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Construction of *m*-Repeated Burst Error Detecting and Correcting Non-binary Linear Codes

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Abstract

Error correcting codes are required to ensure reliable communication of digitally encoded information. One of the areas of practical importance in which a parallel growth of the subject error correcting codes took place is that of burst error detecting and correcting codes. The nature of burst errors differs from channel to channel depending upon the behavior of channels or the kind of errors which occur during the process of transmission. The rate of transmission is efficient if the number of parity-check digits are as minimum as possible. It is usually not possible to give the exact number of parity-check digits required for a given code. However, bounds can be obtained over the number of parity-check digits. An upper bound for a linear code capable of detecting/ correcting burst errors or its variants is many a times established by the technique used to establish Varsharmov-Gilbert-Sacks bound by constructing a parity-check matrix for the requisite code. This technique not only ensures the existence of such a code but also gives a method for constructing such a code. The synthesis method using this technique is cumbersome and to the best of our knowledge, there is no systematic way to construct a parity-check matrix for a burst error correcting non-binary linear code. Extending the algorithm for binary linear codes given by the authors to non-binary codes, the paper proposes a new algorithm for constructing a parity-check matrix for any linear code over $GF(q)$ capable of detecting and correcting a new kind of burst error called `m-repeated burst error of length b or less'. Codes based on the proposed algorithm have been illustrated.

Keywords: Error correcting code; burst error; repeated burst error; parity-check matrix

MSC 2010 No.: 94B20, 94B25, 94B65

1. Introduction

In coding theory one of the important aspects of study is the detection and correction of errors. Codes have been constructed to combat such errors and many of the codes developed have found applications in numerous areas of practical interest. Hamming (1950) dealt with construction of codes capable of detecting and correcting random errors as well as bounds. Extending the work of Hamming, Abramson (1959) constructed a single error and double-adjacent error correcting code which can be considered as the starting point for all the other work on burst codes. Fire (1959) depicted a more general concept of clustered errors and defined two types of burst errors viz., ‘open-loop burst errors’ or simply a burst and ‘closed-loop burst errors’. A burst of length b may be defined as follows (see Fire (1959) and Peterson and Weldon (1972)):

Definition 1.1.

A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last of which is non-zero.

A closed-loop burst error may be defined as follows (see Fire (1959) and Peterson and Weldon (1972)):

Definition 1.2.

Let b be an integer and $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ be a vector in $V^n(q)$, a vector space of n-tuples over GF(q). If $2 \leq b \leq (n + 1)/2$, then ξ is called a `closed-loop burst vector of length b', whenever there is an i such that $1 \leq i \leq b - 1$, $\xi_i . \xi_{n-b+i+1} \neq 0$, $\xi_{i+1} = \xi_{i+2} = \ldots = \xi_{n-b+i} = 0$.

Since the development of various burst error detecting and correcting codes, several variants and modifications of the definition of a burst error came up depending upon the various kinds of channels which were in use. A low-density burst (see Wyner (1963)) may be defined as follows:

Definition 1.3.

A low-density burst of length b with weight w is an n-tuple whose only non-zero components are confined to some b consecutive positions, the first and the last of which are non-zero, with w ($w \leq b$) non-zero components within such b consecutive digits.

Chien and Tang (1965) considered a kind of burst error which has drawn attention of many researchers. Such bursts have been termed as CT bursts defined as follows:

Definition 1.4.

A burst of length b is a vector whose only non-zero components are confined to some b consecutive components, the first of which is non-zero.

Dass (1980) further modified this definition as follows:

Definition 1.5.

A burst of length b (fixed) is an n-tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions in an n-tuple is among the first $n - b + 1$ components.

This definition is useful for channels not producing burst errors near the end of a code word.
Still amongst the various generalizations of bursts, there are codes which have been developed for the correction of multiple bursts, bursts of bursts, random multiple bursts, low-density bursts etc. The initial contributors in the area of burst error detecting/correcting codes have been Bridwell and Wolf (1970), Burton (1969), Dass (1974, 1975, 1980), Elspas and Short (1962), Kasahara and Kasahara (1967), Kasami (1962), Melas (1960), Melas and Gorog (1963), Posner (1965), Reiger (1960), Sharma and Dass (1974) and Stone (1961) amongst others.

It has been observed that in very busy communication channels, errors repeat themselves. So there is a need to develop codes which can detect and correct repeated burst errors. Recently, repeated burst error detecting and correcting codes have been studied by the authors (2008, 2009) and by Berardi et al. (2009). An m-repeated burst of length b has been defined as follows (Dass and Verma (2008)):

Definition 1.6.

An m-repeated burst of length b is a vector of length n whose only non-zero components are confined to m distinct sets of b consecutive components, the first and the last component of each set being non-zero.

For example, (00102002410031403010 0) is a 4-repeated burst of length 3 over $GF(5)$.

Lower and upper bounds on the number of parity-check digits required for a linear code that is capable of detecting errors which are 2-repeated burst errors have been obtained by Berardi et al. (2009). In the same paper, codes capable of detecting and simultaneously correcting such errors have been dealt with. A lower bound on the number of parity-check digits required for m-repeated burst error detecting linear code and for codes capable of detecting and simultaneously correcting such errors have also been dealt with by the authors (2009). Lower and upper bounds on the number of parity-check digits for codes that can correct 2-repeated bursts of length b or less have been obtained by the authors (2008) and then codes capable of correcting m-repeated bursts have also been considered. Bounds on the number of check digits required for codes correcting such errors have been obtained. In the same paper, the authors have also considered an illustration of a (5b, b) binary code correcting 2-repeated bursts of length b or less with parity-check matrix

$$H = \begin{bmatrix} & I_b \\ I_{4b} & I_b \\ & I_b \\ & I_b \end{bmatrix}.$$

To the best of the knowledge of the authors, there was no systematic way to construct a parity-check matrix for a burst error correcting linear code. Binary codes capable of correcting *m*-repeated burst errors have been constructed by the authors (2012). Illustrations with particular value of *m* have also been provided. **For *m* = 1, the code constructed by this technique helps in resolving a long standing problem of devising a systematic algorithm for the construction of a burst error correcting code (see Example 2.4, Dass and Verma (2012)).** There is no need to construct parity-check matrix by the laid down synthesis procedure as given by Campopiano (1962) and Theorem 4.17 (Peterson and Weldon (1972)). **Such a matrix can always be constructed once the value of *b* is known.**

The study of these codes is important not only from a mathematical point of view as a generalization of burst but also because of the occurrence of such errors in other subject areas. In a recent study by Srinivas et al. (2007), the changes in the neuronal network properties during epileptiform activity in vitro in planar two-dimensional neuronal networks cultured on a multielectrode array, using the in vitro model of stroke-induced epilepsy have been explored. Neuronal networks in culture show spontaneous firing activity with short phases of synchronized firing known as network bursts. A network burst represents the period of synchronized activity in the network. For the detection of network burst, a threshold value was calculated as product of number of spikes per time bin and number of active channels. An active channel was defined as a channel presenting two bursts in an acquisition time of 300 sec. In other words, an active channel presents 2-repeated burst in an acquisition time of 300 sec. Further, experiments performed on hippocampal networks showed synchronized network bursts of similar duration. Glutamate-injured networks also showed network bursts of similar durations, but the bursts occurred more frequently. In other words, the value of *m* for *m*-repeated bursts is much higher in glutamate-injured networks as compared to control networks.

The purpose of this paper is to present an easy and new method for the construction of a parity-check matrix of *m*-repeated burst error correcting linear code in the non-binary case. The parity-check matrix of 2*m*-repeated burst error detecting non-binary code has been deduced as a particular case from the case of *m*-repeated burst error correcting code. In Section 2, we state the results that are required to construct the required parity-check matrix. In Section 3, illustrations of parity-check matrix have been provided. Section 4 consists of conclusion. In what follows, a linear code will be considered as a subspace of the space of all *n*-tuples over $GF(q)$. The distance between two vectors shall be considered in the Hamming's sense.

2. Results

In this section we state few results that are required for the construction of codes capable of detecting and correcting repeated burst errors. An upper bound on the number of parity-check digits for a code capable of detecting a burst error is as follows (see Theorem 4.14, Peterson and Weldon (1972)):

Result 2.1.

For detecting all burst errors of length *l* or less with a linear block code of length *n*, *l* parity check symbols are necessary and sufficient.

An upper bound on the number of parity-check digits for a burst error correcting code was obtained by Campopiano (1962) which may be stated as follows:

Result 2.2.

There exists an (*n*, *k*) linear code that corrects any single burst of length $b < n/2$ or less provided that

$$q^{n-k} > q^{(2b-1)}[(q-1)(n-2b+1)+1].$$

This bound was derived by constructing a parity-check matrix of the code by a synthesis procedure. Generalizing Result 2.1 and Result 2.2 for the detection and correction of *m*-repeated burst errors respectively as obtained in Theorem 4.1 and Theorem 3.2 by Dass and Verma (2008), we have the following:

Result 2.3.

There shall always exist an (*n*, *k*) linear code over *GF(q)* that has no *m*-repeated burst of length *b* or less as a code word provided that

$$q^{n-k} > q^{m(b-1)}\left(\binom{n-mb+(m-1)}{m-1}(q-1)^{m-1} + \sum_{l=0}^{m-2}\binom{n-mb+l}{l}(q-1)^{l}(q-1)^{m-2-l}\right). \quad (2.1)$$

Result 2.4.

There shall always exist an (*n*, *k*) linear code over *GF(q)* that corrects all *m*-repeated bursts of length *b* or less as ($n > 2mb$) provided that

$$q^{n-k} > q^{2m(b-1)}\left(\binom{n-2mb+(2m-1)}{2m-1}(q-1)^{2m-1} + \sum_{l=0}^{2m-2}\binom{n-2mb+l}{l}(q-1)^{l}(q-1)^{2m-2-l}\right). \quad (2.2)$$

The existence of such codes has again been proved as in Result 2.1 and Result 2.2 by constructing a parity-check matrix for *m*-repeated burst error detecting and correcting code respectively by using the technique to establish Varsharmov-Gilbert-Sacks bound (Theorem 4.17, Peterson and Weldon (1972)).

3. Illustrations of Repeated Burst Error Detecting and Correcting Non-binary Linear Codes

In this section we provide an example of non-binary linear code that can correct *m*-repeated burst errors of any length followed by illustrations with a specified value of *m* and length of burst. We shall also consider repeated burst error detecting non-binary linear codes for even values of *m*.

Example 3.1(a).

Consider the following matrix H over $GF(q)$:

$$H = \begin{bmatrix} 1 & 0 & \cdots & 0 & I_b & I_b \\ 0 & 1 & \cdots & 0 & I_b & 2I_b \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & I_b & (q-1)I_b \end{bmatrix}, \ (q-1) \geq 2m. \quad (3.1)$$

Such a matrix considered as a parity-check matrix shall give rise to a $((q+1)b, 2b)$ linear code. Such a code corrects *m*-repeated bursts of length b or less over $GF(q)$ for $(q-1) \geq 2m$.

Justification:

For $(q-1) = m$, according to the condition laid down in Result 2.4 (see Theorem 3.2, Dass and Verma (2008)), a column h_j can be added to H provided that it is not a linear combination of immediately preceding $b-1$ or fewer columns together with any $(2m-1)b$ or less consecutive columns from the remaining first $j-b$ columns.

So we start with (100...0) and keep on adding the columns in H to get a $2mb$ X $(2m+2)b$ matrix as defined above. It can be easily verified that the requisite condition is satisfied. Now we prove that none of the $q^{2mb} - 1$ non-zero tuples over $GF(q)$ can be added further to H to get $((2m + 2)b + 1)$ –th column. Now a q^{2mb} non-zero tuple can be considered as consisting of $2m$ sets of b consecutive components. Let a_i denote the i-th digit of any q^{2mb} tuple. We consider the following cases.

Case 1.

When $a_i = 0$ for at least one i, $i = 1, b + 1, 2b + 1, \ldots, (2m - 1)b + 1$.

The digits $a_{i+1}, a_{i+2} \ldots$ and $a_{i+(b-1)}$ are linear combination of $h_{(2m+1)b+2}, \ldots, h_{(2m+2)b-1}$ and $h_{(2m+2)b}$. The remaining $(2m - 1)$ components $a_1, a_2, \ldots, a_{i-1}$, $a_{i+b}, a_{i+b+1}, \ldots, a_{2mb}$ will be a linear combination of at most $(2m - 1)$ distinct sets of b or less consecutive columns amongst the first $2mb$ columns of H. Thus, any tuple with 0 at a_i-th digit, $i = 1, b + 1, 2b + 1, \ldots, (2m - 1)b + 1$ cannot be added as $(2m + 2)b + 1$-th column of H.

Case 2.

When $a_i \neq 0$ for each i, $i = 1, b+1, 2b+1, \ldots, (2m-1)b+1$. This case is further divided into following two subcases.

Case 2(a).

When $a_i = a_j \neq 0$ for some $i, j = 1, b + 1, 2b + 1, \ldots, (2m - 1)b + 1$, $i \neq j$.

The two sets of b consecutive digits viz., a_i, $a_{i+1}, a_{i+2} \ldots$ and $a_{i+(b-1)}$, $a_j, a_{j+1}, a_{j+2} \ldots$ and $a_{j+(b-1)}$ are linear combination of $h_{2mb+1}, h_{2mb+2}, \ldots, h_{(2m+1)b}$, $h_{(2m+1)b+2}, \ldots, h_{(2m+2)b-1}$ and $h_{(2m+2)b}$. The remaining $(2m - 2)b$ components viz., $a_1, a_2, \ldots, a_{i-1}$, $a_{i+b}, a_{i+b+1}, \ldots, a_{j-1}, a_{j+b}, a_{j+b+1} \ldots$, a_{2mb} will be a linear combination of at most $(2m - 2)$ distinct sets of b or less consecutive columns amongst the first $2mb$ columns of H.

Case 2(b).

When $a_i \neq 0$ is distinct for each i, $i = 1, b + 1, 2b + 1, \ldots, (2m - 1)b + 1$.

We observe that a_i, a_j for some $i, j = 1, b + 1, 2b + 1, \ldots, (2m - 1)b + 1$, $i \neq j$ coincide with a multiple of $h_{(2m+1)b+1}$. The two sets of b consecutive digits viz., a_i, $a_{i+1}, a_{i+2} \ldots$ and $a_{i+(b-1)}$, a_j, $a_{j+1}, a_{j+2} \ldots$ and $a_{j+(b-1)}$ are linear combination of $h_{2mb+2}, \ldots, h_{(2m+1)b}$, $h_{(2m+1)b+1}, h_{(2m+1)b+2}, \ldots, h_{(2m+2)b-1}$ and $h_{(2m+2)b}$. The remaining $(2m - 2)b$ components viz., $a_1, a_2, \ldots, a_{i-1}$, $a_{i+b}, a_{i+b+1}, \ldots, a_{j-1}, a_{j+b}, a_{j+b+1} \ldots, a_{2mb}$ will be a linear combination of at most $(2m - 2)$ distinct sets of b or less consecutive columns amongst the first $2mb$ columns of H.

Thus, any tuple with non-zero digit as a_i-th digit , $i = 1, b + 1, 2b + 1, \ldots, (2m - 1)b + 1$ cannot be added as $(2m + 2)b + 1$-th column of H.

Therefore, no more columns can be added to H.

Thus, the $((2m + 2)b, 2b)$ binary code which is the null space of the matrix H as constructed above will correct all m-repeated bursts of length b or less over $GF(q)$.

Similarly, for $(q - 1) > 2m$, matrix can be constructed and condition is satisfied.

Example 3.1(b).

Consider the following matrix H over $GF(q)$:

$$H = \begin{bmatrix} I_{2mb} & \begin{matrix} I_b \\ I_b \\ \vdots \\ I_b \end{matrix} \end{bmatrix}, \ (q-1) < 2m. \tag{3.2}$$

Such a matrix considered as a parity-check matrix shall give rise to a $((2m + 1)b, b)$ linear code. Such a code corrects m-repeated bursts of length b or less over $GF(q)$ for $(q - 1) < 2m$.

Justification:

For $(q - 1) < 2m$, Case 2(b) is not possible. Thus the matrix reduces to (3.2).

It is worthwhile to note that such a code can serve dual purpose, i.e., it can be used to correct m-repeated bursts of length b or less or it can be used to detect a $2m$-repeated burst of length b or less (see Remark 4.4, Dass and Verma (2008)).

Example 3.2.

For $m = 2$, $b = 3$ and $q = 5$, the parity-check matrix for such a code is

$$H = \begin{bmatrix} I_{12} & \begin{matrix} I_3 & I_3 \\ I_3 & 2I_3 \\ I_3 & 3I_3 \\ I_3 & 4I_3 \end{matrix} \end{bmatrix}.$$

It may be easily verified that this code corrects 2-repeated bursts of length 3 or less over $GF(5)$. The various linear combinations of columns of parity-check matrix corresponding to the required digits of vector according to Case 1 and Case 2 as discussed above in Example 3.1(a) have been provided in the following tables.

Case 1.

At least one of the digits a_1, a_4, a_7 and a_{10} is zero

Table 3.1: Linear combinations of columns of parity-check matrix for Case 1of Example 3.2.

For $a_1 = 0$

a_2	a_3	h_{17}	h_{18}
0	0	0	0
0	1	0	1
0	2	0	2
0	3	0	3
0	4	0	4
1	0	1	0
1	1	1	1
1	2	1	2
1	3	1	3
1	4	1	4
2	0	2	0
2	1	2	1
2	2	2	2
2	3	2	3
2	4	2	4
3	0	3	0
3	1	3	1
3	2	3	2
3	3	3	3
3	4	3	4
4	0	4	0
4	1	4	1
4	2	4	2
4	3	4	3
4	4	4	4

For $a_4 = 0$

a_5	a_6	h_{17}	h_{18}
0	0	0	0
0	1	0	3
0	2	0	1
0	3	0	4
0	4	0	2
1	0	3	0
1	1	3	3
1	2	3	1
1	3	3	4
1	4	3	2
2	0	1	0
2	1	1	3
2	2	1	1
2	3	1	4
2	4	1	2
3	0	4	0
3	1	4	3
3	2	4	1
3	3	4	4
3	4	4	2
4	0	2	0
4	1	2	3
4	2	2	1
4	3	2	4
4	4	2	2

For $a_7 = 0$

a_8	a_9	h_{17}	h_{18}
0	0	0	0
0	1	0	2
0	2	0	4
0	3	0	1
0	4	0	3
1	0	2	0
1	1	2	2
1	2	2	4
1	3	2	1
1	4	2	3
2	0	4	0
2	1	4	2
2	2	4	4
2	3	4	1
2	4	4	3
3	0	1	0
3	1	1	2
3	2	1	4
3	3	1	1
3	4	1	3
4	0	3	0
4	1	3	2
4	2	3	4
4	3	3	1
4	4	3	3

For $a_{10} = 0$

a_{11}	a_{12}	h_{17}	h_{18}
0	0	0	0
0	1	0	4
0	2	0	3
0	3	0	2
0	4	0	1
1	0	4	0
1	1	4	4
1	2	4	3
1	3	4	2
1	4	4	1
2	0	3	0
2	1	3	4
2	2	3	3
2	3	3	2
2	4	3	1
3	0	2	0
3	1	2	4
3	2	2	3
3	3	2	2
3	4	2	1
4	0	1	0
4	1	1	4
4	2	1	3
4	3	1	2
4	4	1	1

Case 2.

At least one of the pair of digits a_1, a_4; a_1, a_7; a_1, a_{10}; a_4, a_7; a_4, a_{10} and a_7, a_{10} is non-zero.

Table 3.2. Linear combinations of columns of parity-check matrix for Case 2 of Example 3.2.

For $a_1 \neq 0, a_4 \neq 0$

a_2	a_5	h_{14}	h_{17}
0	0	0	0
0	1	4	1
0	2	3	2
0	3	2	3
0	4	1	4
1	0	2	4
1	1	1	0
1	2	0	1
1	3	4	2
1	4	3	3
2	0	4	3
2	1	3	4
2	2	2	0
2	3	1	1
2	4	0	2
3	0	1	2
3	1	0	3
3	2	4	4
3	3	3	0
3	4	2	1
4	0	3	1
4	1	2	2
4	2	1	3
4	3	0	4
4	4	4	0

For $a_1 \neq 0, a_7 \neq 0$

a_2	a_8	h_{14}	h_{17}
0	0	0	0
0	1	2	3
0	2	4	1
0	3	1	4
0	4	3	2
1	0	4	2
1	1	1	0
1	2	3	3
1	3	0	1
1	4	2	4
2	0	3	4
2	1	0	2
2	2	2	0
2	3	4	3
2	4	1	1
3	0	2	1
3	1	4	4
3	2	1	2
3	3	3	0
3	4	0	3
4	0	1	3
4	1	3	1
4	2	0	4
4	3	2	2
4	4	4	0

For $a_1 \neq 0, a_{10} \neq 0$

a_2	a_{11}	h_{14}	h_{17}
0	0	0	0
0	1	3	2
0	2	1	4
0	3	4	1
0	4	2	3
1	0	3	3
1	1	1	0
1	2	4	2
1	3	2	4
1	4	0	1
2	0	1	1
2	1	4	3
2	2	2	0
2	3	0	2
2	4	3	4
3	0	4	4
3	1	2	1
3	2	0	3
3	3	3	0
3	4	1	2
4	0	2	2
4	1	0	4
4	2	3	1
4	3	1	3
4	4	4	0

For $a_4 \neq 0, a_7 \neq 0$

a_5	a_8	h_{14}	h_{17}
0	0	0	0
0	1	3	1
0	2	1	2
0	3	4	3
0	4	2	4
1	0	3	4
1	1	1	0
1	2	4	1
1	3	2	2
1	4	0	3
2	0	1	3
2	1	4	4
2	2	2	0
2	3	0	1
2	4	3	2
3	0	4	2
3	1	2	3
3	2	0	4
3	3	3	0
3	4	1	1
4	0	2	1
4	1	0	2
4	2	3	3
4	3	1	4
4	4	4	0

For $a_4 \neq 0, a_{10} \neq 0$

a_5	a_{11}	h_{14}	h_{17}
0	0	0	0
0	1	4	3
0	2	3	1
0	3	2	4
0	4	1	2
1	0	2	2
1	1	1	0
1	2	0	3
1	3	4	1
1	4	3	4
2	0	4	4
2	1	3	2
2	2	2	0
2	3	1	3
2	4	0	1
3	0	1	1
3	1	0	4
3	2	4	2
3	3	3	0
3	4	2	3
4	0	3	3
4	1	2	1
4	2	1	4
4	3	0	2
4	4	4	0

For $a_7 \neq 0, a_{10} \neq 0$

a_8	a_{11}	h_{14}	h_{17}
0	0	0	0
0	1	2	1
0	2	4	2
0	3	1	3
0	4	3	4
1	0	4	4
1	1	1	0
1	2	3	1
1	3	0	2
1	4	2	3
2	0	3	3
2	1	0	4
2	2	2	0
2	3	4	1
2	4	1	2
3	0	2	2
3	1	4	3
3	2	1	4
3	3	3	0
3	4	0	1
4	0	1	1
4	1	3	2
4	2	0	3
4	3	2	4
4	4	4	0

The linear combinations for pair of digits a_3, a_6; a_3, a_9; a_3, a_{12}; a_6, a_9; a_6, a_{12} and a_9, a_{12} are obtained by replacing h_{14} and h_{17} with h_{15} and h_{18} respectively in above tables.

Example 3.3.

For $q = 3$, Case 2(b) is not possible. Therefore the parity-check matrix for such a code is

$$H = \begin{bmatrix} I_{2mb} & \begin{matrix} I_b \\ I_b \\ \vdots \\ I_b \end{matrix} \end{bmatrix}.$$

Remark

The parity-check matrix for m-repeated burst error correcting linear code is same for $q = 2$ (see Dass and Verma (2012)) and $q = 3$.

Example 3.4.

For m = 2, b = 3 and q = 3, the parity-check matrix for such a code is

$$H = \begin{bmatrix} & I_3 \\ I_{12} & I_3 \\ & I_3 \\ & I_3 \end{bmatrix}.$$

It has been verified that this matrix corrects 2-repeated bursts of length 3 or less over $GF(3)$.

4. Conclusion

The main purpose of the paper is to present an algorithm for constructing a parity-check matrix for any non-binary linear code capable of correcting m-repeated burst error of any given length b. The algorithm not only simplifies the construction of such codes but also provides a systematic way for the construction of parity-check matrix for linear codes dealing with detection and correction of repeated burst errors and in particular, burst errors. The cumbersome synthesis procedure given in Theorem 3.2, Dass and Verma (2008) to construct a parity-check matrix for the requisite code using the technique to establish Varsharmov-Gilbert-Sacks bound has been replaced by considering a matrix of the type given in Examples 3.1(a) and 3.1(b). It is clear that for given feasible integer values of the parameters q, m and b, a matrix of the type H as in (3.1) and (3.2) can always be constructed.

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