Applications and Applied Mathematics: An International Journal (AAM)

Volume 14 | Issue 2 | Article 16

12-2019

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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 14, Issue 2 (December 2019), pp. 875 – 889

On the Weighted Pseudo Almost Periodic Solutions of Nicholson's Blowflies Equation

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Received: March 23, 2019; Accepted: August 7, 2019

Abstract

This study is concerned with the existence, uniqueness and global exponential stability of weighted pseudo almost periodic solutions of a generalized Nicholson's blowflies equation with mixed delays. Using some differential inequalities and a fixed point theorem, sufficient conditions were obtained for the existence, uniqueness of at the least a weighted pseudo almost periodic solutions and global exponential stability of this solution. The results of this study are new and complementary to the previous ones can be found in the literature. At the end of the study an example is given to show the accuracy of our results.

Keywords: Weighted pseudo almost periodic solution; Exponential stability; Nicholson's blowflies; Fixed point

MSC 2010 No.: 34K14, 47H10, 92D25

1. Introduction

Because of their equivalence in the real world, non-linear functional biological models have attracted great interest from researchers (see Cooke et al. (1999), Huang et al. (2011), Kuang (1993)). One of these interesting biological models is Nicholson's blowflies model, which describes the population of Australian sheep. Is there a differential equation that corresponds to this population model? In response to this question, Gurney et al. (1980) proposed delay differential equation as Nicholson's population model. This differential equation and its modifications have been extensively studied since this equation and its modifications describe the population growth of other species (see Saker and Agarwal (2002), Li and Fan (2007), Alzabut (2010) and the references therein). Some of these comprehensive studies done about almost periodic solutions or pseudo-almost periodic solutions (see Dads et al. (2009), Wang et al. (2011), Liu and Tunç (2015) and the references therein). In particular, Long (2012) established some criteria about positive almost periodic solutions for the generalized Nicholson's blowflies model with a linear harvesting term and multiple time-varying delays. Further, Cherif (2015) discussed the existence pseudo almost periodic solutions for Nicholson's blowflies model with mixed delays, which has a more general form than Long (2012). Recently, a lot of researchers have started to work on weighted pseudo almost periodic functions (see Zhang and Xu (2007), Liang et al. (2010), Coronel et al. (2016)). A weighted pseudo almost periodic function, which can be represented as an almost periodic function plus a weighted ergodic component, is more common than pseudo almost periodic phenomenon (see Diagana (2009), Xiaoxing and Yang (2011), Zhao et al. (2012)). Many researchers have also focused on the periodic phenomenon in delayed dynamic systems and have done a lot of work (see Xu (2017), Zhou and Shao (2018),Yang and Wan (2018)).

Motivated by the studies mentioned, we study some dynamics behaviors such as the existence, uniqueness and global exponential stability of weighted pseudo almost periodic solutions of the Nicholson's blowflies model with variable time delays which more general than the above mentioned equations. As far as we know, there are no studies related to weighted pseudo almost periodic solutions for nonlinear functional Nicholson's blowflies model. Our aim is to study these facts for weighted pseudo almost periodic solutions of the generalized Nicholson's blowflies model. The proof is based on the properties of weighted pseudo almost periodic functions and a fixed point theorem. Our results are new and complementary to the previously known results in the literature.

2. Preliminaries

$BUC(R,R)$ represents the set of bounded and uniformly continuous functions from R to R. Note that $(BC(R,R), \|.\|_\infty)$ is a Banach space and $\|.\|_\infty$ denotes the norm $\|F\|_\infty := \sup_{t\in R} \|F(t)\|$. In this work, for a given a bounded continuous F defined on R, let F^+ and F^- be defined as

$$F^+ = \sup_{t\in R} |F(t)|, \quad F^- = \inf_{t\in R} |F(t)|.$$

Let $\xi = max\{max_{1\le j\le m} \tau_j^+, \sigma^+, \tau^+\}$.

Let U show the collection of weight functions $\rho : R \to (0,\infty)$ which are locally integrable over R such that $\rho > 0$. If $\rho \in U$, then we set

$$\mu(T,\rho) = \int_{-T}^{T} \rho(x)dx, \quad T > 0.$$

In the particular case, when $\rho(x) = 1$ for each $x \in R$, we are exclusively interested in those weights ρ, for which $\lim_{T\to\infty} \mu(T, \rho) = \infty$.

Let

$$U_\infty := \{\rho \in U : \lim_{T\to\infty} \mu(T, \rho) = \infty\},$$

and

$$U_B := \{\rho \in U : \rho \quad \text{is bounded with} \quad \inf_{t\in R} \rho(x) > 0\}.$$

Definition 2.1. (Corduneanu (1989))

For any $\epsilon > 0$, there exists a number $l(\epsilon) > 0$ with the property that any interval of length $l(\epsilon)$ of the real line contains at least one point with τ such that:

$$\mid f(x+\tau) - f(t) \mid < \epsilon, \qquad -\infty < x < \infty.$$

This definition of almost periodic functions is given by H. Bohr. The number τ is called an $\epsilon-$ translation number of $f(x)$ corresponding to ϵ or an ϵ translation number. $AP(X, R)$ means the set all almost periodic functions.

We need to define $PAP_0(R, R^n, \rho)$ the weighted ergodic space, because this is necessary for weighted pseudo almost periodic functions. These functions will then appear as perturbations of almost periodic functions by elements of $PAP_0(R, R^n, \rho)$. For $\rho \in U_\infty$ ergodic space is

$$PAP_0(R, R^n, \rho) := \{f \in BC(R, R^n) : \lim_{T\to\infty} \frac{1}{\mu(T, \rho)} \int_{-T}^{T} \|f(x)\|\rho(x)dx = 0\}.$$

Definition 2.2. (Diagana (2006))

Let $\rho \in U_\infty$. A function $f \in BC(R, R^n)$ is called weighted pseudo almost periodic if it can be expressed as $f = g + \varphi$, where $g \in AP(R, R^n)$ and $\varphi \in PAP_0(R, R^n, \rho)$. The collection of such functions will be denoted by $PAP(R, R^n, \rho)$.

Remark 2.3.

$PAP(R, R)$ and $AP(R, R)$ are proper subspaces of $PAP(R, R, \rho)$ since the function $f(t) = \sin t + \sin\sqrt{2}t + e^{-t}$ is weighted pseudo almost periodic function but not almost periodic and pseudo almost periodic.

Lemma 2.4. (Zhou and Shao (2018))

Let $\rho : R \to (0, \infty), \rho \in U_\infty$ be a continuous function and assume that

$$\sup_{t\in R} \left[\frac{\rho(t+r)}{\rho(t)}\right] < \infty,$$

and

$$\sup_{T>0} \left[\frac{\mu(T + r, \rho(t))}{\mu(T, \rho(t))}\right] < \infty \quad \text{for each} \quad t, r \in R.$$

If $\varphi(.) \in PAP(R,R,\rho)$, then $\varphi(.-h) \in PAP(R,R,\rho)$.

Lemma 2.5. (Diagana (2009))

If $\varphi, \psi \in PAP(R,R,\rho)$, then $\varphi \times \psi \in PAP(R,R,\rho)$.

Lemma 2.6. (Xu (2017))

If $f(t) \in PAP(R,R,\rho)$ and $\tau(t) \in C^1(R,R)$ also $\tau(t) \geq 0, \tau'(t) \leq 1$, then $f(t-\tau(t)) \in PAP(R,R,\rho)$.

Definition 2.7.

The almost periodic solution $x^*(t)$ of Equation (3) is said to be global exponentially stable, if there exist constants $\epsilon > 0$ and $K > 0$ such that

$$\|\phi - x^*\| = \max_{1\leq i\leq n}\{\sup_{-\tau\leq s\leq 0}|\phi_i(s) - x_i^*(s)|\} \leq Ke^{-\epsilon(t-t_0)}, \qquad \text{for all} \quad t > t_0.$$

3. Existence and uniqueness of weighted pseudo almost periodic solutions for the model

We have discussed the following Nicholson's blowflies model with variable time delays:

$$\begin{aligned} x'(t) = &-\alpha(t)x(t) + \sum_{j=1}^{m}\beta_j(t)x(t-\tau_j(t))e^{-\omega_j(t)x(t-\tau_j(t))} - H(t)x(t-\sigma(t)) \\ &+ p(t)\int_{-\tau(t)}^{0}K(t,s)x(t+s)e^{-x(t+s)}ds, \end{aligned} \tag{1}$$

where $t \in R$. Because of the biological interpretation of Equation (1), only positive solutions are significant and therefore admissible. Thus, we just consider the admissible initial conditions

$$x(s) = \phi(s), \quad \phi(0) > 0, \quad s \in [-\tau^+, 0], \tag{2}$$

where $\phi \in BC([-\tau^+, 0], R^+)$.

Assume that the following conditions hold.

(C1) For $\alpha > 0$, the function $\alpha(.)$ is almost periodic and for all $1 \leq j \leq m$, the functions $H, p, \tau, \beta_j, \omega_j \in PAP(R, R^+, \rho)$.

(C2) $\rho : R \to (0,\infty), \rho \in U_\infty$ is continuous and there are K and L constants such that $\rho(t+r) \leq K\rho(t)$ and $\mu(T+r,\rho) \leq L\mu(T,\rho)$.

(C3)

$$r = \frac{1}{e^2\alpha^-}\left[H^+e^2 + p^+\frac{c(1-e^{-\gamma\tau})}{\gamma} + \sum_{j=1}^{m}\beta_j^+\right] < 1.$$

(C4) $K(.,.)$ satisfies the following conditions:

1. $s \in [t-\tau^+, t]$ and for all $t \in R^+$, $K(t,s)$ is non-negative and almost periodic.

2. There exist constants $c > 0$ and $\gamma > 0$ such that for all $t \in R^+$ and $s \in [t-\tau^+, t]$,

$$K(t,s) \leq ce^{-\gamma(t-s)}.$$

Lemma 3.1.

Assume that assumptions (C1)-(C4) hold and $\varphi \in PAP(R, R^+, \rho)$. Then, the function

$$\mu(t) = \int_{-\tau(t)}^{t} K(t,s)\varphi(s+t)e^{-\varphi(s+t)}ds \in PAP(R, R^+, \rho).$$

Theorem 3.2.

Let assumptions (C1)-(C4) hold and

$$\sup_{T>0}\left\{\int_{-T}^{T} e^{-\alpha^+(T+t)}\rho(t)dt\right\} < \infty.$$

Define the operator Θ by

$$\begin{aligned}(\Theta\phi)(t) = \int_{-\infty}^{t} & e^{-\int_s^t \alpha(\sigma)d\sigma} \sum_{j=1}^{m} \beta_j(s)\phi(s-\tau_j(s))e^{-\omega_j(s)\phi(s-\tau_j(s))} \\ & -H(s)\phi(s-\sigma(s))ds \\ & +p(s)\int_{-\tau(s)}^{0} K(t,\xi)\phi(t+\xi)e^{-\phi(t+\xi)}d\xi.\end{aligned}$$

Then, $\Theta \in PAP(R, R^+, \rho)$.

Proof:

$PAP(R, R^+, \rho)$ is a translation invariant closed subspace of $BC(R, R^+)$ and by Lemma 2.3 the function $F(\phi) = \phi(.-h) \in PAP(R, R, \rho)$. Also, from the composition theorem of weighted pseudo almost periodic functions (see Yazgan and Tunç (2017)), $\phi(t+\xi)e^{-\phi(t+\xi)} \in PAP(R, R, \rho)$.

Therefore,

$$\begin{aligned}Q(s) = & \sum_{j=1}^{m} \beta_j(s)\phi(s-\tau_j(s))e^{-\omega_j(s)\phi(s-\tau_j(s))} - H(s)\phi(s-\sigma(s)) \\ & \in PAP(R, R^+, \rho).\end{aligned}$$

We can write

$$Q = Q_1 + Q_2,$$

where $Q_1 \in AP(R, R^+)$ and $Q_2 \in PAP_0(R, R^+, \rho)$. So, one can deduce

$$\begin{aligned}(\Gamma Q)(t) &= \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q(s)ds \\ &= \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s)ds + \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_2(s)ds \\ &= (\Gamma Q_1)(t) + (\Gamma Q_2)(t).\end{aligned}$$

Firstly, we prove that $(\Gamma Q_1)(t) = \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s)ds$ is almost periodic. Given $\epsilon > 0$, in view of the almost periodicity of the functions α and Q_1, there exists a number l_ϵ such that in any interval $[\delta, \delta + l_\epsilon]$ one finds a number h, satisfying

$$\sup_{\theta} |\alpha(\theta + h) - \alpha(\theta)| < \frac{\epsilon\alpha^-}{2|Q_1|_\infty},$$

and

$$\sup_{\theta} |Q_1(\theta + h) - Q_1(\theta)| < \frac{\epsilon\alpha^-}{2}.$$

It is obvious that

$$\begin{aligned}(\Gamma Q_1)(t+h) - (\Gamma Q_1)(t) &= \int_{-\infty}^{t+h} e^{-\int_s^{t+h} \alpha(\sigma)d\sigma} Q_1(s)ds - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s)ds \\ &= \int_{-\infty}^{t+h} e^{-\int_{s-h}^t \alpha(\sigma+h)d\sigma} Q_1(s)ds - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s)ds \\ &= \int_{-\infty}^{t+h} e^{-\int_u^t \alpha(\sigma+h)d\sigma} Q_1(u+h)ds - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s)ds \\ &= \int_{-\infty}^{t+h} e^{-\int_s^t \alpha(\sigma+h)d\sigma} Q_1(s+h)ds - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s+h)ds \\ &\quad + \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s+h)ds - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} Q_1(s)ds.\end{aligned}$$

So there exists $\zeta \in [0, 1]$ such that

$$\begin{aligned}|(\Gamma Q_1)(t+h) - (\Gamma Q_1)(t)| &\leq |Q_1|_\infty \int_{-\infty}^{t} |e^{-\int_s^t \alpha(\sigma+h)d\sigma} - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma}|ds \\ &\quad + \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} |Q_1(s+h) - Q_1(s)|ds \\ &\leq \frac{\epsilon\alpha^-}{2} \int_{-\infty}^{t} e^{-\alpha^-(t-s)}(t-s)ds - \frac{\epsilon\alpha^-}{2} \int_{-\infty}^{t} e^{-\alpha^-(t-s)}ds \\ &= \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.\end{aligned}$$

Therefore, the function ΓQ_1 belongs to $AP(R,R)$. Now, let us show that ΓQ_2 belongs to $PAP_0(R,R^+,\rho)$.

We see immediately

$$\begin{aligned}
\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}|(\Gamma Q_2)(t)|\rho(t)dt &= \lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}|\int_{-\infty}^{t}e^{-\int_s^t\alpha(\sigma)d\sigma}Q_2(s)ds|\rho(t)dt\\
&\leq \lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\int_{-\infty}^{t}e^{-\int_s^t\alpha(\sigma)d\sigma}|Q_2(s)|ds\rho(t)dt\\
&\leq \lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\left(\int_{-\infty}^{t}e^{-\alpha^-(t-s)}|Q_2(s)|ds\right)\rho(t)dt\\
&\leq A_1+A_2,
\end{aligned}$$

where

$$A_1=\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\left(\int_{-T}^{t}e^{-\alpha^-(t-s)}|Q_2(s)|ds\right)\rho(t)dt,$$

and

$$A_2=\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\left(\int_{-\infty}^{T}e^{-\alpha^-(t-s)}|Q_2(s)|ds\right)\rho(t)dt.$$

Let $m=t-s$. Then,

$$\begin{aligned}
A_1&=\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\left(\int_{-T}^{t}e^{-\alpha^-(t-s)}|Q_2(s)|ds\right)\rho(t)dt\\
&=\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\left(\int_{0}^{t+T}e^{-\alpha^-(t-s)}|Q_2(t-m)|dm\right)\rho(t)dt\\
&\leq\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}\left(\int_{0}^{+\infty}e^{-\alpha^-m}|Q_2(t-m)|dm\right)\rho(t)dt\\
&\leq\int_{0}^{+\infty}e^{-\alpha^-m}\left(\lim_{T\longrightarrow\infty}\frac{1}{\mu(T,\rho)}\int_{-T}^{T}|Q_2(t-m)|\rho(t)dt\right)dm\\
&\leq\int_{0}^{+\infty}e^{-\alpha^-m}\left(\lim_{T\longrightarrow\infty}\frac{\mu(T+m,\rho)}{\mu(T,\rho)}\frac{1}{\mu(T+m,\rho)}\int_{-T-m}^{T+m}|Q_2(t)|\rho(t)dt\right)dm.
\end{aligned}$$

Since $Q_2(t)\in PAP_0(R,R^+,\rho)$, then,

$$\frac{\mu(T+m,\rho)}{\mu(T,\rho)}\frac{1}{\mu(T+m,\rho)}\int_{-T-m}^{T+m}|Q_2(t)|\rho(t)dt,$$

is bounded and

$$\lim_{T\longrightarrow\infty}\frac{\mu(T+m,\rho)}{\mu(T,\rho)}\frac{1}{\mu(T+m,\rho)}\int_{-T-m}^{T+m}|Q_2(t)|\rho(t)dt=0.$$

Consequently, by the Lebesgue dominated convergence theorem, we obtain

$$A_1 = \lim_{T \longrightarrow \infty} \frac{1}{\mu(T,\rho)} \int_{-T}^{T} \left(\int_{-T}^{t} e^{-\alpha^-(t-s)} |Q_2(s)| ds \right) \rho(t) dt = 0.$$

Also from $|Q_2|_\infty = \sup_{t \in R} |Q_2(t)| < \infty$, we obtain

$$\begin{aligned} A_2 &= \lim_{T \longrightarrow \infty} \frac{1}{\mu(T,\rho)} \int_{-T}^{T} \left(\int_{-\infty}^{T} e^{-\alpha^-(t-s)} |Q_2(s)| ds \right) \rho(t) dt \\ &\leq \lim_{T \longrightarrow \infty} \frac{|Q_2|_\infty}{\mu(T,\rho)} \int_{-\infty}^{T} e^{sa} ds \int_{-T}^{T} e^{-t\alpha^-} \rho(t) dt \\ &= \lim_{T \longrightarrow \infty} \frac{|Q_2|_\infty}{\mu(T,\rho)} \int_{-T}^{T} e^{-(t+T)\alpha^-} \rho(t) dt = 0, \end{aligned}$$

which implies the required result. ■

Theorem 3.3.

Let assumptions (C1)-(C4)) and Theorem 3.2 hold. Then, there exists one and only one weighted pseudo almost periodic solution of the Nicholson's blowflies model with mixed delays Equation (1) in the region

$$B = \{\varphi \in PAP(R, R^+, \rho), \qquad M_1 \leq |\varphi| \leq M_2\},$$

where $M_1 = \left[\sum_{j=1}^m \beta_j^- \frac{M_2}{\alpha^+} e^{-\omega_j^+ M_2} - H^+ \frac{M_2}{\alpha^+} \right]$ and $M_2 = \frac{1}{e\alpha^-} \sum_{j=1}^m \frac{\beta_j^+}{\omega_j^+} + \frac{p^+ k}{e\gamma\alpha^-}$.

Proof:

We can observe

$$\begin{aligned} (\Theta\phi)(t) = \int_{-\infty}^{t} & e^{-\int_s^t \alpha(\sigma) d\sigma} \sum_{j=1}^{m} \beta_j(s) \phi(s - \tau_j(s)) e^{-\omega_j(s)\phi(s-\tau_j(s))} \\ & -H(s)\phi(s - \sigma(s)) ds + p(s) \int_{-\tau(s)}^{0} K(t,\xi) \phi(t+\xi) e^{-\phi(t+\xi)} d\xi, \end{aligned}$$

$$\begin{aligned} |(\Theta x)(t)| &\geq \int_{-\infty}^{t} e^{-\alpha^+(t-s)} \times \sum_{j=1}^{m} \beta_j^- M_2 e^{-\omega_j^+ M_2} + p^- \int_{-\tau(s)}^{0} K(t,\xi)\phi(t+\xi) e^{-\phi(t+s)} d\xi - H^+ M_2] ds \\ &\geq \int_{-\infty}^{t} e^{-\alpha^+(t-s)} \times \left[\sum_{j=1}^{m} \beta_j^- M_2 e^{-\omega_j^+ M_2} - H^+ M_2 \right] ds \\ &= \left[\sum_{j=1}^{m} \frac{\beta_j^- M_2}{\alpha^+} e^{-\omega_j^+ M_2} - H^+ \frac{M_2}{\alpha^+} \right] = M_1. \end{aligned}$$

$$
\begin{aligned}
|(\Theta x)(t)| \leq & \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} \times [\sum_{j=1}^{m} \frac{\beta_j(s)}{\omega_j(s)} \omega_j(s)\phi(t-\tau_j(s))e^{-\omega_j(s)\phi(s-\tau_j(s))} \\
& + p(s) \int_{-\tau(s)}^{0} K(t,\xi)\phi(t+\xi)e^{-\phi(t+s)}d\xi]ds \\
& \leq \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} \times [\sum_{j=1}^{m} \frac{\beta_j(s)}{\omega_j(s)e} + \frac{p^+k}{e} \int_{-\tau(s)}^{0} e^{-\gamma(t-\xi)d\xi}]ds \\
& = \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} \times [\sum_{j=1}^{m} (\frac{\beta_j}{\omega_j})^+ \frac{1}{e} + \frac{p^+k}{e\gamma}(1-e^{-\xi\tau^+})]ds \\
& \leq [\sum_{j=1}^{m} (\frac{\beta_j}{\omega_j})^+ \frac{1}{e\alpha^-} + \frac{p^+k}{e\gamma\alpha^-}] = M_2,
\end{aligned}
$$

Remark now that for all $u, v \in [1, +\infty]$

$$
|ue^{-u} - ve^{-v}| = e^{-(u+\lambda(v-u)}|1-(u+\lambda(v-u)||u-v| \leq \frac{1}{e^2}|u-v|,
$$

where $\lambda \in [0,1]$. For $x, y \in B$

$$
\begin{aligned}
|(\Theta)(t) - (\Theta y)(t)| = & | \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} \times [\sum_{j=1}^{m} \beta_j(s)x(t-\tau_j(s))e^{-\omega_j(s)x(s-\tau_j(s))} \\
& + p(s) \int_{-\tau(s)}^{0} K(t,\xi)x(t+\xi)e^{-x(t+s)}d\xi - H(s)x(s-\sigma(s)) \\
& - \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma} \times [\sum_{j=1}^{m} \beta_j(s)y(t-\tau_j(s))e^{-\omega_j(s)y(s-\tau_j(s))} \\
& + p(s) \int_{-\tau(s)}^{0} K(t,\xi)y(t+\xi)e^{-y(t+s)}d\xi|ds - H(s)y(s-\sigma(s)) \\
& + \sup_{t\in R} \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma}(|p(s)| \int_{-\tau(s)}^{0} K(t,\xi)|x(t+\xi)e^{-x(t+s)} \\
& - y(t+\xi)e^{-y(t+s)}|d\xi + H(s)|x(s-\sigma(s)) - y(s-\sigma(s))| \\
& + \sum_{j=1}^{m} \beta_j(s)|x(t-\tau_j(s))e^{-\omega_j(s)x(s-\tau_j(s))} - y(t-\tau_j(s))e^{-\omega_j(s)y(s-\tau_j(s))}| \\
& \leq \sup_{t\in R} \int_{-\infty}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma}[H^+e^2 + p^+\frac{(1-e^{-\gamma\tau^-})}{+} \sum_{j=1}^{m} \beta_j^+]|x-y|_\infty \\
& \leq \frac{1}{e^2\alpha^-}[H^+e^2 + p^+\frac{(1-e^{-\gamma\tau^-})}{+} \sum_{j=1}^{m} \beta_j^+]|x-y|_\infty = r.
\end{aligned}
$$

Noting that $0 < r < 1$, it is clear that the operator Θ is a contraction mapping. Thus, applying the Banach fixed point theorem, the mapping Θ possesses a unique fixed point $z^* \in B$, that is,

$\Theta(z^*) = z^*$. Therefore, z^* is the unique weighted pseudo almost periodic solution of Equation (1) in B. This completes the proof. ■

Theorem 3.4.

Suppose that assumptions (C1)-(C2) hold and

$$\sup_{t\in R}\left[-(\alpha(t)-\lambda)+\frac{1}{e^2}\sum_{j=1}^{m}\beta_j(t)e^{\tau_j(t)}+H(t)e^{\sigma(t)}+p(t)e^{\tau(t)}\right]<0,$$

where $\lambda \in [0,1]$. Then, weighted pseudo almost periodic solution of Eq. (1) is global exponential stability.

Proof:

Let $y(t) = x(t) - x^*(t)$. Then,

$$\begin{aligned} y'(t) = &-\alpha(t)y(t) + \sum_{j=1}^{m}\beta_j(t)[(x(t-\tau_j(t))e^{-\omega_j(t)x(t-\tau_j(t))} \\ &-x^*(t-\tau_j(t))e^{-\omega_j(t)x^*(t-\tau_j(t))}] - H(t)\left(x(t-\sigma(t)) - x^*(t-\sigma(t))\right) \\ &+p(t)\int_{-\tau(t)}^{0}K(t,s)\left(x(t+s)e^{-x(t+s)} - x^*(t+s)e^{-x^*(t+s)}\right)ds, \end{aligned}$$

and

$$\begin{aligned} y(t) = &\int_{t_0}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma}[\sum_{j=1}^{m}\beta_j(s)(x(s-\tau_j(s))e^{-\omega_j(s)x(s-\tau_j(s))} \\ &-x^*(s-\tau_j(s))e^{-\omega_j(s)x^*(s-\tau_j(s))})]ds \\ &-\int_{t_0}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma}\left[H(s)\left(x(s-\sigma(s)) - x^*(s-\sigma(s))\right)\right]ds \\ &+\int_{t_0}^{t} e^{-\int_s^t \alpha(\sigma)d\sigma}[p(s)\int_{-\tau(s)}^{0}K(s,v) \\ &\times(x(s+v)e^{-x(s+v)} - x^*(s+v)e^{-x^*(s+v)})dv]ds. \end{aligned}$$

Let $V(t) = |y(t)|e^{\lambda t}$. A direct calculation of the left upper derivate gives

$$\begin{aligned} D^-(V(t)) \le &-\alpha(t)|y(t)|e^{\lambda t} + \sum_{j=1}^{m}\beta_j(s)|x(s-\tau_j(s))e^{-\omega_j(s)x(s-\tau_j(s)} \\ &-x^*(s-\tau_j(s)e^{-\omega_j(s)x^*(s-\tau_j(s)}|e^{\lambda t} \\ &+H(t)\left|y(t-\sigma(t))\right|e^{\lambda t} + \lambda|y(t)|e^{\lambda t} \\ &+p(t)\int_{-\tau(s)}^{0}K(t,s)\left|x(t+s)e^{-x(t+s)} - x^*(t+s)e^{-x^*(t+s)}\right|dse^{\lambda t}. \end{aligned}$$

Let

$$e^{\lambda t}\times\left(\max_{t\in[t_0-\zeta,t_\varphi]}|x(t)-x^*(t)|+1\right):=C_\varphi \quad ,$$

and for every $t>t_\varphi$

$$V(t)=|y(t)|e^{\lambda t}<C_\varphi.$$

Suppose that this is not true. Then, there is a $t_1>t_\varphi$ such that

$$V(t_1)=C_\varphi,$$

and

$$V(t)=|y(t)|e^{\lambda t}<C_\varphi \ \text{ for } t\in[t_0-\zeta,t_1).$$

Hence,

$$\begin{aligned}
D^-(V(t_1)) \leq & -\alpha(t_1)|y(t_1)|e^{\lambda t_1}+\sum_{j=1}^{m}\beta_j(t_1)|x(t_1-\tau_j(t_1))e^{-\omega_j(t_1)x(t_1-\tau_j(t_1))}\\
& -x^*(t_1-\tau_j(t_1))e^{-\omega_j(t_1)x^*(t_1-\tau_j(t_1))}|e^{\lambda t_1}\\
& +H(t_1)\left|y(t_1-\sigma(t_1))\right|e^{\lambda t_1}+\lambda|y(t_1)|e^{\lambda t_1}\\
& +p(t_1)\int_{-\tau(t_1)}^{0}K(t_1,s)\left|x(t_1+s)e^{-x(t_1+s)}-x^*(t_1+s)e^{-x^*(t_1+s)}\right|dse^{\lambda t_1}\\
\leq & -\alpha(t_1)-\lambda)|y(t_1)|e^{\lambda t_1}+\sum_{j=1}^{m}\frac{\beta_j(t_1)}{\omega_j(t_1)}|\omega_j(t_1)x(t_1-\tau_j(t_1)e^{-\omega_j(t_1)x(t_1-\tau_j(t_1)}\\
& -\omega_j(t_1)x^*(t_1-\tau_j(t_1)e^{-\omega_j(t_1)x^*(t_1-\tau_j(t_1)}|e^{\lambda t_1}\\
& +p(t_1)\int_{-\tau(t_1)}^{0}K(t_1,s)\left|x(t_1+s)e^{-x(t_1+s)}-x^*(t_1+s)e^{-x^*(t_1+s)}\right|dse^{\lambda t_1}\\
& +H(t_1)\left|y(t_1-\sigma(t_1))\right|e^{\lambda t_1}\\
\leq & -(\alpha(t_1)-\lambda)|y(t_1)|e^{\lambda t_1}+\frac{1}{e^2}\sum_{j=1}^{m}\beta_j(t_1)\left|y(t_1-\sigma(t_1))\right|e^{\lambda(t_1-\tau_j(t_1))}e^{\tau_j(t_1)}\\
& +H(t_1)\left|y(t_1-\sigma(t_1))\right|e^{\lambda(t_1-\sigma_j(t_1))}e^{\sigma_j(t_1)}\\
& +p(t_1)\int_{-\tau(t_1)}^{0}K(t_1,s)\left|y(t_1+s)\right|e^{\lambda(t_1-\tau(t_1))}e^{\tau(t_1)}ds\\
\leq & \left[-(\alpha(t_1)-\lambda)+\frac{1}{e^2}\sum_{j=1}^{m}\beta_j(t_1)e^{\lambda\zeta}+H(t_1)e^{\lambda\zeta}+p(t_1)e^{\lambda\zeta}\right]C_\varphi,
\end{aligned}$$

which gives a contradiction. Then,

$$V(t)=|y(t)|e^{\lambda t}<C_\varphi,$$

holds. ∎

Example 3.5.

Let us take a look the following Nicholson's blowflies model with a linear harvesting term and mixed delays

$$x'(t) = -\alpha(t)x(t) + \sum_{j=1}^{3} \beta_j(t)x(t-\tau_j(t))e^{-\omega_j(t)x(t-\tau_j(t))} - H(t)x(t-\sigma(t)$$

$$+ p(t)\int_{-\tau(t)}^{0} K(t,s)x(t+s)e^{-x(t+s)}ds, \tag{3}$$

where $\alpha(t) = 5 + \sin^2\sqrt{2}t + \sin^2 \pi t$, $p(t) = \frac{\sin^2\sqrt{5}t + \sin^2\sqrt{3}t}{100}$, $H(t) = \frac{\sin^2\sqrt{5}t+\sin^2\sqrt{6}t}{100}$,

for $j = 1,2,3, \tau_j(t) = \sin t + \sin\sqrt{2}t, \omega_j = 1, \tau(t) = \sin t, k = 2, \gamma = 1,$

$$\begin{pmatrix} \beta_1(t) \\ \beta_2(t) \\ \beta_3(t) \end{pmatrix} = \frac{1}{100}\begin{pmatrix} 1 + \left|\sin\sqrt{2}t\right| + e^{-t} \\ 1 + |\cos t| + e^{-t} \\ 1 + \left|\sin\sqrt{3}t\right| + e^{-t} \end{pmatrix}.$$

Then,

$$\sup_{T>0}\left\{\int_{-T}^{T} e^{-\alpha^-(T+t)}\rho(t)dt\right\} < \infty,$$

$$r = \frac{1}{e^2\alpha^-}[H^+e^2 + p^+\frac{k(1-e^{-\gamma\tau})}{\gamma} + \sum_{j=1}^{m}\beta_j^+]$$

$$= \frac{1}{500}\left[2e^2 + 4(1-\frac{1}{e}) + 9\right] \approx 0.007120 < 1,$$

$$M_1 = \left[\sum_{j=1}^{m}\beta_j^-\frac{M_2}{\alpha^+}e^{-\omega_j^+M_2} - H^+\frac{M_2}{\alpha^+}\right] = \frac{6}{5e} + \frac{4}{500e} \approx 0.665126,$$

and

$$M_2 = \frac{1}{e\alpha^-}\left[\sum_{j=1}^{m}\frac{\beta_j^+}{\omega_j^+} + \frac{p^+k}{e\gamma\alpha^-}\right] = \frac{3M_2}{7}e^{-M_2} - \frac{2}{700}M_2 \approx 1.024140.$$

Therefore, all conditions of Theorem 3.3 are satisfied. Then, Equation (3) has a unique weighted pseudo almost periodic solution in $[M_1, M_2]$, which implies that all conditions of Theorem 3.4 are satisfied. Hence, Equation (3) has an weighted pseudo almost periodic solution, which is global exponential stable.

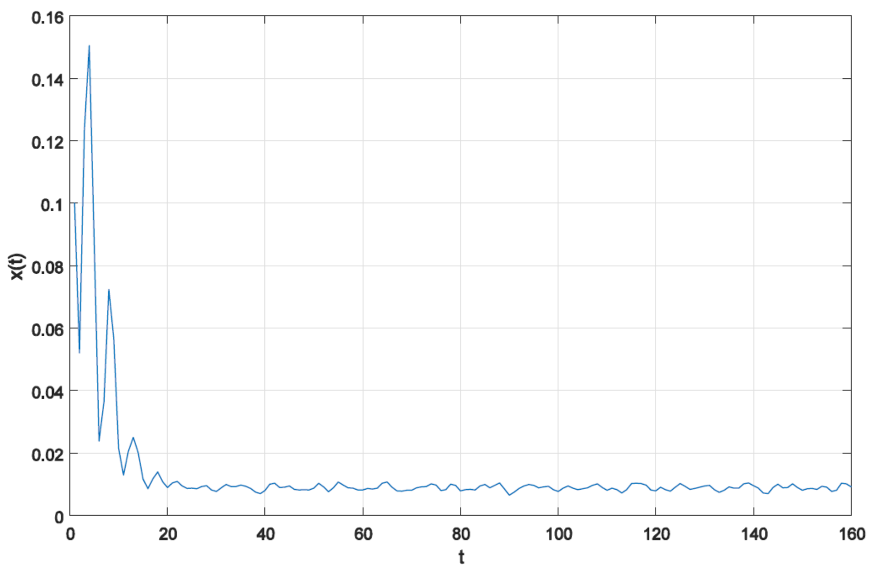

Figure 1. Curve of the weighted pseudo almost periodic solution of model (5)

4. Conclusion

This study deals with the weighted pseudo almost periodic solutions of Nicholson's blowflies differential equation with mixed delay by using properties of weighted pseudo almost periodic functions, some differential inequalities and Banach fixed point theorem. Also we give an example to show the correctness of our results.

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