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Application of Bernoulli Sub-ODE Method For Finding Travelling Wave Solutions of Schrödinger Equation Power Law Nonlinearity

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Abstract

In this paper, the exact travelling wave solution of the Schrödinger equation with power law nonlinearity is studied by the Sub-ODE method. It is shown that the method is one of the most effective approaches for finding exact solutions of nonlinear differential equations.

Keywords: Bernoulli Sub-ODE Method; Travelling Wave Solutions; Nonlinear Evolution Equation; Schrödinger Equation with Power Law Nonlinearity

MSC 2010 No.: 35C07, 35C08, 35L05, 35Q55

1. Introduction

The Schrödinger equation is one of the important partial differential equations and plays a vital role in various areas of physical, biological, and engineering sciences. It appears in the study of nonlinear optics, plasma physics, mathematical bioscience, quantum mechanics, and several other disciplines. One of the considerable cases of the nonlinear Schrödinger equations is power law nonlinearity which was studied by Wazwaz (2009).

Recently, there has been a growing interest in finding exact analytical solutions to nonlinear wave equations by using appropriate techniques. The investigation of exact travelling wave solutions for nonlinear partial differential equations (NLPDEs) plays an important role in the study of nonlinear physical phenomena. Many powerful methods to construct exact solutions of NLPDEs have been established and developed, which lead to one of the most excited advances of nonlinear science and theoretical physics. In fact, many kinds of exact soliton solutions have been obtained by using, for example, the homogeneous balance method (see Wang (1995), Zayed et al. (2004)), the hyperbolic tangent expansion method (see Yang et al. (2001), Zayed et al. (2004)), the trial function method (see Inc and Evans (2004)), the tanh-method (see Abdou (2007), Fan (2000), Malfliet (1992)), the non-linear transform method (see Hu (2004)), the inverse scattering transform (see Ablowitz and Clarkson (1991)), the Backlund transform (see Miura (1978), Rogers and Shadwick (1982)), the Hirota's bilinear method (see Hirota (1973), Hirota and Satsuma (1981)), the generalized Riccati equation method (see Yan and Zhang (2001), Porubov (1996)), the Jacobi elliptic functions method (see Liu et al. (2001), Yan (2003)), F-expansion method (see Wang and Li (2005), Wang and Li (2005)), and so on. The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli Sub-ODE method to obtain travelling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent section, we will apply the method for finding exact travelling wave solutions for the nonlinear Schrödinger equation with power law nonlinearity. In the last section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE Method

In this section we present the solutions of the following ODE,

$$G' + \lambda G = \mu G^2, \quad (1)$$

where $\lambda \neq 0$, $G = G(\xi)$. When $\mu \neq 0$, equation (1) is the type of Bernoulli equation, and we can obtain the solution as:

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \quad (2)$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, \dots) = 0, \quad (3)$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of equation (1), we can construct a series of exact solutions of nonlinear equations:

Step 1. We suppose that

$$u(x, y, t) = u(\xi), \quad \xi = \xi(x, y, t), \quad (4)$$

the travelling wave variable (4) permits us reducing equation (3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', \dots) = 0. \quad (5)$$

Step 2. Suppose the solution of (5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + \dots, \quad (6)$$

where $G = G(\xi)$ satisfies equation (3), and $\alpha_m, \alpha_{m-1}, \dots$ are constants to be determined later, $\alpha_m \neq 0$, m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (5).

Step 3. Substituting (6) into (5) and using (1), collecting all the terms with the same order of G together, the left hand side of equation (5) is converted into another polynomial in G . Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, \dots, \lambda, \mu$.

Step 4. Solving the algebraic equations system in step 3, and by using the solutions of equation (1), we can construct the travelling wave solutions of the nonlinear evolution equations (5).

3. Application of the Bernoulli Sub-ODE Method For Schrödinger equation with power law nonlinearity

In this section, we will consider the following NLS equation with power law nonlinearity

$$iw_t + w_{xx} + a|w|^{2n+1}w = 0, \quad (7)$$

where a is a real parameter and $w = w(x, t)$ is a complex-valued function of two real variable x, t . We use the transformation

$$w(x, t) = \phi(\xi) \exp[i(\alpha x + \beta t)], \quad \xi = k(x - 2\alpha t), \quad (8)$$

where k, α and β are constants, all of them are to be determined.

Substitution (8) into (7), we obtain ordinary differential equation:

$$-(\beta + \alpha^2)\phi + k^2\phi_{\xi\xi} + a\phi^{2n+1} = 0. \quad (9)$$

Due to the difficulty in obtaining the Sub-ODE of equation (9), we suppose a transformation denoted by $u = v^{\frac{1}{n}}$. Therefore, equation (9) is converted to

$$nv''v + (1 - n)(v')^2 - n^2Av^2 + n^2Bv^4 = 0, \quad (10)$$

where $A = \frac{\beta + \alpha^2}{k^2}$, $B = \frac{a}{k^2}$.

Suppose that the solution of (10) can be expressed by a polynomial in G as follows:

$$v(\xi) = \sum_{i=0}^L b_i G^i, \quad (11)$$

where b_i are constants and $G = G(\xi)$ satisfies equation (1).

Balancing the order of v^4 with the order of $(v')^2$ in equation (11), we find $L = 1$. So the solution takes on the form

$$v(\xi) = b_1 G + b_0, \quad b_1 \neq 0, \quad (12)$$

where b_0 and b_1 are constants to be determined later.

Substituting (12) into the ODE (10) and collecting all the terms with the same power of G together and equating each coefficient to zero, yields a set of the simultaneous algebraic equations as follows:

$$\begin{aligned} G^4 &: 2nb_1^2\mu^2 + b_1^2\mu^2 - nb_1^2\mu^2 + n^2Bb_1^4 = 0, \\ G^3 &: -3b_1^2n\mu\lambda + 2b_0b_1n\mu^2 - 2b_1^2\mu\lambda + 2b_1^2n\mu\lambda + 4n^2Bb_0b_1^3 = 0, \\ G^2 &: nb_1^2\lambda^2 - 3nb_0b_1\mu\lambda + b_1^2\lambda^2 - nb_1^2\lambda^2 - n^2Ab_1^2 + 6b_0^2b_1^2n^2B = 0, \\ G^1 &: nb_0b_1\lambda^2 - 2n^2Ab_0b_1 + 4b_0^3b_1n^2B = 0, \\ G^0 &: -n^2Ab_0^2 + n^2Bb_0^4 = 0. \end{aligned}$$

Solving the algebraic equations above, yields:

Case 1

$$b_0 = \sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad b_1 = \sqrt{\frac{n+1}{\beta}} \frac{\mu}{n}, \quad \beta = b_0^2\alpha - \alpha^2. \quad (13)$$

Substituting (13) into (12), we have

$$v_1(\xi) = \sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} G + \sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad \xi = k(x - 2\alpha t). \quad (14)$$

Combining with equation (2) and considering $\phi = v^{\frac{1}{n}}$, we can obtain the travelling wave solutions of NLS equation with power law nonlinearity as follows:

$$\phi_1(\xi) = \left(\sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right) + \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}}, \quad \xi = k(x - 2\alpha t), \quad (15)$$

where d is an arbitrary constant.

Substituting (15) into (8), we have

$$w_1(\xi) = \left(\sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right) + \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + \beta t)], \quad \xi = k(x - 2\alpha t).$$

Then we have

$$w_1(x, t) = \left(\sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k(x-2\alpha t)}} \right) + \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + (b_0^2\alpha - \alpha^2)t)].$$

Case 2

$$b_0 = -\sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad b_1 = \sqrt{\frac{n+1}{\beta}} \frac{\mu}{n}, \quad \beta = b_0^2\alpha - \alpha^2. \quad (16)$$

Substituting (16) into (12), we have

$$v_2(\xi) = \sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} G - \sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad \xi = k(x - 2\alpha t). \quad (17)$$

Combining with equation (2) and considering $\phi = v^{\frac{1}{n}}$, we can obtain the travelling wave solutions of NLS equation with power law nonlinearity as follows:

$$\phi_2(\xi) = \left(\sqrt{\frac{n+1}{\beta} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right)} - \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}}, \quad \xi = k(x - 2\alpha t), \quad (18)$$

where d is an arbitrary constant.

Substituting (18) into (8), we have

$$w_2(\xi) = \left(\sqrt{\frac{n+1}{\beta} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right)} - \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + \beta t)], \quad \xi = k(x - 2\alpha t).$$

Then we have

$$w_2(x, t) = \left(\sqrt{\frac{n+1}{\beta} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k(x-2\alpha t)}} \right)} - \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + (b_0^2 \alpha - \alpha^2)t)].$$

Case 3

$$b_0 = \sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad b_1 = -\sqrt{\frac{n+1}{\beta} \frac{\mu}{n}}, \quad \beta = b_0^2 \alpha - \alpha^2. \quad (19)$$

Substituting (19) into (12), we have

$$v_3(\xi) = -\sqrt{\frac{n+1}{\beta} \frac{\mu}{n}} G + \sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad \xi = k(x - 2\alpha t). \quad (20)$$

Combining with equation (2) and considering $\phi = v^{\frac{1}{n}}$, we can obtain the travelling wave solutions of NLS equation with power law nonlinearity as follows:

$$\phi_3(\xi) = \left(-\sqrt{\frac{n+1}{\beta} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right)} + \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}}, \quad \xi = k(x - 2\alpha t), \quad (21)$$

where d is an arbitrary constant.

Substituting (21) into (8), we have

$$w_3(\xi) = \left(-\sqrt{\frac{n+1}{\beta} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right)} + \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + \beta t)], \quad \xi = k(x - 2\alpha t).$$

Then we have

$$w_3(x, t) = \left(-\sqrt{\frac{n+1}{\beta} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k(x-2\alpha t)}} \right)} + \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + (b_0^2 \alpha - \alpha^2)t)].$$

Case 4

$$b_0 = -\sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad b_1 = -\sqrt{\frac{n+1}{\beta} \frac{\mu}{n}}, \quad \beta = b_0^2 \alpha - \alpha^2. \quad (22)$$

Substituting (22) into (20), we have

$$v_4(\xi) = -\sqrt{\frac{n+1}{\beta} \frac{\mu}{n}} G - \sqrt{\frac{\beta + \alpha^2}{\alpha}}, \quad \xi = k(x - 2\alpha t). \quad (23)$$

Combining with equation (2) and considering $\phi = v^{\frac{1}{n}}$, we can obtain the travelling wave solutions of NLS equation with power law nonlinearity as follows:

$$\phi_4(\xi) = \left(-\sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right) - \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}}, \quad \xi = k(x - 2\alpha t), \quad (24)$$

where d is an arbitrary constant.

Substituting (24) into (8), we have

$$w_4(\xi) = \left(-\sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}} \right) - \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + \beta t)], \quad \xi = k(x - 2\alpha t).$$

Then we have

$$w_4(x, t) = \left(-\sqrt{\frac{n+1}{\beta}} \frac{\mu}{n} \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k(x-2\alpha t)}} \right) - \sqrt{\frac{\beta + \alpha^2}{\alpha}} \right)^{\frac{1}{n}} \exp[i(\alpha x + (b_0^2 \alpha - \alpha^2)t)].$$

4. Conclusion

In this paper, we have seen that some new traveling wave solutions of NLS equation with power law nonlinearity are successfully found by using the Bernoulli sub-ODE method. This method is concise and effective and can be fulfilled by the aid of the mathematical software Maple. Also this method can be applied to many other nonlinear problems.

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REFERENCES

- Abdou, M. A. (2007). The extended tanh-method and its applications for solving nonlinear physical models, Appl. Math. Comput., Vol. 190, pp. 988-996.
- Ablowitz, M. J. and Clarkson, P. A. (1991). *Solitons, Non-linear Evolution Equations and Inverse Scattering Transform*, Cambridge University Press, Cambridge.
- Bhrawy, A. H., Alshaary, A. A., Hilal, E. M., Millovic, D., Moraru, L., Savescu, M. and Biswas, A. (2014). Optical solitons with polynomial and triple power law nonlinearities and spatio-temporal dispersion, Proceedings of the Romanian Academy, Series A, Vol. 15, pp. 235-240.
- Biswas, A., Bhrawy, A. H., Alshaery, A. A. and Hilal, E. M. (2014). Thirring optical solitons with Kerr law nonlinearity, Optik, Vol. 125, pp. 4946-4948.
- Biswas, A. (2003). Quasistationary optical solitons with power law nonlinearity, Journal of Physics A, Vol. 36, pp. 4581-4589.

- Fan, E. G. (2000). Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A, Vol. 277, pp. 212-218.
- Hu, J. L. (2004). A new method of exact traveling wave solution for coupled nonlinear differential equations, Phys. Lett. A, Vol. 322, pp. 211-216.
- Hirota, R. (1973). Exact envelope soliton solutions of a nonlinear wave equation, J. Math. Phys., Vol. 14, pp. 805-810.
- Hirota, R. and Satsuma, J. (1981). Soliton solution of a coupled KdV equation, Phys. Lett. A, Vol. 85, pp. 407-408.
- Inc, M. and Evans, D. J. (2004). On traveling wave solutions of some nonlinear evolution equations, Int. J. Comput. Math., Vol. 81, pp. 191-202.
- Khalique, C. M. and Biswas, A. (2009). Optical solitons with power law nonlinearity using lie group analysis, Physics Letters A, Vol. 373, pp. 2047-2049.
- Liu, S. K., Fu, Z. T., Liu, S. D. and Zhao, Q. (2001). Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A, Vol. 289, pp. 69-74.
- Malfliet, W. (1992). Solitary wave solutions of nonlinear wave equations, Am. J. Phys., Vol. 60, pp. 650-654.
- Miura, M. R. (1978). *Backlund Transformation*, Springer-Verlag, Berlin.
- Mirzazadeh, M., Eslami, M., Fathi Vajargah, B. and Biswas, A. (2014). Optical solitons and optical rogons of generalized resonant dispersive nonlinear Schrodinger's equation with power law nonlinearity, Optik, Vol. 125, pp. 4246-4256.
- Mirzazadeh, M., Zhou, Q., Biswas, A. and Belic, M. (2015). Optical solitons of generalized Resonant dispersive nonlinear Schrodinger's equation with power law nonlinearity, Opto-electronics and Advanced Materials-Rapid Communications, Vol. 9, pp. 1100-1103.
- Porubov, A. V. (1996). Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, Phys. Lett. A, Vol. 221, pp. 391-394.
- Rogers, C. and Shadwick, W. F. (1982). *Backlund Transformations*, Academic Press, New York.
- Savescu, M., Khan, K. R., Naruka, P., Jafari, H., Moraru, L. and Biswas, A. (2013). Optical solitons in photonic nanowaveguides with an improved nonlinear Schrodinger's equation, Journal of Computational and Theoretical Nanoscience, Vol. 10, pp. 1182-1191.
- Wazwaz, A. M. (2009). Partial differential equations and solitary wave theory: Schrodinger equation with power law nonlinearity, Springer-Verlag, Berlin, pp. 690-691.
- Wang, M. L. (1995). Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A, Vol. 199, pp. 169-172.
- Wang, M. L. and Li, X. Z. (2005). Extended F-expansion and periodic wave solutions for the generalized Zakharov equations, Phys. Lett. A, Vol. 343, pp. 48-54.
- Wang, M. L. and Li, X. Z. (2005). Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation, Chaos, Solitons and Frac., Vol. 24, pp. 1257-1268.
- Yang, L., Liu, J. and Yang, K. (2001). Exact solutions of nonlinear PDE nonlinear transformations and reduction of nonlinear PDE to a quadrature, Phys. Lett. A, Vol. 278, pp. 267-270.
- Yan, Z. Y. and Zhang, H. Q. (2001). New explicit solitary wave solutions and periodic wave solutions for Whitham-Broer-Kaup equation in shallow water, Phys. Lett. A, Vol. 285, pp. 355-362.
- Yan, Z. (2003). Abundant families of Jacobi elliptic functions of the (2+1)-dimensional inte-

grable Davey-Stewartson-type equation via a new method, *Chaos, Solitons and Frac.*, Vol. 18, pp. 299-309.

Zayed, E. M. E., Zedan, H. A. and Gepreel, K. A. (2004). Group analysis and modified tanh-function to find the invariant solutions and soliton solution for nonlinear Euler equations, *Int. J. Nonlinear Sci. Numer. Simul.*, Vol. 5, pp. 221-234.

Zayed, E. M. E., Zedan, H. A. and Gepreel, K. A. (2004). On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, *Chaos, Solitons and Frac.*, Vol. 22, pp. 285-303.