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Interactions of Thermoelastic Beam in Modified Couple Stress Theory

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Abstract

This paper is concerned with the study of thermoelastic beam in modified couple stress theory. The governing equations of motion for modified couple stress theory and heat conduction equation for non-Fourier (non-classical process) are investigated to model the vibrations in a homogeneous isotropic thin beam in a closed form by employing the Euler Bernoulli beam theory. The generalized theories of thermoelasticity with one and two relaxation times are used to model the problem. Both ends of the beam are simply supported. The Laplace transform technique applied to solve the system of equations which are written in dimensionless form. A general algorithm of the inverse Laplace transform is developed. The thermal moment is approximated as the difference between the upper and the lower surfaces of the beam. The analytical results have been analyzed numerically with the help of MATLAB software. The lateral deflection, thermal moment, axial stress average due to normal heat flux in the beam are derived and computed numerically. Numerical inversion technique has been applied to recover the results in a physical domain. The effect of couple stress on the resulting quantities are depicted graphically for a specific model. Comparisons are made with the results of different theories in the absence and presence of couple stress parameter. Particular cases of interest are also derived. The present study may find applications in medical science, engineering, accelerometers, sensors, resonators etc. The study of lateral deflection, thermal moment and axial stress average is a significant problem of continuum mechanics.

Keywords: Thermoelasticity; Beam; Modified couple stress theory; Euler Bernoulli theory; Laplace transform; Thermal and Mechanical conditions; Normal heat flux

MSC 2010 No.: 65N25; 74B05; 74B10

1. Introduction

The concept of couple stress linear theory of elasticity was originally introduced by Voigt (1887) and then extended by Cosserat and Cosserat (1909). Couple-stress theory is an extended continuum theory that includes the effects of a couple per unit area on a material volume, in addition to the classical direct and shear forces per unit area. This immediately admits the possibility of asymmetric stress tensor, since shear stress no longer have to be conjugate in order to ensure rotational equilibrium. Mindlin and Tiersten (1962) derived the associative constitutive equations for finite deformation of perfectly elastic materials. Toupin (1962) formulated a linearized theory of couple stress elasticity. Making use of this theory by Toupin (1962), the effect of couple stresses were studied on surface waves in elastic media and propagation of waves in an elastic layer by Sengupta and Ghosh (1972a, 1972b). Marin (1995) proved existence and uniqueness theorems in thermoelasticity with micropolar bodies. Marin and Marinescu (1998) investigated the asymptotic position of energies for the solutions of the mixed initial boundary value problem in the context of thermoelasticity of initially stressed bodies.

Yang et al. (2002) modified the classical couple stress theory and proposed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length parameter is needed to capture the size effect, which is caused by microstructure. Bernoulli–Euler beam model is based on a modified couple stress theory studied by Park and Gao (2006). Sun et al. (2008) used Laplace transform technique to study the vibration phenomena due to pulsed laser heating of a microbeam under different boundary conditions. Marin (2010) discussed the problem of vibrations in thermoelasticity of dipolar bodies. Thermoelastic beams with voids were studied by Sharma and Grover (2011). Sharma (2011) derived governing equations of flexural vibrations in a transversely isotropic, thermoelastic beam in closed form which is based on Euler-Bernoulli theory and was used to study thermoelastic damping (TED) and frequency shift (FS) of vibrations in clamped and simply supported beam structures.

Zang and Fu (2012) developed a new beam model for a viscoelastic micro-beam based on a modified couple stress theory. Rezazadeh et al. (2012) discussed problem of thermoelastic damping in a micro-beam resonator using modified couple stress theory. An eigenvalue formulation and Galerkin finite element method were used to evaluate the problem of thermoelastic damping in vented micro-electromechanical systems (MEMS) beam resonators presented by Guo et al. (2013). Shaat et al. (2014) investigated the bending and vibration of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory. Darijani and Shahdadi (2015) investigated the effect of shear deformation on the static bending and vibration responses of a simply supported microplate by using modified couple stress theory. Recently, Gang et al. (2015) presented a nonlinear bending and post-buckling of extensible microscale beams based on modified couple stress theory where the effects of the material length scale parameter and the Poisson ratio on the bending and thermal post-buckling behaviors of microbeams are discussed in detail. El-Karamany and Ezzat (2011) discussed general models of fractional heat conduction for non-homogeneous anisotropic elastic solids and obtained the constitutive equations for thermoelasticity theory. Ezzat et al. (2016, 2017) studied different problems in thermoelasticity theory.

Sharma and Kaur (2014) studied transverse vibrations in thermoelastic-diffusive thin micro beam based on Euler-Bernoulli theory under clamped-clamped boundary conditions. Allam and Abouelregal (2014) investigated the thermoelastic waves induced by pulsed laser and varying heat of homogeneous microscale beam resonators. The analytical solution to the problem was obtained using Laplace transform technique. Abouelregal and Zenkour (2014) discussed the problem of an axially moving microbeam subjected to sinusoidal pulse heating and an external transverse excitation with one relaxation time by using Laplace transform and also studied the effects of the pulse-width of thermal vibration, moving speed and the transverse excitation. Zenkour and Abouelregal (2015) studied the problem of thermoelastic vibration of an axially moving microbeam subjected to sinusoidal pulse heating. The mathematical model of fractional magneto-thermo-viscoelasticity for isotropic perfectly conducting media was presented by Ezzat and El-Bary (2016).

In the present study, the solutions for coupled thermoelastic beam are derived. A numerical technique based on the Laplace transformation is used to calculate the lateral deflection, thermal moment and axial stress average. A general algorithm of the inverse Laplace transform is developed. The effect of couple stress on lateral deflection, thermal moment and axial stress average for both L-S and G-L theories are computed numerically and shown graphically.

2. Basic Equations

Following Yang et al.(2002), Lord-Shulman (1967) and Green-Lindsay (1972), the constitutive relations, equations of motion and equation of heat conduction in modified couple stress generalized thermoelastic medium in absence of body forces and heat sources are:

(i) Constitutive relations

$$t_{ij} = \lambda e \delta_{ij} + 2\mu e_{ij} - \frac{1}{2} e_{kij} m_{lk,l} - \beta \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \delta_{ij}, \tag{1}$$

$$m_{ij} = 2\alpha \chi_{ij}, \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \tag{3}$$

$$\omega_i = \frac{1}{2} e_{ipq} u_{q,p}, \quad i, j, k = 1, 2, 3. \tag{4}$$

(ii) Equations of motion

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right)\nabla(\nabla.\boldsymbol{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right)\nabla^2 \boldsymbol{u} - \beta\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla T = \rho \ddot{\boldsymbol{u}}, \tag{5}$$

(iii) Equation of heat conduction

$$K\Delta T - \rho c_e \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T = T_0 \beta \left(\frac{\partial}{\partial t} + \tau_0 \eta_0 \frac{\partial^2}{\partial t^2}\right)(\nabla.\boldsymbol{u}), \tag{6}$$

where t_{ij} are the components of stress tensor, λ and μ are Lame$'$s constants, δ_{ij} is Kronecker's delta, e_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, $\beta=(3\lambda+2\mu)\alpha_T$, Here, α_T is the coefficient of linear thermal expansion, T is the temperature change, α is the couple stress parameter, χ_{ij} is symmetric curvature, ω_i is the rotational vector. $\boldsymbol{u}=(u_1, u_2, u_3)$ is the components of displacement vector, ρ is the density, Δ is the Laplacian operator, ∇ and is del operator. K is the coefficient of the thermal conductivity, c_e is the specific heat at constant strain, T_0 is the reference temperature assumed such that $T/T_0 << 1$, τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $\eta_0=1$, $\tau_1=0$, for Lord-Shulman (L-S, 1967) model and $\eta_0=0, \tau_1>0$, for Green Lindsay (G-L, 1972) model.

3. Formulation of the problem

Let us consider a homogeneous isotropic, rectangular modified couple stress thermoelastic beam of length $(0 \leq x \leq L)$, width $(-d/2 \leq y \leq d/2)$ and thickness $(-h/2 \leq z \leq h/2)$, where x, y and z are Cartesian axes lying along the length, width and thickness of the beam so that $x-$axis coincides with the beam axis and y, z axes coincide with the end $(x=0)$ with origin located at the axis of the beam.

According to the fundamental Euler-Bernoulli theory for small deflection of a simple bending problem, the displacement components are given by

$$u=-z\frac{\partial w}{\partial x},\ v=0,\ w(x,y,z,t)=w(x,t), \tag{7}$$

where $w(x,t)$ is the lateral deflection of the beam and t is the time.

The constitutive relation (1) in one-dimension with the help of Equation (7), we obtain

$$t_x=-(\lambda+2\mu)z\frac{\partial^2 w}{\partial x^2}-\beta\left(1+\tau_1\frac{\partial}{\partial t}\right)M_T. \tag{8}$$

The bending moment resultant of the beam M can be evaluated via the following relation:

$$M=M_\sigma+M_m=\int_{\frac{-h}{2}}^{\frac{h}{2}} dt_x z\, dz+\int_{\frac{-h}{2}}^{\frac{h}{2}} dm_{xy} dz, \tag{9}$$

where M_σ and M_m are the components of the bending moment due to the classic stress and couple stress tensors, respectively.

Making use of Euler-Bernoulli assumption (7) and with the aid of (8) in (9), we obtain

$$M = -\left(\lambda + 2\mu\right) I \frac{\partial^2 w}{\partial x^2} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) M_T - \alpha A \frac{\partial^2 w}{\partial x^2}. \tag{10}$$

Here, I is the second moment of the cross-section area of the beam and M_T is the thermal moment and I, M_T are given as

$$I = \int_{\frac{-h}{2}}^{\frac{h}{2}} dz^2 dz = \frac{dh^3}{12},\ M_T = \beta d \int_{\frac{-h}{2}}^{\frac{h}{2}} Tz\ dz. \tag{11}$$

The equation of transverse deflections of the beam is given by Rao (2007)

$$\frac{\partial^2 M}{\partial x^2} - \rho A \frac{\partial^2 w}{\partial t^2} = 0, \tag{12}$$

where $A = dh$ is the cross sectional area of the beam.

From Equations (10) and (12), yield

$$\left(\left(\lambda + 2\mu\right) I + \alpha A\right) \frac{\partial^4 w}{\partial x^4} + \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial^2 M_T}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} = 0, \tag{13}$$

and the heat conduction equation can be written as

$$\nabla^2 T - \frac{\rho c_e}{K}\left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \frac{\beta T_0}{K}\left(\frac{\partial}{\partial t} + \tau_0 \eta_0 \frac{\partial^2}{\partial t^2}\right) z \frac{\partial^2 w}{\partial x^2} = 0. \tag{14}$$

Multiplying (14) by zdz and integrating from interval $\left(-h/2,\ h/2\right)$, yields

$$\left(\frac{\partial^2 M_T}{\partial x^2} - \frac{12}{h^2} M_T\right)\left(x,t\right) + \frac{h\beta d}{2}\left\{\frac{\partial T}{\partial z}\left(x, \frac{h}{2}, t\right) + \frac{\partial T}{\partial z}\left(x, -\frac{h}{2}, t\right)\right\} - \frac{\rho c_e \tau_t^1}{K} M_T - \frac{\beta^2 T_0 I}{K} \tau_\eta^0 \frac{\partial^2 w}{\partial x^2} = 0, \tag{15}$$

where M_T is mathematically approximated as the difference between the temperatures at the upper and bottom surfaces of the beam. The temperature is assumed to vary linearly through the thickness of the beam and thus, we have

$$T\left(x,\ \frac{h}{2},\ t\right) - T\left(x, -\frac{h}{2},\ t\right) = \frac{12}{\beta d h^2} M_T\left(x,\ t\right). \tag{16}$$

Introduce the dimensionless quantities:

$$x' = \frac{x}{L},\ z' = \frac{z}{L},\ w' = \frac{w}{L},\ t' = \frac{vt}{L},\ \tau_0' = \frac{v\tau_0}{L},\ \tau_1' = \frac{v\tau_1}{L},\ T' = \frac{\beta T}{E},\ M' = \frac{M}{dEh^2},\ M_T' = \frac{M_T}{dEh^2},\ v^2 = \frac{E}{\rho},\ t_x' = \frac{t_x}{E}, \tag{17}$$

where E is the Young modulus, ν is the Poisson ratio, respectively.

Using (17) in (13) and (15), after dropping the dashes for convenience, we obtain

$$\frac{\partial^4 w}{\partial x^4} + a_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial^2 M_T}{\partial x^2} + a_2\frac{\partial^2 w}{\partial t^2} = 0, \tag{18}$$

$$\left(\frac{\partial^2 M_T}{\partial x^2} - 12a_3^2 M_T\right)(x,t) + \frac{a_3}{2}\left\{\frac{\partial T}{\partial z}\left(x,\frac{h}{2},t\right) + \frac{\partial T}{\partial z}\left(x,-\frac{h}{2},t\right)\right\} - a_4\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)M_T + a_5\left(\frac{\partial}{\partial t} + \tau_0\eta_0\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 w}{\partial x^2} = 0, \tag{19}$$

where

$$a_1 = \frac{dEh^2L}{(EI+\alpha A)},\ a_2 = \frac{\rho A v^2 L^2}{(EI+\alpha h)},\ a_3 = \frac{L}{h},\ a_4 = \frac{\rho c_e v L}{K},\ a_5 = \frac{T_0\beta^2 I v}{KdEh^2}.$$

4. Solution in the Laplace Domain

We define the Laplace transform as

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s), \tag{20}$$

where s is the Laplace transform parameter.
Making use of (20) in Equations (18) and (19), we obtain

$$\left(D^4 + a_2 s^2\right)\bar{w} = -a_1\xi_1 D^2 \bar{M}_T, \tag{21}$$

$$\left(D^2 - \Gamma_1\right)\bar{M}_T + \Gamma_2 D^2 \bar{w} = -\bar{Q}(s), \tag{22}$$

where

$$\begin{gathered} D = \frac{d}{dx},\ \Gamma_1 = \left(12a_3^2 + a_4\xi_2\right),\ \Gamma_2 = a_5\xi_3,\ \xi_1 = (1+\tau_1 s),\ \xi_2 = \left(s+\tau_0 s^2\right),\ \xi_3 = \left(s+\tau_0\eta_0 s^2\right), \\ \bar{Q}(s) = \frac{a_3}{2}\left\{\frac{d\bar{T}}{dz}\left(x,\ \frac{h}{2},\ s\right) + \frac{d\bar{T}}{dz}\left(x, -\frac{h}{2},\ s\right)\right\}. \end{gathered} \tag{23}$$

The differential equation of the lateral deflection $\bar{w}$ and the thermal moment $\bar{M}_T$ are

$$\left\{D^6 - pD^4 + qD^2 - r\right\}\begin{bmatrix}\bar{w} \\ \bar{M}_T\end{bmatrix} = \begin{bmatrix}0 \\ -a_2 s^2 \bar{Q}\end{bmatrix}, \tag{24}$$

where

$$p = \left(\Gamma_1 + a_1\xi_1\Gamma_2\right),\ q = a_2 s^2,\ r = q\Gamma_1.$$

The differential equation governing the lateral deflection $\bar{w}$ can take the form

$$\left\{\left(D^2-\lambda_1^2\right)\left(D^2-\lambda_2^2\right)\left(D^2-\lambda_3^2\right)\right\}\bar{w} = 0, \tag{25}$$

where $\pm\lambda_1, \pm\lambda_2$ and $\pm\lambda_3$ are the characteristic roots of the equation:

$$\lambda^6 - p\lambda^4 + q\lambda^2 - r = 0, \tag{26}$$

and satisfy the well-known relations:

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = p,\ \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 = q,\ \lambda_1^2\lambda_2^2\lambda_3^2 = r, \tag{27}$$

where p, q, r are the sum of all the roots, sum of the roots taken two at a time and product of all the roots, respectively.

Then the lateral deflection is given by

$$\bar{w}(x, s) = \sum_{i=1}^{3}\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right), \tag{28}$$

where A_i and B_i, $i = 1, 2, 3,$ are constant coefficients and are dependent on the Laplace variable s.

The thermal moment is given by

$$\bar{M}_T(x, s) = \sum_{i=1}^{3}\left(A_i' e^{\lambda_i x} + B_i' e^{-\lambda_i x}\right) + \frac{\bar{Q}}{\Gamma_1}, \tag{29}$$

where A_i, A_i' and B_i, B_i' $i = 1, 2, 3,$ are constant coefficients and are depending on the Laplace variable s. Substituting (28) and (29) in (22), yields

$$\begin{pmatrix} A_i' \\ B_i' \end{pmatrix} = \frac{\Gamma_2\lambda_i^2}{\left(\Gamma_1 - \lambda_i^2\right)}\begin{pmatrix} A_i \\ B_i \end{pmatrix}, \quad i = 1,2,3. \tag{30}$$

Making use of (30) in Equation (29), we obtain

$$\bar{M}_T(x, s) = \sum_{i=1}^{3}\frac{\Gamma_2\lambda_i^2}{\left(\Gamma_1 - \lambda_i^2\right)}\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right) + \frac{\bar{Q}}{\Gamma_1}. \tag{31}$$

Using (8), (16), (17) and (20) and with the aid of (29) and (31), the axial stresses can be written as

$$\bar{T}_x(x, s) = \sum_{i=1}^{3}\left[\lambda_i^2\left(\frac{-h(\lambda + 2\mu)}{EL} + \frac{12\xi_1\Gamma_2}{\left(\lambda_i^2 - \Gamma_1\right)}\right)\right]\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right) - \frac{12\xi_1\bar{Q}}{\Gamma_1}, \tag{32}$$

where

$$\bar{T}_x(x, s) = \bar{t}_{x_{avg}}(x, s) = \bar{t}_x\left(x, \frac{h}{2}, s\right) - \bar{t}_x\left(x, -\frac{h}{2}, s\right). \tag{33}$$

5. Application

We will discuss thermal loads over the upper surface of the beam:

The constant heat flux $(-q_0)$ is normal to the upper surface $\left(z=\frac{h}{2}\right)$ of the beam and the bottom surface $\left(z=-\frac{h}{2}\right)$ is at zero temperature gradient. The boundary conditions on the upper and bottom surfaces of heat conduction equation is

$$q_0 = K\frac{\partial T}{\partial z}\left(x,\ \frac{h}{2},\ t\right),\ \frac{\partial T}{\partial z}\left(x,-\frac{h}{2},\ t\right)=0. \tag{34}$$

Appling (17) and (20) on (34), we obtain

$$\frac{d\bar{T}}{dz}\left(x,\ \frac{h}{2},\ s\right)=\frac{q_0}{K},\ \frac{d\bar{T}}{dz}\left(x,-\frac{h}{2},\ s\right)=0. \tag{35}$$

6. Boundary Conditions

Mechanical and thermal conditions are:

$$w(0,\ t)=0,\ \frac{\partial^2 w(0,\ t)}{\partial x^2}=0,\ M_T\left(0,\ t\right)=0, \tag{36}$$

$$w(L,\ t)=0,\ \frac{\partial^2 w(L,\ t)}{\partial x^2}=0,\ M_T\left(L,\ t\right)=0. \tag{37}$$

From (23) and (35), the thermal influence is given by

$$\bar{Q}=\frac{a_3 q_0}{2K}. \tag{38}$$

Using (17) and (20) in the boundary conditions (36) and (37), yield

$$\bar{w}(0,\ s)=0,\ \frac{d^2\bar{w}(0,\ s)}{dx^2}=0,\ \bar{M}_T\left(0,\ s\right)=0, \tag{39}$$

$$\bar{w}(1,\ s)=0,\ \frac{d^2\bar{w}(1,\ s)}{dx^2}=0,\ \bar{M}_T\left(1,\ s\right)=0. \tag{40}$$

Substituting the values of $\bar{w}$ and $\bar{M}_T$ from (28) and (31) in the boundary conditions (39) and (40), with the aid of (38), after some simplification, we obtain the expressions of lateral deflection, thermal moment and axial stress average as

$$\bar{w}(x,\ s)=\sum_{i=1}^{3}\left(A_i e^{\lambda_i x}+B_i e^{-\lambda_i x}\right), \tag{41}$$

$$\bar{M}_T\left(x,\ s\right)=\sum_{i=1}^{3}M_i\left(A_i e^{\lambda_i x}+B_i e^{-\lambda_i x}\right)+\frac{\bar{Q}}{\Gamma_1}, \tag{42}$$

$$\bar{T}_x(x,s)=\sum_{i=1}^{3} N_i\left(A_i e^{\lambda_i x}+B_i e^{-\lambda_i x}\right)-\frac{12\bar{Q}}{\Gamma_1}, \tag{43}$$

where

$$A_1=\frac{\Delta_1}{\Delta},\ A_2=\frac{\Delta_2}{\Delta},\ A_3=\frac{\Delta_3}{\Delta},\ B_1=\frac{\Delta_4}{\Delta},\ B_2=\frac{\Delta_5}{\Delta},\ B_3=\frac{\Delta_6}{\Delta}.$$

and

$$\Delta=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} & e^{-\lambda_1} & e^{-\lambda_2} & e^{-\lambda_3} \\ \lambda_1^2 e^{\lambda_1} & \lambda_2^2 e^{\lambda_2} & \lambda_3^2 e^{\lambda_3} & \lambda_1^2 e^{-\lambda_1} & \lambda_2^2 e^{-\lambda_2} & \lambda_3^2 e^{-\lambda_3} \\ M_1 & M_2 & M_3 & M_1 & M_2 & M_3 \\ M_1 e^{\lambda_1} & M_2 e^{\lambda_2} & M_3 e^{\lambda_3} & M_1 e^{-\lambda_1} & M_2 e^{-\lambda_2} & M_3 e^{-\lambda_3} \end{bmatrix}$$

$$M_i=\frac{\Gamma_2\lambda_i^2}{\left(\Gamma_1-\lambda_i^2\right)},\ N_i=\lambda_i^2\left(\frac{-h(\lambda+2\mu)}{EL}+\frac{12\Gamma_2}{\left(\lambda_i^2-\Gamma_1\right)}\right)$$

$\Delta_i\ (i=1,...,6)$ are obtain by replacing 1st, 2nd, 3rd, 4th, 5th and 6th column by $\left[0,0,0,0,\left(-\frac{Q}{\Gamma_1}\right),\left(-\frac{Q}{\Gamma_1}\right)\right]^T$ in Δ_i.

7. Particular cases

(i) If $\tau_1=\tau^1=0, \eta_0=1, \gamma=\tau_0$, In Equations (41) - (43), we obtain the corresponding results for modified couple stress thermoelastic beam for Lord Shulman (L-S) model.

(ii) If $\eta_0=0,\ \gamma=\tau^0$, In Equations (41) - (43), we obtain the corresponding results for modified couple stress thermoelastic beam for Green Lindsay (G-L) model.

(iii) If $\alpha=0$, in Equations (41) - (43), we obtain the results for lateral deflection, thermal moment and axial stress average in a generalized thermoelastic beam and these results are similar as obtained by Sirafy et al. (2014) in a specific case.

8. Inversion of the Laplace Transform

To obtain the solution of the present application in the physical domain, we first apply the well-known formula:

$$f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\bar{f}(s)\,e^{-st}ds, \tag{44}$$

Secondly, we adopt a numerical inversion method on the Fourier series expansion, by which the integral (47) can be approximated as a series

$$f(t)=\frac{e^{ct}}{t_1}\left[-\frac{1}{2}\mathrm{Re}\,\bar{f}(c)+\sum_{j=0}^{\infty}\mathrm{Re}\left(\bar{f}\left(c+\frac{ij\pi}{t_1}\right)\right)\cos\left(\frac{j\pi}{t_1}\right)-\sum_{j=0}^{\infty}\mathrm{Im}\left(\bar{f}\left(c+\frac{ij\pi}{t_1}\right)\right)\sin\left(\frac{j\pi}{t_1}\right)\right]$$
$$-\sum_{j=1}^{\infty}\mathrm{e}^{-2cjt_1}f(2jt_1+t), \tag{45}$$

for $0\le t\le 2t_1$. The above series (45) is called the Durbin formula and the last term is called the discretization error. Honig and Hirdes (1984) developed a method for accelerating the convergence of the Fourier series and a procedure that computes approximately the best choice of the free parameters.

9. Numerical results and Discussion

For the purpose of numerical computations, we take the magnesium material. The physical data chosen for magnesium are taken as Sirafy et al. (2014) and Daliwal and Singh (1980).

$\lambda=2.696\times10^{10}$ Kg m^{-1} s^{-2}, $\mu=1.639\times10^{10}$ Kg m^{-1} s^{-2}, $\rho=1.74\times10^{3}$ Kg m^{-3}, $T_0=0.293\times10^{3}$ K,

$E=45\times10^{3}$GPa, $\alpha_t=1.78\times10^{-5}$ K^{-1}, $\nu=0.35$, $c_e=1.04\times10^{3}$ J Kg^{-1} K^{-1}, $q_0=2.25\times10^{11}\mathrm{Wm}^{-2}$,

$L/h=10$, b/h=0.5, h=10μm, $\alpha=2.5$ Kg m s^{-2}, $K=1.7\times10^{2}$ Wm^{-1}K^{-1}, $t=0.1s$, $\tau_0=0.01s$,

$\tau_1=0.03s$.

The software MATLAB 7.10.4 has been used to determine the absence and presence of couple stress on lateral deflection, thermal moment and axial stress average for both L-S and G-L theories in the absence and presence of couple stress are computed numerically and shown graphically in Figures 1-3, respectively.

In all these Figures, solid line (—) corresponds to L-S $(\alpha=0)$, solid line with centre symbol (—∗—) corresponds to L-S$(\alpha=2.5)$, small dash line (----) corresponds to G-L$(\alpha=0)$, small dash line with centre symbol (---∗---) corresponds to G-L$(\alpha=2.5)$.

Figure 1 shows the variation of lateral deflection w with respect to length. It is evident that lateral deflection decreases gradually for the range $0\le x<3$ and then oscillates in the remaining range for both cases and both theories of thermoelasticity. Figure 2 depicts the variation of thermal moment M_T with respect to length. It is noticed that the value of thermal moment decreases smoothly in the absence and presence of both L-S and G-L theories of thermoelasticity. On the other hand, the value of thermal moment for G-L theory is higher as compared to L-S theory and reversed behavior is observed in the absence and presence of couple stress. Figure 3 represents the variation of average of axial stress T_x with respect to length. The value of axial stress increases monotonically in the whole range for both L-S and G-L theories. Also, the value of axial stress for G-L theory is more in comparison to L-S theory for both cases.

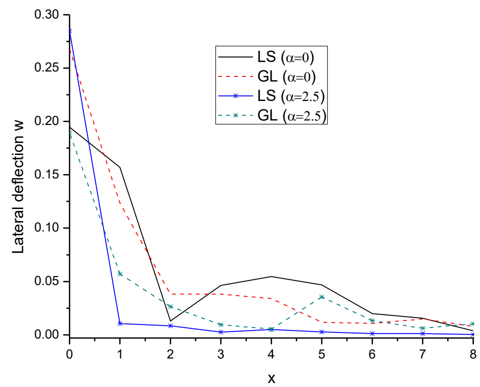

Figure 1. Variation of lateral deflection with length

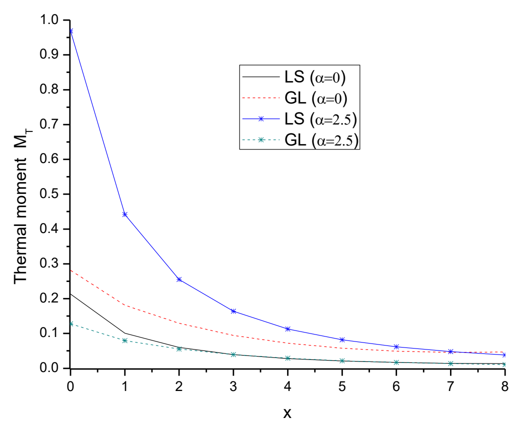

Figure 2. Variation of thermal moment with length

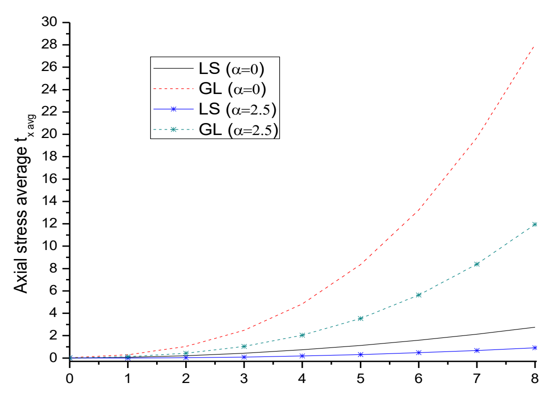

Figure 3. Variation of axial stress average with length

10. Conclusion

The interactions of thermoelastic beam in modified couple stress theory in the context of L-S and G-L theories of thermoelasticity have been investigated by using Euler-Bernoulli theory and Laplace transform technique. A numerical technique has been used to recover the solutions in the physical domain. The expressions for lateral deflection, thermal moment and axial stress average have been derived successfully and shown graphically to depict the effect of couple stress.

It is noticed from the figure that the behavior and variation of lateral deflection is oscillatory for absence and presence of couple stress. In the presence of couple stress, the value of thermal moment for L-S theory is higher in comparison to G-L theory and reverse behavior is observed in the absence of couple stress. The value of axial stress is more for G-L theory in comparison to L-S theory due to couple stress. The results obtained in the study should be beneficial for people working in medical science, thermomechanical engineering, accelerometers, and in the field of thermoelastic beam in modified couple stress theory. Analysis of lateral deflection, thermal moment and axial stress average is a significant problem of solid mechanics. The resulting quantities are observed to be very sensitive to the couple stress parameters.

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