Chemistry, Biology and Engineering science are modeled mathematically by systems of ordinary differential equations, e.g., series circuits, mechanical systems with several springs attached in series lead to a system of differential equations. In this work we consider He's variational iteration method as a well known method for finding both analytical and approximate solutions of systems of differential equations. This technique was developed by the Chinese mathematician He (2006).

The variational iteration method is used for solving autonomous ordinary differential system in He (2000). Application of this method to the Helmholtz equation is investigated in Momani et al. (2006). This method is used for solving Burgers and coupled Burgers equations in Abdou et al. (2005). In Abdou et al. (2005), the applications of the present method to coupled Schrodinger *KdV* equations and shallow water equations are provided. Most realistic differential equations do not have exact analytic solutions approximation and numerical techniques, therefore, are used extensively. This new iterative method has proven rather successful in dealing with both linear as well as non-linear problems, as it yields analytical solutions and offers certain advantages over standard numerical methods. Biazar et al (2004) have applied this method to a system of ordinary differential equations. In the present paper we use the revised variational iteration method to obtain solutions of systems of linear/ non-linear ordinary differential equations. We demonstrate that the series solution thus obtained converges faster relative to the series obtained by standard VIM. Several illustrative examples have been presented.

The present paper has been organized as follows. The well known He's variational iteration method is reviewed in Section 2. Section 3 deals with the analysis of VIM applied to a system of ordinary differential equations. In sections 4, we introduce revised VIM for systems of ordinary differential equations, respectively. Section 6 compares the revised VIM and standard VIM with illustrative examples. This is followed by the conclusions in Section 7.

2.   He's Variational Iteration Method

According to the variational iteration method, we consider the following differential equation:

$$Lu + Nu = g(t), \qquad\qquad 1$$

where L is a linear operator, N a non-linear operator and $g(t)$ is the source inhomogeneous term. According to the variational iteration method, we can construct a Correction functional as follow

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\tau)\{Lu_n(\tau) + N\widetilde{u}_n(\tau) - g(\tau)\}d\tau, \qquad\qquad 2$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory, the second term on the right is called the correction and $\tilde{u}_n$ is considered as a restricted variation, i.e., $\delta \tilde{u}_n = 0$.

3.   VIM for System of Ordinary Differential Equations

Consider the following system of ordinary differential equations:

$$L_1(u_1, u_2, ..., u_m) + N_1(u_1, u_2, ..., u_m) = g_1,$$
$$L_2(u_1, u_2, ..., u_m) + N_2(u_1, u_2, ..., u_m) = g_2,$$
$$\vdots$$
$$L_m(u_1, u_2, ..., u_m) + N_m(u_1, u_2, ..., u_m) = g_m.$$

where $L_1, L_2, ..., L_m$ are linear operators, $N_1, N_2, ..., N_m$ are non-linear operators, $u_m = u_m(x, y, t)$ and $g_m = g_m(x, y, t)$. We can construct a correction functional as follows:

$$u_{1,n+1} = u_{1,n} + \int_0^t \lambda_1 [L_1(u_{1,n}, u_{2,n}, ..., u_{m,n}) + N_1(\tilde{u}_{1,n}, \tilde{u}_{2,n}, ..., \tilde{u}_{m,n}) - g_1] d\tau,$$
$$u_{2,n+1} = u_{2,n} + \int_0^t \lambda_2 [L_2(u_{1,n}, u_{2,n}, ..., u_{m,n}) + N_2(\tilde{u}_{1,n}, \tilde{u}_{2,n}, ..., \tilde{u}_{m,n}) - g_2] d\tau,$$
$$u_{m,n+1} = u_{m,n} + \int_0^t \lambda_m [L_m(u_{1,n}, u_{2,n}, ..., u_{m,n}) + N_m(\tilde{u}_{1,n}, \tilde{u}_{2,n}, ..., \tilde{u}_{m,n}) - g_m] d\tau,$$

where $\lambda_1, \lambda_2, ..., \lambda_m$ are a general Lagrangian multiplies which can be identified optimally via the variational theory by using the stationary conditions, the subscript n denotes the nth-order approximation, $\tilde{u}_n$ is considered as a restricted variation, i.e., $\delta \tilde{u}_n = 0$.

4.   Revised VIM for a System of Ordinary Differential Equations

In this section we propose a modification of the Variational iteration, we rewrite equations in the form

$$L_i(u_i) + N_i(u_1, ..., u_m) = g_i(x), \qquad i = 1, 2, ..., m, \qquad\qquad 3$$

where L_i are linear operators, N_i are non-linear operators, in this case the functional are obtained as

$$u_{i,n+1} = u_{i,n} + \int_0^t \lambda_i [L_i(u_{i,n}(\tau) + N_i(\tilde{u}_{1,n}(\tau), ..., \tilde{u}_{m,n}(\tau)) - g_i(\tau)) d\tau, \qquad\qquad 4$$

where λ_i is a general Lagrange multipliers, $\delta u_{i,n+1} = 0$, $i = 1, 2, ..., m$,

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 5$$

$$u_{i,n+1} = u_{i,n} + \int_0^t \lambda_i [L_i(u_{i,n}(\tau)) + N_i(u_{1,n+1}(\tau), ..., u_{i-1,n+1}(\tau), u_{i,n}(\tau), ..., u_{m,n}(\tau)) - g_i(\tau)) d\tau,$$

Applications and Applied Mathematics: An International Journal (AAM), Vol. 5 [2010], Iss. 3, Art. 10

AAM: Intern. J., Special Issue No. 1 (August 2010) 113

for $i = 2, ..., m$. In fact the updated values $u_{1,n+1}, u_{2,n+1}, ..., u_{i-1,n+1}$ are used for finding $u_{i,n+1}$. This technique accelerates the convergence of the system of sequences.

5.   Illustrative Examples

To give a clear overview of the revised method, we present the following examples. We apply the revised VIM and compare the results with the standard VIM.

(i) Consider the following system of linear equations:

$$
\begin{aligned}
y_1' &= y_3 - \cos x, & y_1(0) &= 1, \\
y_2' &= y_3 - e^x, & y_2(0) &= 0, \\
y_3' &= y_1 - y_2, & y_3(0) &= 2.
\end{aligned}
$$

The correction functionals for above system read

$$ y_{1,n+1}(x) = y_{1,n}(x) + \int_0^x \lambda_1(s)\{y_{1,n}'(s) - y_{3,n}(s) + \cos s\}ds, $$

$$ y_{2,n+1}(x) = y_{2,n}(x) + \int_0^x \lambda_2(s)\{y_{2,n}'(s) - y_{3,n}(s) + \exp(s)\}ds, $$

$$ y_{3,n+1}(x) = y_{3,n}(x) + \int_0^x \lambda_3(s)\{y_{3,n}'(s) - y_{1,n}(s) + y_{2,n}(s)\}ds. $$

This yield the stationary conditions

$$ 1 + \lambda_1(s) = 0, \qquad \lambda_1'|_{s=x} = 0, $$

$$ 1 + \lambda_2(s) = 0, \qquad \lambda_2'|_{s=x} = 0, $$

$$ 1 + \lambda_3(s) = 0, \qquad \lambda_3'|_{s=x} = 0. $$

As a result we find

$$ \lambda_1(s) = \lambda_2(s) = \lambda_3(s) = -1. $$

Substituting these values of the Lagrange multipliers into the functionals above gives the iteration formulas

$$
\begin{aligned}
y_{1,n+1}(x) &= y_{1,n}(x) - \int_0^x \{y_{1,n}'(s) - y_{3,n}(s) + \cos s\}ds, \\
y_{2,n+1}(x) &= y_{2,n}(x) - \int_0^x \{y_{2,n}'(s) - y_{3,n}(s) + \exp(s)\}ds, \\
y_{3,n+1}(x) &= y_{3,n}(x) - \int_0^x \{y_{3,n}'(s) - y_{1,n}(s) + y_{2,n}(s)\}ds, \qquad n \geq 0.
\end{aligned}
$$

We get the exact solution $y_1 = e^x$, $y_2 = \sin x$ and $y_3 = e^x + \cos x$.. Accordingly, we obtain the following successive approximations:

$$y_{1,0} = 1,$$
$$y_{2,0} = 0,$$
$$y_{3,0} = 2,$$
$$y_{1,1} = 1 + 2x - \sin x,$$
$$y_{2,1} = 1 - e^x + 2x,$$
$$y_{3,1} = 2 + x,$$
$$y_{1,2} = 1 + 2x + \frac{x^2}{2} - \sin x,$$
$$y_{2,2} = 1 - e^x + 2x + \frac{x^2}{2},$$
$$y_{3,2} = e^x + \cos x,$$
$$y_{1,3} = e^x,$$
$$y_{2,3} = \sin x,$$
$$y_{3,3} = e^x + \cos x,$$
$$\vdots$$

The use of the modified method results in the sequences

$$\bar{y}_{1,n+1}(x) = \bar{y}_{1,n}(x) - \int_0^x \{\bar{y}'_{1,n}(s) - \bar{y}_{3,n}(s) + \cos s\}ds, \quad \bar{y}_1(0) = 1,$$
$$\bar{y}_{2,n+1}(x) = \bar{y}_{2,n+1}(x) \int_0^x \{\bar{y}'_{2,n}(s) - \bar{y}_{3,n}(s) + \exp(s)\}ds, \quad \bar{y}_2(0) = 0,$$
$$\bar{y}_{3,n+1}(x) = \bar{y}_{3,n}(x) - \int_0^x \{\bar{y}'_{3,n}(s) - \bar{y}_{1,n+1}(s) + \bar{y}_{2,n+1}(s)\}ds, \quad \bar{y}_3(0) = 2, n \geq 0.$$

We obtain the following successive approximations:

$$y_{1,0} = 1,$$
$$y_{2,0} = 0,$$
$$y_{3,0} = 2,$$
$$y_{1,1} = 1 + 2x - \sin x,$$
$$y_{2,1} = 1 - e^x + 2x,$$
$$y_{3,1} = e^x + \cos x,$$
$$y_{1,2} = e^x,$$
$$y_{2,2} = \sin x,$$
$$y_{3,2} = e^x + \cos x,$$
$$y_{1,3} = e^x,$$

Applications and Applied Mathematics: An International Journal (AAM), Vol. 5 [2010], Iss. 3, Art. 10

AAM: Intern. J., Special Issue No. 1 (August 2010) 115

$$y_{2,3} = \sin x,$$

$$y_{3,3} = e^x + \cos x,$$

$$\vdots$$

(ii) Consider the following system of non-linear differential equations:

$$y_1' = 2y_2^2, \qquad y_1(0) = 1,$$
$$y_2' = e^{-x}y_1, \qquad y_2(0) = 1,$$
$$y_3' = y_2 + y_3, \qquad y_3(0) = 0.$$

The correction functionals for above system read

$$y_{1,n+1}(x) = y_{1,n}(x) + \int_0^x \lambda_1(s)\{y_{1,n}'(s) - 2y_{2,n}^2(s)\}ds,$$

$$y_{2,n+1}(x) = y_{2,n}(x) + \int_0^x \lambda_2(s)\{y_{2,n}'(s) - \exp(-s)y_{1,n}(s)\}ds,$$

$$y_{3,n+1}(x) = y_{3,n}(x) + \int_0^x \lambda_3(s)\{y_{3,n}'(s) - y_{2,n}(s) - y_{3,n}(s)\}ds.$$

This yields the stationary conditions

$$1 + \lambda_1(s) = 0, \quad \lambda_1'|_{s=x} = 0,$$
$$1 + \lambda_2(s) = 0, \quad \lambda_2'|_{s=x} = 0,$$
$$1 + \lambda_3(s) = 0, \quad \lambda_3'|_{s=x} = 0.$$

As a result we find

$$\lambda_1(s) = \lambda_2(s) = \lambda_3(s) = -1.$$

Substituting these values of the Lagrange multipliers into the functionals above gives the iteration formulas

$$y_{1,n+1}(x) = y_{1,n}(x) - \int_0^x \{y_{1,n}'(s) - 2y_{2,n}^2(s)\}ds,$$

$$y_{2,n+1}(x) = y_{2,n}(x) - \int_0^x \{y_{2,n}'(s) - \exp(-s)y_{1,n}(s)\}ds,$$

$$y_{3,n+1}(x) = y_{3,n}(x) - \int_0^x \{y_{3,n}'(s) - y_{2,n}(s) - y_{3,n}(s)\}ds.$$

We get exact solution $y_1 = e^{2x}$, $y_2 = e^x$ and $y_3 = xe^x$. Accordingly, we obtain the following successive approximations:

$$y_{1,0} = 1,$$
$$y_{2,0} = 1,$$
$$y_{3,0} = 0,$$
$$y_{1,1} = 1 + 2x,$$
$$y_{2,1} = 2 - e^{-x},$$

$$y_{3,1} = x,$$

$$y_{1,2} = 1 + 2x + 2(-4 + 3x + e^{-x}(4 + \sinh x)),$$

$$y_{2,2} = 2 - e^{-x} + 2(1 - e^{-x}(1 + x)),$$

$$y_{3,2} = -1 + e^{-x} + 2x + \frac{x^2}{2},$$

$$y_{1,3} = 1 + 2x + 8(-7 + 3x) + e^{-2x}(-4(2 + x)^2) + 8e^x(9 + 4x) + 2(-4 + 3x + e^{-x}(4 + \sinh x)),$$

$$y_{2,3} = 2 - e^{-x} + \frac{1}{3}e^{-3x}(1 + e^x(-12 + e^x(3 + 8e^x - 18x))) + 2(1 - e^{-x}(1 + x)),$$

$$y_{3,3} = -4 + e^x + 3x + x^2 + \frac{x^3}{6} + e^{-x}(3 + 2x),$$

$$\vdots$$

The use of the modified method results in the sequences

$$\overline{y}_{1,n+1}(x) = \overline{y}_{1,n}(x) - \int_0^x \{\overline{y}'_{1,n}(s) - 2\overline{y}^2_{2,n}(s)\}ds, \qquad \overline{y}_1(0) = 1,$$

$$\overline{y}_{2,n+1}(x) = \overline{y}_{2,n}(x) - \int_0^x \{\overline{y}'_{2,n}(s) - \exp(-s)\overline{y}_{1,n+1}(s)\}ds, \quad \overline{y}_2(0) = 1,$$

$$\overline{y}_{3,n+1}(x) = \overline{y}_{3,n}(x) - \int_0^x \{\overline{y}'_{3,n}(s) - \overline{y}_{2,n+1}(s) - \overline{y}_{3,n}(s)\}ds, \quad \overline{y}_3(0) = 0, n \geq 0.$$

We obtain the following successive approximations:

$$y_{1,0} = 1,$$

$$y_{2,0} = 1,$$

$$y_{3,0} = 0,$$

$$y_{1,1} = 1 + 2x,$$

$$y_{2,1} = 1 + e^x(-3 + 3e^x - 2x),$$

$$y_{3,1} = -5 + 4x + e^{-x}(5 + 2x),$$

$$y_{1,2} = 1 + 2x + e^{-2x}(-17 - 16x - 4x^2 + 16e^x(5 + 2x) + e^{2x}(-63 + 30x)),$$

$$y_{2,2} = 1 + e^{-x}(-3 + 3e^x - 2x) + \frac{1}{27}e^{-3x}(209 + 196e^{3x} + e^{2x}(891 - 810x) + 168x$$

$$+ 36x^2 - 432e^x(3 + x)),$$

$$y_{3,2} = -5 + 4x + e^{-x}(5 + 2x) + \frac{1}{27}e^{-3x}(-91 - 64x - 12x^2 + 108e^x(7 + 2x)$$

$$+ 54e^{2x}(-5 + 14x) + e^{3x}(-395 + 61x + 54x^2)),$$

$$\vdots$$

State if for non-linear terms replace Adomian polynomial

$$\overline{y}_{1,n+1}(x) = \overline{y}_{1,n}(x) - \int_0^x \{\overline{y}'_{1,n}(s) - 2A_{2,n}(s)\}ds, \qquad \overline{y}_1(0) = 1,$$

$$\overline{y}_{2,n+1}(x) = \overline{y}_{2,n}(x) - \int_0^x \{\overline{y}'_{2,n}(s) - \exp(-s)\overline{y}_{1,n+1}(s)\}ds, \quad \overline{y}_2(0) = 1,$$

Applications and Applied Mathematics: An International Journal (AAM), Vol. 5 [2010], Iss. 3, Art. 10

AAM: Intern. J., Special Issue No. 1 (August 2010) 117

$$\bar{y}_{3,n+1}(x) = \bar{y}_{3,n}(x) - \int_0^x \{\bar{y}'_{3,n}(s) - \bar{y}_{2,n+1}(s) - \bar{y}_{3,n}(s)\}ds, \quad \bar{y}_3(0) = 0, n \geq 0.$$

where

$$A_{2,0} = y_{2,0}^2$$
$$A_{2,1} = 2y_{2,0}y_{2,1}$$
$$A_{2,2} = 2y_{2,0}y_{2,2} + y_{2,1}^2$$
$$A_{2,3} = 2y_{2,0}y_{2,3} + 2y_{2,1}y_{2,2}$$
$$\vdots$$

We obtain the following successive approximations:

$$y_{1,0} = 1,$$
$$y_{2,0} = 1,$$
$$y_{3,0} = 0,$$
$$y_{1,1} = 1 + 2x,$$
$$y_{2,1} = 1 + e^x(-3 + 3e^x - 2x),$$
$$y_{3,1} = -5 + 4x + e^{-x}(5 + 2x),$$
$$y_{1,2} = 1 + 2x + e^{-x}(20 + 8x + 2e^x(-10 + 7x)),$$
$$y_{2,2} = 7 + e^{-x}(-3 + 3e^x - 2x) + e^{-2x}(e^x(6 - 14x) - 4(3 + x)),$$
$$y_{3,2} = -13 + 5x + e^{-x}(5 + 2x) + e^{-2x}(7 + 2x + e^x(1 + 2x(6 + e^x x))),$$
$$y_{1,3} = 1 + 2x + e^{-x}(20 + 8x + 2e^x(-10 + 7x)) + e^{-2x}(11 - 8x - 4x^2 + 8e^x(14$$
$$\quad + 11x) + e^{2x}(-123 + 56x)),$$
$$y_{2,3} = 7 + e^{-x}(-3 + 3e^x - 2x) + e^{-2x}(e^x(6 - 14x) - 4(3 + x)) + \frac{1}{27}e^{-3x}$$
$$\quad (-67 + 364e^{3x} + 96x + 36x^2 - 54e^x(39 + 22x) - 27e^{2x}(-67 + 56x)),$$
$$y_{3,3} = -13 + 5x + e^{-x}(5 + 2x) + \frac{1}{54}e^{-3x}(18 - 80x - 24x^2 + 216e^{2x}(-6 + 11x)$$
$$\quad + 54e^x(46 + 21x) + e^{3x}(-1206 + 296x + 27x^2 + 36x^3)) + e^{-2x}$$
$$\quad (7 + 2x + e^x(1 + 2x(6 + e^x x))),$$
$$\vdots$$

We draw below graphs of $y_1(x), y_2(x)$ and $y_3(x)$ and compare the exact solution, solution given by the standard VIM method and solution given by the revised VIM method. In Figs 1,2 and 3, we put e^{2x}, e^x, xe^x, which are the exact solutions, y_1, y_2, y_3 denote solutions obtained by revised VIM and y_1^*, y_2^*, y_3^* denote solutions obtained by the standard VIM.
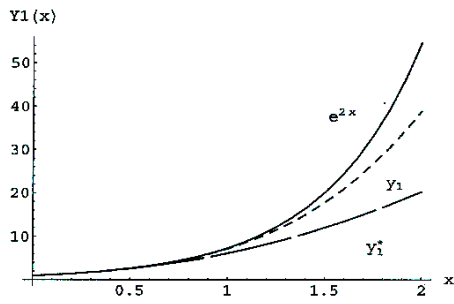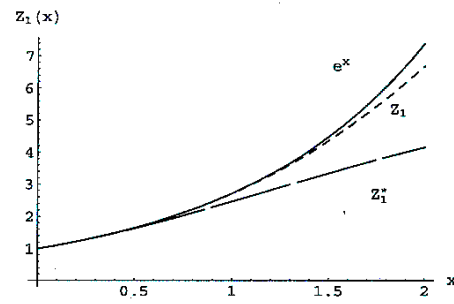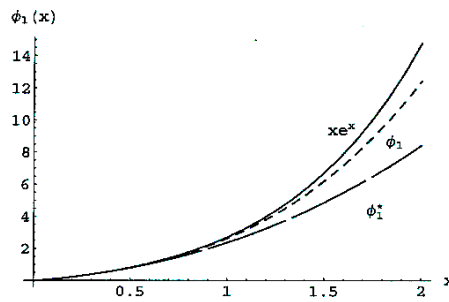
Fig. 1

Fig. 2

Fig. 3

(iii) Consider the following system of linear differential equations:

$$y_1' = y_1 + y_2, \qquad y_1(0) = 0,$$
$$y_2' = -y_1 + y_2, \qquad y_2(0) = 1.$$

The correction functionals for above system read

$$y_{1,n+1}(x) = y_{1,n}(x) + \int_0^x \lambda_1(s)\{y_{1,n}'(s) - y_{1,n}(s) - y_{2,n}(s)\}ds,$$
$$y_{2,n+1}(x) = y_{2,n}(x) + \int_0^x \lambda_2(s)\{y_{2,n}'(s) + y_{1,n}(s) - y_{2,n}(s)\}ds.$$

This yields the stationary conditions

$$1 + \lambda_1(s) = 0, \qquad \lambda_1'|_{s=x} + \lambda_1(s) = 0,$$
$$1 + \lambda_2(s) = 0, \qquad \lambda_2'|_{s=x} + \lambda_2(s) = 0.$$

As a result we find

$$\lambda_1(s) = \lambda_2(s) = -e^{x-s}.$$

Substituting these values of the Lagrange multipliers into the functionals above gives the iteration formulas

$$y_{1,n+1}(x) = y_{1,n}(x) - \int_0^x e^{x-s}\{y_{1,n}'(s) - y_{1,n}(s) - y_{2,n}(s)\}ds,$$

Applications and Applied Mathematics: An International Journal (AAM), Vol. 5 [2010], Iss. 3, Art. 10

AAM: Intern. J., Special Issue No. 1 (August 2010) 119

$$y_{2,n+1}(x) = y_{2,n}(x) - \int_0^x e^{x-s}\{y'_{2,n}(s) + y_{1,n}(s) - y_{2,n}(s)\}ds,$$

We get the exact solution $y_1 = e^x$ and $y_2 = e^x \cos x$. accordingly, we obtain the following successive approximations:

$$y_{1,0} = 0,$$

$$y_{2,0} = 1,$$

$$y_{1,1} = -1 + e^x,$$

$$y_{2,1} = e^x,$$

$$y_{1,2} = e^x + e^x(-1+x),$$

$$y_{2,2} = -1 + e^x - e^x(-1+x),$$

$$y_{1,3} = 1 + e^x + e^x(-1+x) + e^x(-1+x-\frac{x^2}{2}),$$

$$y_{2,3} = e^x - e^x(-1+x) + e^x(-1+x-\frac{x^2}{2}),$$

$$y_{1,4} = e^x + e^x(-1+x) + e^x(-1+x-\frac{x^2}{2}) - \frac{1}{6}e^x(-6+6x-3x^2+x^3),$$

$$y_{2,4} = e^x - e^x(-1+x) + e^x(-1+x-\frac{x^2}{2}) + \frac{1}{6}(6+e^x(-6+6x-3x^2+x^3)),$$

$$\vdots$$

The use of the modified method results in the sequences

$$\bar{y}_{1,n+1}(x) = \bar{y}_{1,n}(x) - \int_0^x e^{x-s}\{\bar{y}'_{1,n}(s) - \bar{y}_{1,n}(s) - \bar{y}_{2,n}(s)\}ds,$$

$$\bar{y}_{2,n+1}(x) = \bar{y}_{2,n}(x) - \int_0^x e^{x-s}\{\bar{y}'_{2,n}(s) + \bar{y}_{1,n+1}(s) - \bar{y}_{2,n}(s)\}ds.$$

We obtain the following successive approximations:

$$y_{1,0} = 0,$$

$$y_{2,0} = 1,$$

$$y_{1,1} = -1 + e^x,$$

$$y_{2,1} = -1 - e^x(-2+x),$$

$$y_{1,2} = 1 + e^x - \frac{1}{2}e^x(-2+x)^2,$$

$$y_{2,2} = 1 - e^x(-2+x) + e^x(-2+2x-x^2+\frac{x^3}{6}),$$

$$y_{1,3} = -1 + e^x - \frac{1}{2}e^x(-2+x)^2 + e^x(2-2x+x^2-\frac{x^3}{3}+\frac{x^4}{24}),$$

$$y_{2,3} = -1 - e^x(-2 + x) + e^x(-2 + 2x - x^2 + \frac{x^3}{6}) + e^x(2 - 2x + x^2 - \frac{x^3}{3}$$

$$+ \frac{x^4}{12} - \frac{x^5}{120}),$$

$$y_{1,4} = 1 + e^x - \frac{1}{2}e^x(-2 + x)^2 + e^x(2 - 2x + x^2 - \frac{x^3}{3} + \frac{x^4}{24}) + e^x(-2 + 2x - x^2$$

$$+ \frac{x^3}{3} - \frac{x^4}{12} + \frac{x^5}{60} - \frac{x^6}{720}),$$

$$y_{2,4} = 1 - e^x(-2 + x) + e^x(-2 + 2x - x^2 + \frac{x^3}{6}) + e^x(2 - 2x + x^2 - \frac{x^3}{3} +$$

$$\frac{x^4}{12} - \frac{x^5}{120}) + e^x(-2 + 2x - x^2 + \frac{x^3}{3} - \frac{x^4}{12} + \frac{x^5}{60} - \frac{x^6}{360} + \frac{x^7}{5040}),$$

$$\vdots$$

In Figs. 1,2, we plot e^x and $e^x \cos x$ which are the exact solutions, $y_i, i = 1,2$, denote solutions obtained by revised VIM and $y_i^*, i = 1,2$ denotes solutions obtained by the standard VIM.
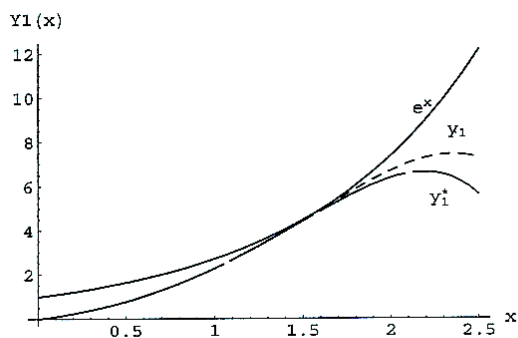

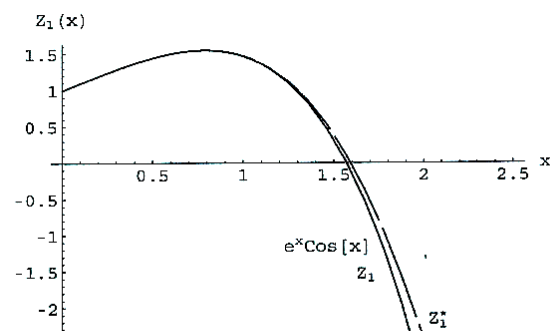Fig. 4   Fig. 5

6. Conclusions

Variational iteration is a powerful method which yields a convergent series solution for linear/non-linear problems. The solution obtained by the variational iteration method is an infinite power series for appropriate initial condition, which can, in turn, be expressed in a closed form, the exact solution. The results show that the variational iteration method is a powerful mathematical tool to solving systems of ordinary differential equations. In our work, we use the Mathematica to calculate the series obtained from the variational iteration method. This method does not require large computer power. In the present paper we employ the revised VIM for solving a system of ordinary differential equations. The revised method yields a series solution which converges faster than the series obtained by standard VIM. The illustrative examples clearly demonstrate this. Mathematica has been used for graphs presented in this paper.

Applications and Applied Mathematics: An International Journal (AAM), Vol. 5 [2010], Iss. 3, Art. 10

AAM: Intern. J., Special Issue No. 1 (August 2010)   121

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