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Application of Taylor-Padé technique for obtaining approximate solution for system of linear Fredholm integro-differential equations

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Abstract

In this article, we introduce a modification of the Taylor matrix method using Padé approximation to obtain an accurate solution of linear system of Fredholm integro-differential equations (FIDEs). This modification is based on, first, taking truncated Taylor series of the functions and then substituting their matrix forms into the given equations. Thereby the equation reduces to a matrix equation, which corresponds to a system of linear algebraic equations with unknown Taylor coefficients. Finally, we use Padé approximation to obtain an accurate numerical solution of the proposed problem. To demonstrate the validity and the applicability of the proposed method, we present some numerical examples. A comparison with the standard Taylor matrix method is given.

Keywords: Taylor matrix method, Padé approximation, System of Fredholm integro-differential equations

MSC 2010 No.: 65N20; 41A30

1. Introduction

There are several approximate methods for solving linear and non-linear integro-differential equations, for example, the variational iteration method (Biazar, et al. (2010)), Galerkin methods with hybrid functions (Maleknejad and Tavassoli (2004)), Tau method (Pour-Mahmoud, et al. (2005)), collocation method (Aky$\ddot{u}$z and Yaslan (2011) & Yusufoglu (2014)), power series method (Gachpazan (2009)) and others (Elzaki and Ezaki (2011), Khader and Ahmed (2015)-Linz (1985) & Mohamed and Khader (2011)).

The Taylor series is a representation of a function as a sum of terms that are calculated from the values of the function's derivatives at a single point. Padé approximation is the best approximation of a function by a rational function of given order with this technique, the approximate power series agrees with the power series of the approximating function. The Padé approximation often gives better approximation of the function than truncating its Taylor series, and it may still work where the Taylor series not convergent.

In this paper, we solve linear system of Fredholm integro-differential equations of the form

$$\begin{aligned} y_r^{(m)}(x) =& f_r(x) + \lambda_r \int_a^b \Big[K_r(x,t)F_r(y_1(t), y_2(t), ..., y_M(t))\Big]dt, \\ & r = 1, 2, ..., M, \quad m \in \mathbb{N}, \end{aligned} \tag{1}$$

with the initial conditions

$$y_r^{(k)}(a) = \gamma_{r,k}, \quad k = 0, 1, ..., m-1, \quad r = 1, 2, ..., M, \tag{2}$$

where m is the order of the derivative, $f_r(x)$ and kernel $K_r(x,t)$ are given functions and λ_r, $\gamma_{r,k}$ are suitable constants. We assume that the solution is expressed in Taylor polynomials

$$y_r(x) = \sum_{n=0}^{N} y_{r,n}(x-c)^n, \quad y_{r,n} = \frac{y_r^{(n)}(c)}{n!}, \quad a \leq c \leq b, \tag{3}$$

so, that the Taylor coefficients to be determined are $y_{r,n},\ r = 1, 2, ..., M,\ n = 0, 1, ..., N$.

2. Matrix relations and the fundamental matrix relation

To construct the matrix form of the proposed problem (1), let us rewrite it in the following form

$$D_r(x) = f_r(x) + \lambda_r J_r(x), \quad r = 1, 2, ..., M, \tag{4}$$

where

$$D_r(x) = y_r^{(m)}(x), \quad \text{and} \quad J_r(x) = \int_a^b \Big[K_r(x,t)F_r(y_1(t), y_2(t), ..., y_M(t))\Big]dt. \tag{5}$$

Now, we convert the solution $y_r(x)$ and its derivative $y_r^{(m)}(x)$, the parts $D_r(x)$ and $J_r(x)$ and the initial conditions to a matrix form.

I. Matrix relation for the differential part $D_r(x)$

We first consider the desired solution $y_r(x)$, $r = 1, 2, ..., M$, of Equation (4) defined by the truncated Taylor series (3). Then, we can write Equation (3) in the form

$$[y_r(x)] = X(x)Y_r, \quad r = 1, 2, ..., M, \tag{6}$$

where

$$X(x) = [\ 1 \quad (x-c) \quad ... \quad (x-c)^N\ ], \qquad Y_r = [\ y_{r,0} \quad y_{r,1} \quad ... \quad y_{r,N}\ ].$$

On the other hand, it is clear that the relation between the matrix $X(x)$ and its first derivative $X^{(1)}(x)$ is

$$X^{(1)}(x) = X(x)B^T, \tag{7}$$

where

$$B = \begin{pmatrix} 0 & 0 & ... & 0 & 0 \\ 1 & 0 & ... & 0 & 0 \\ 0 & 2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & N & 0 \end{pmatrix}.$$

Using the matrix Equation (7), we can deduce the matrix solution of k-th derivative as follows

$$\begin{aligned} X^{(1)}(x) &= X(x)B^T, \\ X^{(2)}(x) &= X^{(1)}(x)B^T = X(x)(B^T)^2, \\ &\vdots \\ X^{(k)}(x) &= X^{(k-1)}(x)(B^T)^{(k-1)} = X(x)(B^T)^k. \end{aligned} \tag{8}$$

Using the relations (6)-(8) we have the matrix relations

$$y_r^{(k)}(x) = X^{(k)}(x)Y_r = X(x)(B^T)^kY_r, \quad k = 1, 2, ..., m, \quad r = 1, 2, ..., M. \tag{9}$$

By substituting the relation (9) into (5), we can construct the formula of the differential part $D_r(x)$ as follows

$$[D_r(x)] = X(x)(B^T)^kY_r, \quad r = 1, 2, ..., M. \tag{10}$$

II. Matrix relation for the integral part $J_r(x)$

The kernel function $K_r(x, t)$ can be approximated by the truncated Taylor series of degree N about $x = c$, $t = c$ in the form

$$K_r(x, t) = \sum_{i=0}^{N} \sum_{j=0}^{N} k_{r,ij}(x-c)^i(t-c)^j, \quad r = 1, 2, ..., M, \tag{11}$$

where $k_{r,ij} = \frac{1}{i!j!}\frac{\partial^{i+j}K_r(c,c)}{\partial x^i \partial t^j}, \quad i,j = 0,1,...,N$. Then, Equation (11) can be written in the form

$$[K_r(x,t)] = X(x)K_r X^T(t), \tag{12}$$

where

$$X(t) = [1 \ \ (t-c) \ \ ... \ \ (t-c)^N], \ \ K_r = [k_{r,ij}], \ \ i,j = 0,1,...,N.$$

By substituting the matrix forms (5) and (12), we have the integral matrix relation

$$[J_r(x)] = \int_a^b X(x)K_r X^T(t)F_r\Big(Y_1, Y_2, ..., Y_M\Big)dt, \quad r = 1,2,...,M. \tag{13}$$

III. Matrix representation of the function $f_r(x)$

The matrix representation of the non-homogenous term of Equation (1) can be written in the form

$$[f_r(x)] = \sum_{n=0}^{N} f_{r,n}(x-c)^n = X(x)F_r, \quad f_{r,n} = \frac{f_r^{(n)}(c)}{n!}, \ F_r = [f_{r,0} \ f_{r,1} \ ... \ f_{r,N}]^T. \tag{14}$$

IV. Matrix relation for the initial-boundary conditions

We can obtain the corresponding matrix forms for the initial-boundary conditions (2) as

$$X(a)(B^T)^k Y_r = [\gamma_{r,k}], \quad k = 0,1,...,m-1, \quad r = 1,2,...,M. \tag{15}$$

3. The Padé approximation of the series solution

The general setup in approximation theory is that a function f is given and that one wants to approximate it with a simpler function g but in such a way that the difference between f and g is small. The advantage is that the simpler function g can be handled without too many difficulties, but the disadvantage is that one loses some information since f and g are different.

Definition 1.

When we obtain the truncated series solution of order at least $L+M$ in x by Taylor method, we use it to obtain the Padé approximation $PA[L/M](x)$, for the function $y(x)$. The Padé approximation is a particular type of rational fraction approximation to the value of the function. The idea is to match the Taylor series expansion as far as possible. We denote $PA[L/M](x)$ to $R(x) = \sum_{i=0}^{\infty} a_i \, x^i$ by

$$PA[L/M](x) = \frac{P_L(x)}{Q_M(x)}, \tag{16}$$

where $P_L(x)$ and $Q_M(x)$ are polynomials of degree at most L and M, respectively

$$\begin{aligned} P_L(x) &= p_0 + p_1\,x + p_2\,x^2 + ... + p_L\,x^L, \\ Q_M(x) &= 1 + q_1\,x + q_2\,x^2 + ... + q_M\,x^M. \end{aligned} \tag{17}$$

To determine the coefficients of $P_L(x)$ and $Q_M(x)$, we may multiply (16) by $Q_M(x)$, which linearizes the coefficient equations. We can write out (16) in more detail as (Baker (1975))

$$
\begin{aligned}
&a_{L+1} + a_L q_1 + ... + a_{L-M+1} q_M = 0,\\
&a_{L+2} + a_{L+1} q_1 + ... + a_{L-M+2} q_M = 0,\\
&...\\
&a_{L+M} + a_{L+M-1} q_1 + ... + a_L q_M = 0,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&a_0 = p_0,\\
&a_1 + a_0 q_1 = p_1,\\
&a_2 + a_1 q_1 + a_0 q_2 = p_2,\\
&...\\
&a_L + a_{L-1} q_1 + ... + a_0 q_L = p_L.
\end{aligned}
\tag{19}
$$

To solve these equations, we start with Equation (18), which is a set of linear equations for all the unknowns $q's$. Once the $q's$ are known, then Equation (19) gives an explicit formula for the unknowns $p's$, which complete the solution. Each choice of L, degree of the numerator and M, degree of the denominator, leads to an approximation. The major difficulty in applying this technique is how to direct the choice in order to obtain the best approximation. This needs the use of a criterion for the choice depending on the shape of the solution. A criterion which has worked well here is the choice of $[L/M]$ approximation such that $L = M$ (Abassy, et al. (2007), Baker (1975), & Yang et al. (2009)).

4. Procedure of solution using Padé-Taylor technique

In spite of the advantages of Taylor approximation, it has some drawbacks. By using this approximation, we can obtain a series, in practice a truncated series solution. Although the series can be rapidly convergent in a very small region, it is very slow convergence rate in the wider region we examine and the truncated series solution is an inaccurate solution in that region, which will greatly restrict the application area of the method. All the truncated series solutions have the same problem. Many examples given can be used to support this assertion (Yang et al. (2009)).

In this section, we present a modification of Taylor approximation by using the Padé approximation and then apply this modification to solve numerically linear system of Fredholm integro-differential equations. The suggested modification of Taylor approximation can be given by using the following algorithm.

Algorithm

Step 1. Solve the system of IDEs using standard Taylor approximation;
Step 2. Truncate the obtained series solution by using Taylor approximation;
Step 3. Compute the Padé approximation of the previous step.

This modification often gives an accurate and efficient solution of the integro-differential equations with high accuracy and enlarge the convergence domain of the truncated Taylor series and can improve greatly the convergence rate of the truncated Taylor series.

Now, we implement this algorithm for some examples of linear system of Fredholm integro-differential equations to illustrate our modification.

5. Numerical examples

In this section, to achieve validity, accuracy and support our theoretical discussion of the proposed technique, we introduce some computational results of numerical examples.

Example 1.

Consider Equation (1) with the following functions and coefficients

$$f_1(x) = -1 + \cos(1) - \sin(x), \quad f_2(x) = \cos(x) - \sin(1), \quad K_1(x,t) = 1, \quad K_2(x,t) = 1,$$
$$F_1(y_1, y_2) = y_2(t), \quad F_2(y_1, y_2) = y_1(t), \quad r = 2, m = 1, \ \lambda_1 = \lambda_2 = 1, \quad a = 0, \ b = 1.$$

In this case, Equation (1) takes the form

$$\begin{aligned} y_1'(x) &= -1 + \cos(1) - \sin(x) + \int_0^1 y_2(t)dt, \\ y_2'(x) &= \cos(x) - \sin(1) + \int_0^1 y_1(t)dt, \end{aligned} \tag{20}$$

with initial conditions $y_1(0) = 1$, $y_2(0) = 0$. The exact solution of this system is $y_1(x) = \cos(x)$, $y_2(x) = \sin(x)$. We apply the suggested algorithm as follows:

1. Solve Equations (20) using Taylor matrix method:

Let us approximate the solution with $N = 4$ as follows

$$y_r(x) = \sum_{n=0}^{4} y_{r,n}(x-c)^n, \quad y_{r,n} = \frac{y_r^{(n)}(c)}{n!}, \ c = 0, \quad r = 1, 2, \tag{21}$$

using Equation (10) we obtain the matrix relation for the differential part as follows

$$D[y_r] = X(x)B^T Y_r, \quad r = 1, 2, \tag{22}$$

where

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix},$$

and the matrix relation for the integral part defined by Equation (13) is given by

$$J=\begin{bmatrix}0&0&0&0&0&1&0.5&0.33&0.25&0.2\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\1&0.5&0.33&0.25&0.2&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\end{bmatrix},$$

and $F=\begin{bmatrix}-0.460&-1&0&0.167&0&0.159&0&-0.5&0&0.042\end{bmatrix}^T$. Then, the system of integro-differential equations can be written as a system of algebraic equations $PY=F$, where

$$P=B^T-J=\begin{bmatrix}0&1&0&0&0&-1&-0.6&-0.33&-0.25&-0.2\\0&0&2&0&0&0&0&0&0&0\\0&0&0&3&0&0&0&0&0&0\\0&0&0&0&4&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\-1&-0.5&-0.33&-0.25&-0.2&0&1&0&0&0\\0&0&0&0&0&0&0&2&0&0\\0&0&0&0&0&0&0&0&3&0\\0&0&0&0&0&0&0&0&0&4\\0&0&0&0&0&0&0&0&0&0\end{bmatrix}.$$

To confirm the initial conditions, we replace the last two rows of the previous matrix with

$$u_{1,0}=[1,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0],$$

$$u_{2,0}=[0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 1,\quad 0,\quad 0,\quad 0,\quad 0].$$

By solving the linear system $PY=F$ for the unknowns $y_{r,i},\ i=0,1,2,3,4,\ r=1,2$ we obtain

$$\begin{array}{lllll}y_{1,0}=1, & y_{1,1}=-0.0017, & y_{1,2}=-0.5, & y_{1,3}=0.0, & y_{1,4}=0.0417,\\ y_{2,0}=0.0, & y_{2,1}=0.9994, & y_{2,2}=0.0, & y_{2,3}=-0.1667, & y_{2,4}=0.0.\end{array}$$

So, the approximate solution using Taylor expansion is given by

$$y_1(x)=1-0.0017x-0.5x^2+0.0417x^4,$$
$$y_2(x)=0.9994x-0.1667x^3.$$

The behavior of the approximate solution and the exact solution is presented in Figure 1.

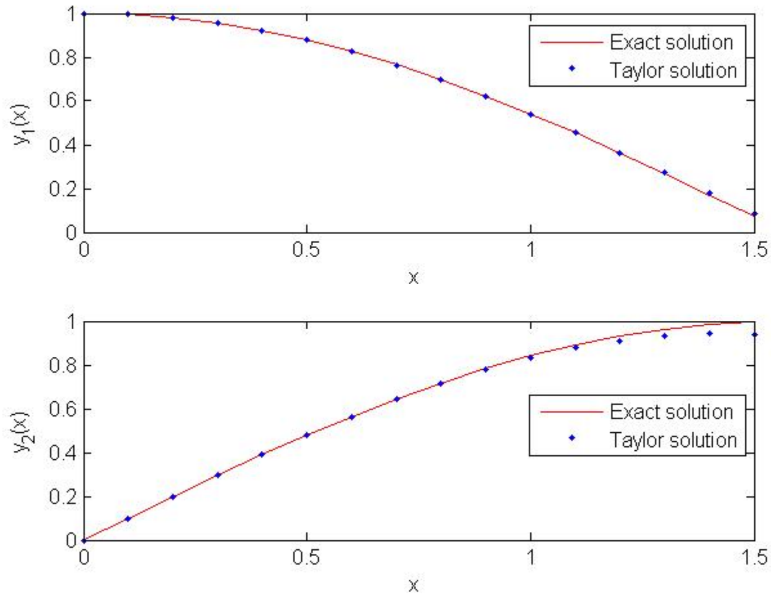

Figure 1: Configuration of Jupiter-Europa System in the framework of the Circular Restricted Three-Body Problem

2. Compute Padé approximation:

The Padé approximation $PA_r[2/2](x)$ is given by

$$PA_1[2/2](x) = \frac{1 - 0.0020x - 0.4166x^2}{1 - 0.0003x + 0.0834x^2},$$
$$PA_2[2/2](x) = \frac{0.9994x}{1 + 0.1668x^2}.$$

The behavior of the exact solution and the approximate solution using Padé approximation is presented in Figure 2. From Figures 1 and 2, we can see that the solution using Padé approximation is in excellent agreement with the exact solution, and more convergent to the exact solution for large domain than Taylor solution. This conclusion ensure the advantages of the proposed technique.

Example 2.

Consider Equation (1) with the following functions and coefficients

$$f_1(x) = e^x - \frac{(-2+e)x}{e}, \quad f_2(x) = -1 + e^{-x} - (-1+e)x, \quad K_1(x,t) = x\,t, \quad K_2(x,t) = x + t,$$

$$F_1(y_1, y_2) = y_2(t), \quad F_2(y_1, y_2) = y_1(t), \quad r = 2, \quad m = 2, \quad \lambda_1 = \lambda_2 = 1, \quad a = 0, \quad b = 1.$$

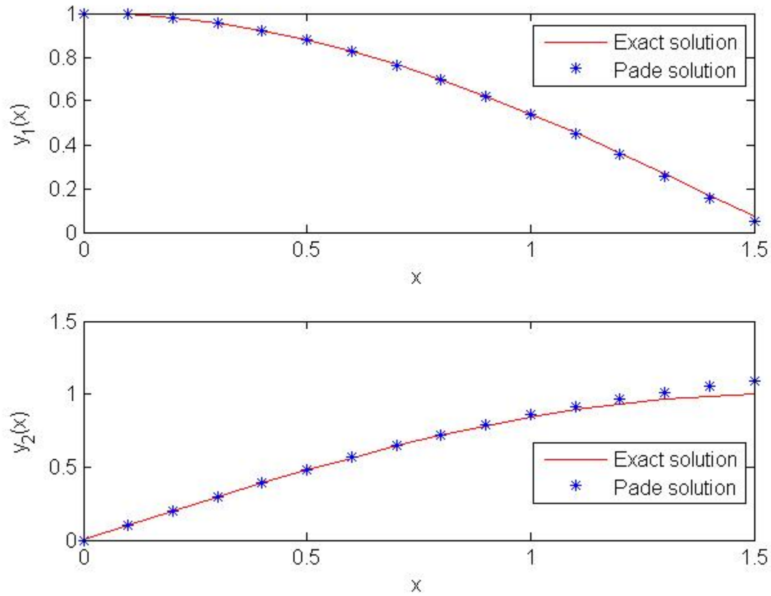

Figure 2: The behavior of the exact solution and the Padé approximate solution at $N = 4$.

In this case, Equation (1) takes the form

$$\begin{aligned} y_1''(x) &= e^x - \frac{(-2+e)x}{e} + \int_0^1 x\, t\, y_2(t)dt, \\ y_2''(x) &= -1 + e^{-x} - (-1+e)x + \int_0^1 (x+t)\, y_1(t)dt, \end{aligned} \tag{23}$$

with initial conditions $y_1(0) = 1$, $y_1'(0) = 1$, $y_2(0) = 1$, $y_2'(0) = -1$. The exact solution is $y_1(x) = e^x$, $y_2(x) = e^{-x}$. We apply the suggested algorithm as follows:

1. Solve Equations (23) using Taylor matrix method:

Let us approximate the solution with $N = 4$ as follows

$$y_r(x) = \sum_{n=0}^{4} y_{r,n}(x-c)^n, \quad y_{r,n} = \frac{y_r^{(n)}(c)}{n!}, \quad c = 0, \quad r = 1, 2. \tag{24}$$

using Equation (10) we obtain the matrix relation for the differential part as follows

$$D[y_r] = X(x)(B^T)^2 Y_r, \quad r = 1, 2, \tag{25}$$

where

$$B=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0&0\\0&2&0&0&0&0&0&0&0&0\\0&0&3&0&0&0&0&0&0&0\\0&0&0&4&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&2&0&0&0\\0&0&0&0&0&0&0&3&0&0\\0&0&0&0&0&0&0&0&4&0\end{bmatrix},$$

the matrix relation for the integral part defined by Equation (13) is given by

$$J=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0.5&0.33&0.25&0.2&0.1667\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0.5&0.33&0.25&0.2&0.1667&0&0&0&0&0\\1&0.5&0.33&0.25&0.2&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\end{bmatrix},$$

and $F=\begin{bmatrix}1 & 0.7358 & 0.50 & 0.1667 & 0.0417 & 0 & -2.7183 & 0.50 & -0.1667 & 0.0417\end{bmatrix}^T$. Then, the system of integro-differential equations can be written as a system of algebraic equations $PY=F$, where

$$P=(B^T)^2-J=\begin{bmatrix}0&0&2&0&0&0&0&0&0&0\\0&0&0&6&0&-0.5&-0.33&-0.25&-0.2&-0.1667\\0&0&0&0&12&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\\-0.5&-0.33&-0.25&-0.2&-0.1667&0&0&2&0&0\\-1&-0.5&-0.33&-0.25&-0.2&0&0&0&6&0\\0&0&0&0&0&0&0&0&0&12\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0\end{bmatrix}.$$

To confirm the initial conditions, we replace the last four rows of the previous matrix with

$$u_{1,0}=[1,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0],$$

$$u_{2,0}=[0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 1,\quad 0,\quad 0,\quad 0,\quad 0],$$

$$u_{1,1}=[0,\quad 1,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0],$$

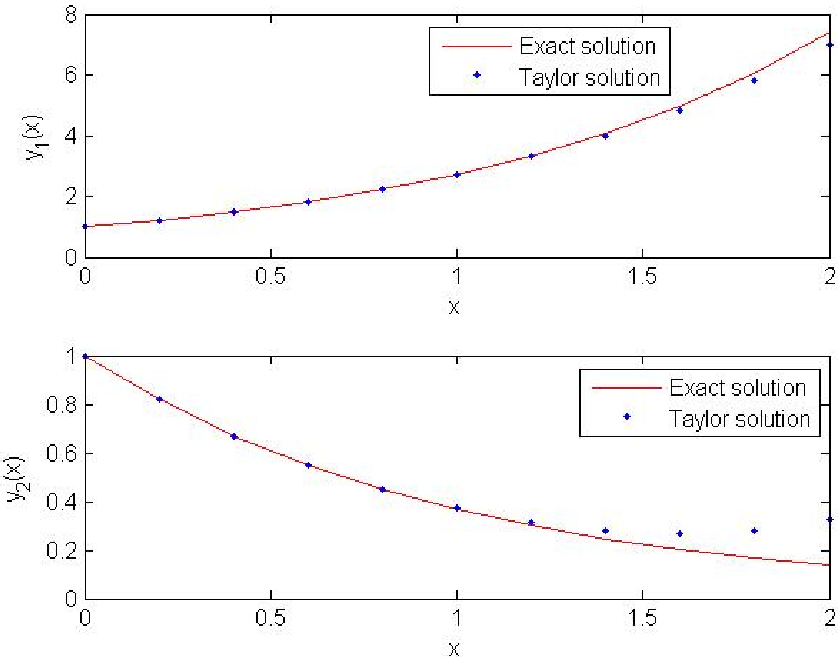

Figure 3: The behavior of the exact solution and the approximate solution using Taylor expansion method at $N = 4$.

$$u_{2,1} = [0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 1, \quad 0, \quad 0, \quad 0].$$

By solving the linear system $PY = F$ for the unknowns $y_{r,i},\ i = 0, 1, 2, 3, 4,\ r = 1, 2$ we obtain

$$\begin{array}{ccccc} y_{1,0} = 1, & y_{1,1} = 1, & y_{1,2} = 0.5, & y_{1,3} = 0.1668, & y_{1,4} = 0.0417, \\ y_{2,0} = 1, & y_{2,1} = -1, & y_{2,2} = 0.4993, & y_{2,3} = -0.1669, & y_{2,4} = 0.0417. \end{array}$$

So, the approximate solution using Taylor expansion is given by

$$y_1(x) = 1 + x + 0.5x^2 + 0.1668x^3 + 0.0417x^4,$$
$$y_2(x) = 1 - x + 0.4993x^2 - 0.1669x^3 + 0.0417x^4.$$

The behavior of the exact solution and the approximate solution is presented in Figure 3.

2. Compute Padé approximation:

The Padé approximation $PA_r[2/2](x)$ is given by

$$PA_1[2/2](x) = \frac{1 + 0.498798x + 0.082599x^2}{1 - 0.501202x + 0.083801x^2},$$

$$PA_2[2/2](x) = \frac{1 - 0.494746x + 0.0794194x^2}{1 + 0.505254x + 0.0853733x^2}.$$

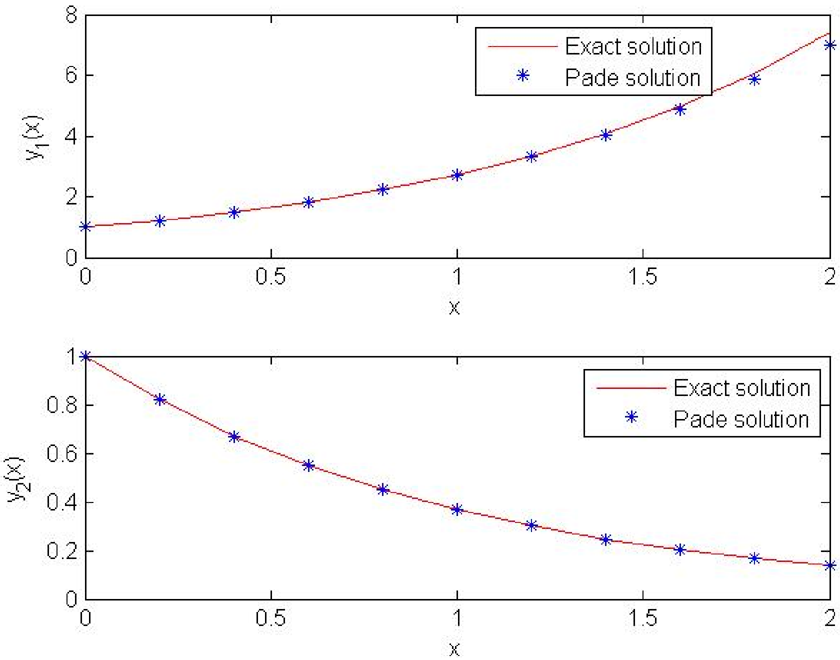

Figure 4: The behavior of the exact solution and the Padé approximate solution at $N = 4$.

The behavior of the exact solution, and the approximate solution using Padé approximation is presented in Figure 4. From Figures 3 and 4, we can see that the solution using Padé approximate is in excellent agreement with the exact solution, and more convergent to the exact solution for large domain than Taylor solution. This conclusion ensure the advantages of the proposed technique.

6. Conclusions

In this article, we presented the numerical solutions for system of integro-differential equations by using Taylor polynomials. Also, to increase the convergence region we used the Padé technique to improve the solution using Taylor matrix method. From the obtained numerical solution of the proposed linear system by using the proposed algorithm, we can conclude that our solutions are excellent agreement with the exact solution and more convergent to the solution using the standard Taylor matrix method with large domain.

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REFERENCES

Abassy, T. A., Magdy A. El-Tawil and El Zoheiry, H. (2007). Solving nonlinear partial differential equations using the modified variational iteration Padé technique, Journal of Computational and Applied Mathematics, Vol. 207, pp. 73–91.

Akyüz, A. and Yaslan, H. (2011). The solution of high-order non-linear ordinary differential equations by Chebyshev series, Applied Mathematics and Computation, Vol. 217, pp. 5658–5666.

Baker, G. A. J. (1975). *Essentials of Padé Approximants*, Academic Press.

Biazar, J. Porshokouhi, M. G. and Ebrahimi, H. (2010). Variational iteration method for linear and non-linear systems of integro-differential equations, International Mathematical Forum, Vol. 5, No. 64, pp. 3185–3191.

Elzaki, T. M. and Ezaki, S. M. (2011). On the solution of integro-differential equation systems by using Elzaki transform, Global Journal of Mathematical Sciences, Vol. 3, No. 1, pp. 13–23.

Gachpazan, M. (2009). Numerical scheme to solve integro-differential equations system, Journal of Advanced Research in Scientific Computing, Vol. 1, pp. 11–21.

Khader, M. M. and Ahmed M. Megahed, (2016). Approximate solutions for the flow and heat transfer due to a stretching sheet embedded in a porous medium with variable thickness, variable thermal conductivity and thermal radiation using Laguerre collocation method, Applications and Applied Mathematics: An International Journal, Vol. 10, No. 2, pp. 817–834.

Khader, M. M. and Borhanifar, A. (2016). Implementation of the matrix differential transformation procedure for obtaining the approximate solution of some nonlinear matrix evolution equations, Applications and Applied Mathematics: An International Journal, Vol. 11, No. 2, pp. 906–918.

Linz, P. (1985). *Analytical and Numerical Methods for Volterra Equations*, SIAM, Philadelphia, PA.

Maleknejad, K. and Tavassoli Kajani, M. (2004). Solving linear integro-differential equation system by Galerkin methods with hybrid functions, Applied Mathematics and Computation, Vol. 159, No. 3, pp. 603–612.

Mohamed, S. T. and Khader, M. M. (2011). Numerical solutions to the second order Fredholm integro-differential equations using the spline functions expansion, Global Journal of Pure and Applied Mathematics, Vol. 34, pp. 21–29.

Pour-Mahmoud, J. Rahimi-Ardabili, M. Y. and Shahmorad, S. (2005). Numerical solution of the system of Fredholm integro-differential equations by the Tau method, Applied Mathematics and Computation, Vol. 168, pp. 465–478.

Yang, P. Chen, Y. and Zhi-Bin Li, (2009). ADM-Padé technique for the nonlinear lattice equations, Applied Mathematics and Computation, Vol. 210, pp. 362–375.

Yusufoglu, E. (2011). Numerical expansion methods for solving systems of Fredholm-Volterra integro-differential equations by using discrete collocation method, The Journal of Mathematics and Computer Science, Vol. 3, No. 4, pp. 382–389.