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Radiation Effects on Boundary Layer Flow of Cu-water and Ag-water Nanofluids over a Stretching Plate with Suction and Heat Transfer with Convective Surface Boundary Condition

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Abstract

This paper deals with boundary layer flow of Cu-water and Ag-water nanofluids past a stretching plate with suction and heat transfer with convective surface boundary condition in the presence of thermal radiation. This flow belongs to the boundary layer flow of Skiadis type. A closed form solution has been obtained for convective heat transfer under the given conditions. We study the flow field with suction on stretching surface, the effect of volume fraction of nano-sized particles of Cu in Cu-water and Ag in Ag-water nanofluids on the temperature field, and the effect of suction parameter on the convective heat transfer. Moreover, the skin friction and Nusselt number both have been calculated and the possible effect of related parameters has been studied.

Keywords: Nanofluids; Boundary layer equations; Radiation flux; Nusselt number

MSC 2010 No.: 76N20, 76N17

1. Introduction

In view of its vast applications, the study of boundary layer flow past a stretching sheet has attracted scholars to do research in several variants. Some of the important applications of this study include aerodynamics extrusion of plastic sheets, formation of boundary layer along liquid film in condensation process, the cooling of metallic plate in a cooling bath, drawing of polymer yarn in textile industry and manufacturing of the glass sheets in glass industry.

In recent years, the conceptual birth of technology came into the existence due to legendry scientist R.P. Feynman. He delivered a famous lecture, “There is plenty of room at the bottom," at the American Physical Society Meeting at Caltech on December 29, 1969. In 1980, the invention of scanning tunnelling microscope accelerated the growth of nanotechnology. Nanofluids is the next existing frontier in the technology, so the study of nanofluids is of the considerable interest because of its ever application and important bearings on several technological processes. The birth of nanofluids is attributed to the revolutionary idea of adding solid particles in Heat Transfer Fluids (HTF) to increase the thermal conductivity. This innovative idea was put forth by Maxwell in 1973. Nanofluids have potential applications in microelectronics, fuel cells, and pharmaceutical industry. The application of nanofluids are largely because of the enhanced thermal conductivity. In particular, some of possible applications are nano drug delivery system, cancer therapeutics because radiation can be administered to the cancer patients using iron based nanoparticles. Nanofluids which have magnetic properties can be used as smart fluids in nuclear reactors and automotive applications. Referring to the reviewed article “An overview of recent nanofluid research" (see Sreelakshmy et al. (2014)), there are all around applications of nanofluids such as in automotive engines to improve the efficiency of the heat transfer, cooling of microchips as applications in electronics where nanofluids act as detergent. Seeing the utility of nanofluids in science and technology, scholars paid their attention to do investigations on nanofluids around the globe.

Vajravelu et al. (2011) studied convective heat transfer in the flow of Cu-water and Ag-water nanofluids over a stretching surface. Yirga et al. (2015) studied MHD flow and heat transfer of nanofluids through a porous media due to a stretching sheet with viscous dissipation and chemical reaction effects. Das (2015) investigated nanofluids flow over a non-linear permeable stretching sheet with partial slip. Das et al. (2015) published numerical simulation of nanofluid flow with convective boundary condition, and Buddakkagari et al. (2015) solved transient boundary layer laminar free convective flow of a nanofluid over a vertical cone/plate.

In this paper, we investigate the closed form solution of steady laminar boundary layer flow of Cu-water and Ag-water nanofluids past a stretching plate with suction and convective surface boundary condition in the presence of thermal radiation. We study the following

(a) flow field with suction on the stretching surface,
(b) the effect of volume fraction of nano-sized particles of Cu in Cu-water and Ag in Ag-water, nanofluids on the temperature field, and
(c) the effect of suction on the convective heat transfer.

The skin friction and Nusselt number both have also been calculated and the possible effect of related parameters has been studied.

2. Mathematical Formulation

Considering two dimensional boundary layer flow over a stretching sheet with pores in a coordinate system where x-axis is along the stretching sheet and y-axis is normal to the stretching sheet in the positive direction. Figure 1 shows the geometry of the problem where the continuous stretching surface is governed by $U(x) = mx$, where m is a positive constant.

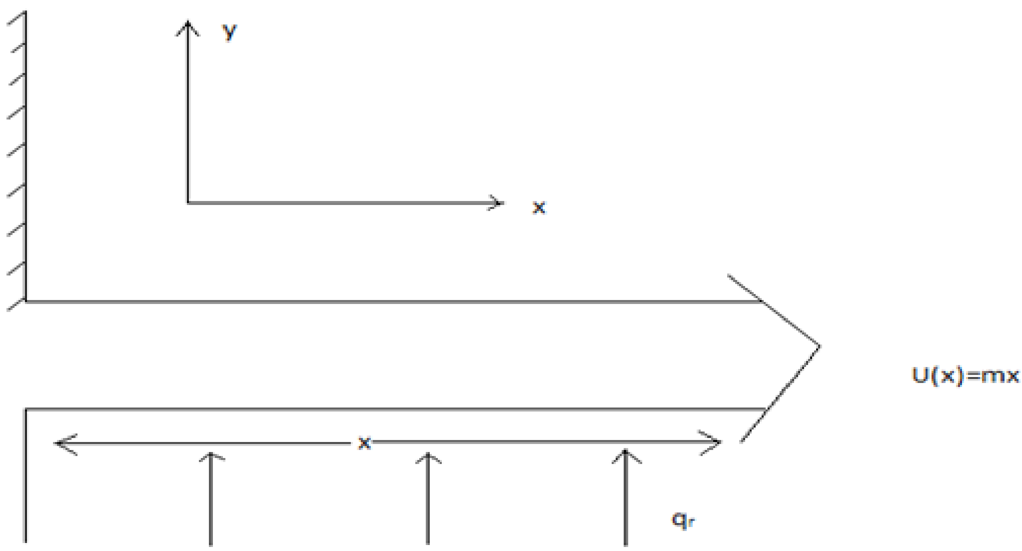

Figure 1. Flow model for stretching plate with convective surface boundary condition

The fluids considered here are Cu-water and Au-water nanofluids. We study the boundary layer flow and heat transfer here under the following assumptions

(a) nanofluids are incompressible,
(b) there is no chemical reaction,
(c) there is negligible viscous dissipation,
(d) nano-sized solid particles and the base fluid both are in thermal equilibrium and no slip occurred between the nano-sized particles and the fluid.

2.1. Boundary Layer Flow problem

The governing equations for steady boundary layer flow of Cu-water and Ag-water nanofluids past a stretching plate are

$$\text{Continuity Equation:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

and

$$\text{Momentum Equation:} \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

where u and v are the velocity components along x and y axes, respectively. μ_{nf} and ρ_{nf} are dynamic viscosity and density of nanofluids, respectively. The appropriate boundary conditions for flow problem are given by

$$u(x,0)=U(x)=mx,\quad v(x,0)=-v_0,\quad \text{and}\quad y\to\infty,\quad u=0, \tag{2.3}$$

where v_0 is the initial strength of the suction. Now, we introduce dimensionless variables as

$$\bar{x}=\frac{x}{h},\quad \bar{y}=\frac{y}{h},\quad \bar{u}=\frac{uh}{\nu_{nf}},\quad \bar{v}=\frac{vh}{\nu_{nf}}, \tag{2.4}$$

where h is characteristic length and ν_{nf} is the kinematic viscosity of the nanofluids. Referring to Ahmad et al. (2000), we get the velocity distribution as

$$u=mxe^{-ry},\qquad v=-\frac{m}{r}\big(1-e^{-ry}\big), \tag{2.5}$$

where

$$r=\frac{v_0+\sqrt{v_0^2+4m}}{2}.$$

2.2. Heat Transfer Problem

The energy equation with convective surface boundary condition is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho {c_p}_{nf}}\frac{\partial q_r}{\partial y}, \tag{2.6}$$

with relevant boundary conditions:

$$y=0,\quad -k_{nf}\frac{\partial T}{\partial y}=h_f\big(T_p-T_\infty\big),\quad \text{and}\quad y\to\infty,\quad T\to T_\infty, \tag{2.7}$$

where k_{nf} is the thermal conductivity of the nanofluid, T_p is temperature of the plate and T_∞ is ambient fluid temperature, that is, the temperature of the fluid far away from the plate, and h_f is heat transfer coefficient. Referring to Rosseland (1936) and Siegel et al. (1936), the radiative heat flux may be considered as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{2.8}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Here we use the approximation as it is being used by Bataller (2008a, 2008b), Pal (2009), Mondal et al. (2009), Mukhopadhyay et al. (2009), Ishak (2010) and very recently Ahmad et al. (2016) as

$$T^4\approx 4T_\infty^3T-3T_\infty^4. \tag{2.9}$$

Using (2.8) and (2.9), we have

$$\begin{aligned}\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}&=\frac{1}{(\rho c_p)_{nf}}\frac{\partial}{\partial y}\left[\frac{-4\sigma^*}{3k^*}\frac{\partial}{\partial y}\big(4T_\infty^3T-3T_\infty^4\big)\right]\\&=-\frac{16\sigma^*T_\infty^3}{3k^*(\rho c_p)_{nf}}\left(\frac{\partial^2 T}{\partial y^2}\right).\end{aligned} \tag{2.10}$$

Thus, the equation (2.6) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha_{nf}+\frac{16\sigma^* T_\infty^3}{3k^*(\rho c_p)_{nf}}\right)\frac{\partial^2 T}{\partial y^2}. \tag{2.11}$$

Now, we define the dimensionless temperature T as

$$\theta(\eta)=\frac{T-T_\infty}{T_p-T_\infty},$$

and assume that $\eta = ry$. Further, we substitute u and v from equation (2.5) into (2.11), we get

$$\theta^{''}+\frac{K_0 m(Pr)_{nf}}{r^2}(1-e^{-\eta})\theta^{'}=0, \tag{2.12}$$

with boundary conditions

$$\theta^{'}(0)=-\frac{h_f}{k_{nf}r},\quad \text{and}\quad \theta\to 0\quad \text{as}\quad \eta\to\infty, \tag{2.13}$$

where

$$(Pr)_{nf}=\frac{\nu_{nf}}{\alpha_{nf}},$$

the Prandlt number of nanofluid, and

$$K_0=\frac{3N}{3N+4},\quad \text{with}\quad N=\frac{k_{nf}k^*}{4\sigma^* T_\infty^3},$$

the radiation parameter. A solution of the equation (2.12) together with boundary condition (2.13) is

$$\theta(\eta)=\frac{2h_f}{k_{nf}(v_0+\sqrt{v_0^2+4m})}e^{\alpha}(\alpha)^{-\alpha}\gamma(\alpha,\alpha e^{-\eta}), \tag{2.14}$$

where

$$\alpha=\frac{4(Pr)_{nf}K_0 m}{(v_0+\sqrt{v_0^2+4m})^2},\quad \text{and}\quad \gamma(a,x)=\int_0^x e^{-t}t^{a-1}dt,$$

the incomplete gamma function. The effective density of nanofluid is given by

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s, \tag{2.15}$$

where φ is the solid volume fraction of nano-particles. Thus, thermal diffusivity of the nanofluid becomes

$$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{2.16}$$

where the heat capacitance of the nanofluid is taken as

$$(\rho c_p)_{nf}=(1-\varphi)\rho c_{p_f}+\varphi\rho c_{p_s}. \tag{2.17}$$

Referring to Brinkman (1952), effective dynamic viscosity of the nanofluid is given by

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}. \tag{2.18}$$

Thus, we have the following thermal conductivity of the nanofluid k_{nf}, which is given by Maxwell (1881),

$$k_{nf}=k_f\left\{\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}\right\}. \tag{2.19}$$

3. Skin Friction and Nusselt Number

In this section, we define both skin friction and Nusselt number. Then we calculate the skin friction for different volume fraction and suction parameter (see Table 1).

Definition 3.1.

The skin friction is a friction between the fluid and enclosed the surface by the fluid. The skin friction coefficient is defined as

$$C_f = -\frac{\mu_{nf}}{\rho_f U^2}(\frac{\partial u}{\partial y})_{y=0} = \frac{\sqrt{\frac{\nu_f}{m}}}{(1-\varphi)^{2.5}}(Re_x)^{-\frac{1}{2}}\left(\frac{v_0+\sqrt{v_0^2+4m}}{2}\right), \tag{3.1}$$

where $Re_x = \frac{U_x}{\nu_f}$ is the local Reynolds number.

Definition 3.2.

The coefficient of convectional heat transfer is called Nusselt number (Nu) and it is defined as

$$Nu = \frac{-(\frac{\partial T}{\partial y})_{y=0}}{T_p - T_\infty} = \frac{h_f}{k_{nf}}. \tag{3.2}$$

Table 1. The skin friction for different volume fraction and suction parameter

φ	ν_0	$C_f(Re_x)^{\frac{1}{2}}$
0.0	0.1	0.000992878
	0.3	0.001096691
	0.5	0.001209636
0.1	0.1	0.001292056
	0.3	0.001427178
	0.5	0.001574161
0.2	0.1	0.001734454
	0.3	0.001915841
	0.5	0.002113151

4. Discussion and Results

The Cu-water and Ag-water nanofluids have been considered for boundary layer flow past a stretching plate and heat transfer with suction and convective surface boundary condition to read the radiation effect. The exact solution to this problem has been obtained. Skin friction and Nusselt number have also been derived. The effect of radiation parameter N, suction parameter v_0 and the volume fraction of nano-sized particles φ have been studied on temperature field through graphs. We summarize the results in the following paragraphs

(a) Reading the graphs in Figure 2, we conclude that as suction parameter v_0 increases the temperature field increases within the boundary layer for given radiation N and volume friction $\varphi = 0$.

Hence, the effect of suction agrees with the findings of Ahmad et al. (2007). Thus, the cooling or heat transfer may be controlled by introducing the suction on the stretching surface. The suction parameter acts as the controller of heat transfer.

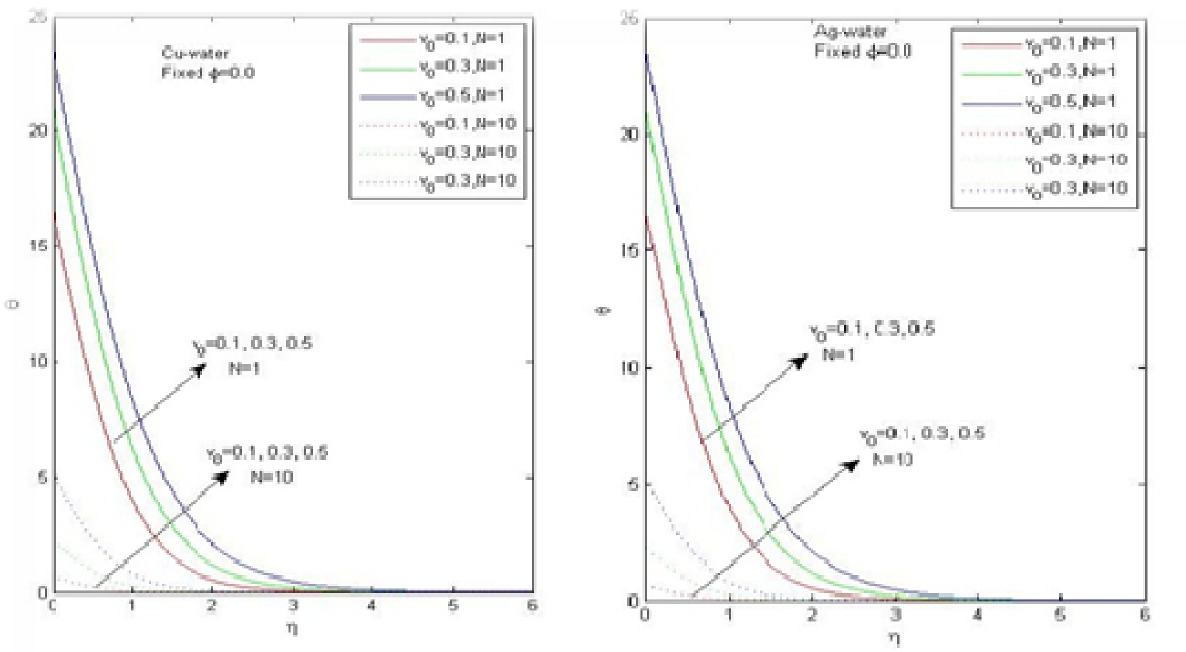

Figure 2. Dependence of temperature field on the suction parameter v_0 for given values of volume fraction $\varphi = 0.0$ and radiation parameter $N = 1$ and $N = 10$

(b) For a given value of volume fraction $\varphi = 0.1$ in Figure 3, we notice that the temperature increases as N, the radiation parameter, decreases. When $N = 100$, the temperature θ is the lowest, that is, heat transfer becomes maximum. On the other hand, when $N = 1$, the temperature is maximum, that is, the transfer of heat is the lowest and the heat within the fluid increases the temperature of fluid.

(c) Reading the graph in Figure 4, we notice that the value of temperature field θ for $N = 1$ is more than the corresponding value of the θ for $N = 10$. Physically, the transfer of heat for $N = 10$ is more than the transfer of heat for $N = 1$. It is also noted that as v_0 increases, the rate of heat transfer may decrease, that is, v_0 acts as a controller of heat transfer. Moreover, we notice that the graph for $N = 1$ is different for the graph for $N = 10$. In case of $N = 1$, we get a point in $0 \leq \eta \leq 1$, where θ is unique for all values of v_0. It is due to the presence of v_0.

(d) In Figure 5, we notice that as v_0 increases, θ also increases. In case $N = 10$, the value of temperature field θ is remarkably high in the immediate neighbourhood of stretching plate. We may express it as in this case rate of heat transfer has been enhanced so that the temperature at stretching plate has been supplemented to get it high value in the immediate neighbourhood of stretching plate. This phenomenon has not been seen in the case $N = 1$. On the other hand, when $N = 1$, v_0 increases θ which attains the same value for all v_0. This point lies in $0 \leq \eta \leq 1.5$ for

$\varphi = 0.2$ while it lies in $0 \leq \eta \leq 1$ for $\varphi = 0.1$.

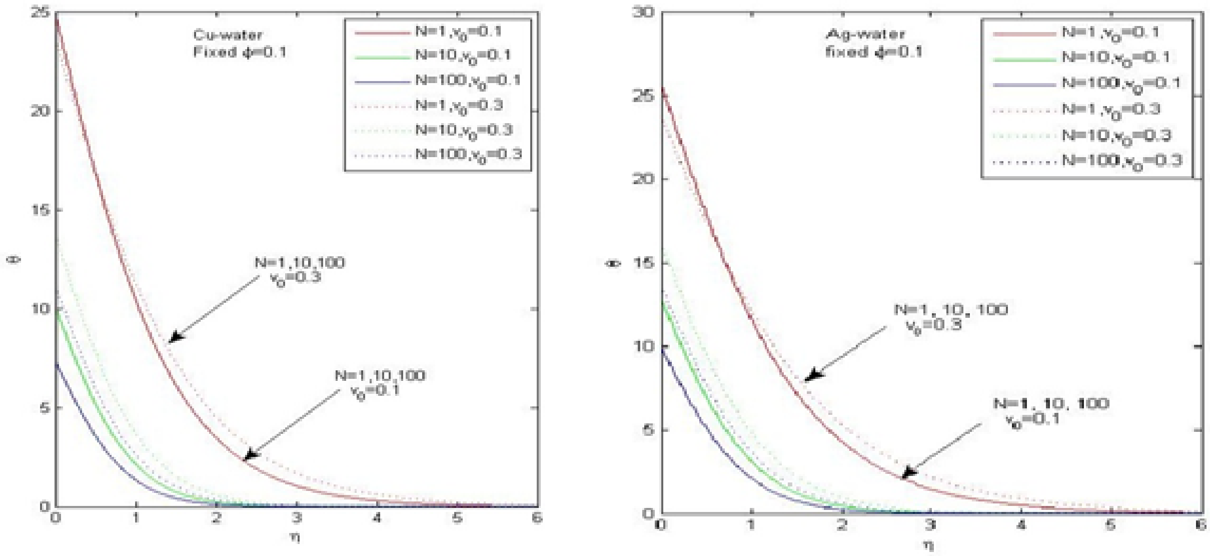

Figure 3. Dependence of temperature field θ on radiation parameter N for given values of suction parameter v_0 and volume fraction φ

(e) By reading Table 1 for skin friction, we conclude that as the volume fraction φ increases, the skin friction increases. In this case, a force called semi-frictional force increases between nanofluid and stretching plate in turn skin friction increases. Also, for some particular φ, skin friction increases as v_0 increases.

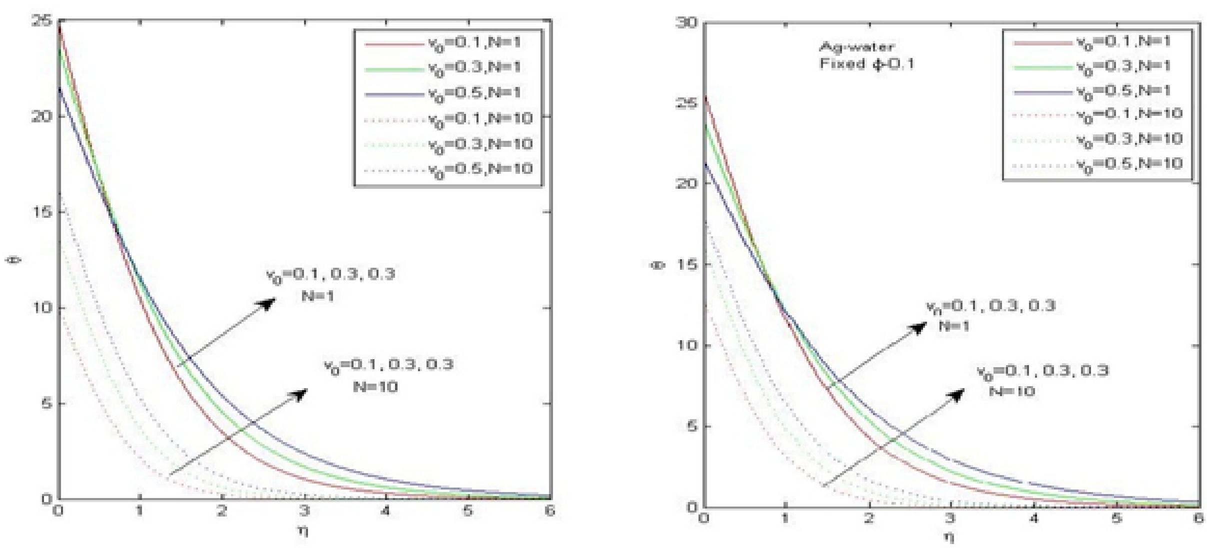

Figure 4. Variation in temperature field θ due to suction parameter v_0 for given volume fraction φ and radiation parameter N

(f) The Nusselt number (Nu) is independent of suction parameter but Nu decreases as thermal conductivity of nanofluid increases. Alternatively, the Nusselt number depends on volume fraction of nano-sized particles of nanofluid.

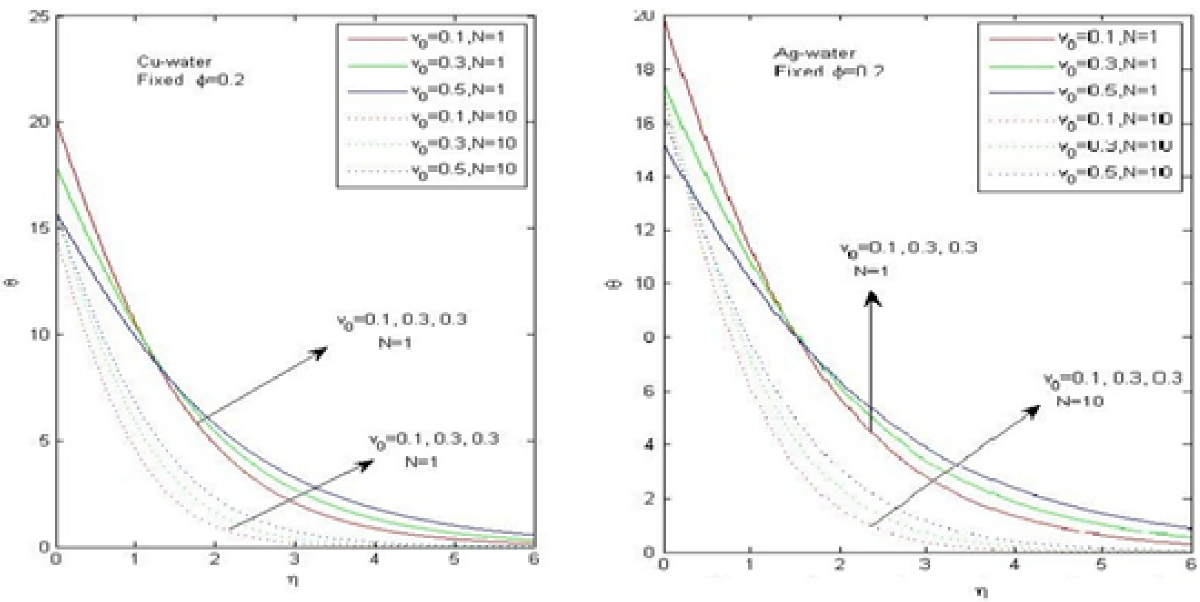

Figure 5. Variation in temperature field θ with suction parameter v_0 when $\varphi = 0.2$ and radiation parameter $N = 1$ and $N = 10$

5. Conclusion

A closed form solution has been obtained to "Radiation effects on boundary layer flow of Cu-water and Ag-water nanofluids over a stretching plate with suction and heat transfer with convective surface boundary." We conclude that v_0, the suction parameter, controls the heat transfer in the boundary layer flow. In nano-fliuds, the volume fraction is one of the factor which contribute to the skin friction and heat transfer both.

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