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Dynamic Network Flows with Uncertain Costs belonging to Interval

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Abstract

This paper considers minimum cost flow problem in dynamic networks with uncertain costs. First, we present a short introduction of dynamic minimum cost flow. Then, we survey discrete and continuous dynamic minimum cost flow problems, their properties and relationships between them. After that, the minimum cost flow problem in discrete dynamic network with uncertainty in the cost vector is considered such that the arc cost can be changed within an interval. Finally, we propose an algorithm to find the optimal solution of the proposed model.

Keywords: Dynamic network flows; Robust optimization; Duality theorem; Uncertain cost; Discrete dynamic network

MSC 2010 No.: 90B10, 90B18

1. Introduction

The network flow problems are divided into static and dynamic problems. A complete survey of static network flow problems has been widely studied by Ahuja et al. (1993). Ford and Fulkerson (1958) introduced dynamic network flow problems. They presented the concept of dynamic flows in networks with the maximal dynamic network flow problems. Their work was extended later in [Aronson (1989), Fonoberova (2007), Glockner and Nemhauser (2000), Hashemi et al. (2010), Hashemi and Nasrabadi (2012), Hoppe (1995), Salehi and Hosseini (2011), Salehi and

Hosseini (2010), Tahmasbi et al. (2013)]. Algorithms, their implementations for dynamic network flow models and a survey of known results were presented by Aronson (1989). Hoppe (1995) provided extension of the dynamic network flows algorithms and proposed the polynomial algorithm to compute the value of a universally maximum dynamic flow. Glockner and Nemhauser (2000) studied dynamic network flow problems with uncertainty of arc capacities. The dynamic network flow problem with nonlinear cost function was considered by Fonoberova (2007). Moreover, she studied one case of the minimum cost flow problem with cost functions dependent on times instead of flows, where capacities of edges, supply and demand functions depend on time. Salehi and Hosseini (2010, 2011) focused on minimum cost flow and maximum flow problems on dynamic generative network flows with time-varying bounds, in which the flow was dynamically generated at a source node and dynamically consumed at a sink node and the arc-flow bounds are time dependent. They presented a method for solving the maximum flows problem in dynamic generative network flows. Nasrabadi and Hashemi (2010) presented an algorithm of time complexity $O\left(VnT\left(n+T\right)\right)$, where V is an upper bound on the total supply, n is the number of nodes and T is a given time horizon for a general minimum cost dynamic network flow problem in a discrete time model. In 2012, Hashemi and Nasrabadi (2012) studied a general class of dynamic network flow problem in the continuous-time model, where the input functions are assumed to be piecewise linear or constant.

In mathematical optimization models, it is commonly assumed that the data inputs are known. In recent years, it has been recognized that dealing with uncertain data is a major challenge in optimization. The approach to address data uncertainty has developed under the name Robust optimization. Robust optimization is a recent optimization approach that deals with data uncertainty and does not assume probability distributions of random parameters. In general, a robust solution is not optimal for all realizations of the uncertain data, but performs well even for the worst case scenario.

The first robust approach was proposed by Soyster (1973), where he considered inexact linear programming with convex uncertainty sets. The approach of Soyster (1973) can directly be applied to minimum cost flow problems with uncertain demand. Kouvelis and Yu (1997) proposed a general scenario-based approach for robust optimization in discrete optimization problems. Their approach is also a scenario-based one. For a specific scenario *S*, their proposed optimization problem has been given, as follows:

$$\min_{X\in F_s} f(X,D^s), \tag{1}$$

where *X* is the set of decision variables, F_s is the set of all feasible decisions, when scenario *S* is realized, D^S is the input data scenario *S*, and *f* is the function which evaluates the quality of the decision *X* dependent on the input data instance D^S. In 1998, Ben-Tal and Nemirovski (1998) suggested a model for uncertain linear problems with ellipsoidal uncertainty. Their approach assumes that the linear programming problems are uncertain and have hard constraints. This is an approach which deals with linear problems in general form and can be applied to the network flows problems. For more details, we refer the reader to Ben Tal and Nemirovski (1998, 1999 and 2000).

The robust approach of Bertsimas and Sim (2003) examines the minimum cost flow problem with uncertainty in the cost vector. They proposed a general approach to obtain a solution of robust optimization, which leads to an efficient algorithm for solving minimum cost flow problem with uncertainty in the cost vector. Their approach conserves the network structure, such that the robust counterpart can be solved by solving several minimum cost flow problems. In addition, they introduce a parameter Γ to control the price of robustness, the trade-off between the degree of uncertainty taking into account and the cost of this additional feature. Tahmasebi et al. (2013) studied the maximum flow problem in stochastic networks with random arc failures. They presented the concept of expected value of a given flow and expected capacity of a given cut.

In this paper, dynamic network flows with uncertain costs belonging to interval is studied. In Section 2, discrete and continuous minimum cost flow models are presented. Moreover, the relationships between them are mentioned. The formulation of the dynamic minimum cost flow with uncertain cost is given in Section 3. Finally, the new algorithm for solving the proposed formulation is given in Section 4.

2. Preliminary

In this section, some basic concepts, formulations and theorems of dynamic network problems are reviewed. For more details see [Fonoberova (2007), Ford and Fulkerson (1958), Hoppe (1995), Klinz and Woeginger (2004), Salehi and Hosseini (2011)]. Let $G=\left(N,A\right)$ be a directed network, where N and A are sets of nodes and sets of directed arcs, respectively. In Subsection 2.1, the discrete dynamic minimum cost flow and its formulations will be presented, where the time t is dependent on the discrete values of the time $\tau=\{0,1,2,...,T-1\}$ and $T\geq 0$ is the maximum allowable time. In Subsection 2.2, the dynamic minimum cost flow will be surveyed, such that the problem parameters change in continuous time interval $[0,T]$. For finding a solution of this problem, the time interval $[0,T]$ can be divided into a finite number of subintervals and an approximation discrete structure can be obtained. In Subsection 2.3, discrete and continuous models relationships will be reviewed.

2.1. Discrete minimum cost flow in dynamic networks

Consider $G=\left(N,A\right)$ to be a directed dynamic network flow. Capacity function is defined as $U:A\times\tau\to R$ such that $U_{ij}(t)$ shows arc capacity (i,j) at the moment t, in which t changes in $\tau=\{0,1,2,...,T-1\}$. Moreover, flow function is defined as $X:A\times\tau\to R$, where $X_{ij}(t)$ is the flow on the arc (i,j) at the moment t. Cost function and transmission time function are also defined as $C:A\times\tau\to R$, $\lambda:A\times\tau\to R$, respectively. If it is assumed that arcs transmission time are non-negative integers, then the discrete minimum cost flow problem on the dynamic network G can be formulated, as follows:

$$\min \sum_{(i,j)\in A} \sum_{t=0}^{T-1} C_{ij}(t)\, x_{ij}(t), \tag{2}$$

$$\sum_{j} \sum_{t=0}^{\theta} x_{ij}(t) - \sum_{j} \sum_{t=0}^{\theta} \sum_{t':t'+\lambda_{ji}(t')=t} x_{ji}(t-\lambda_{ji}(t')) = 0, \qquad \forall i \in N, \theta \in \tau, \tag{3}$$

$$\sum_{j} \sum_{t=0}^{T-1} x_{ij}(t) - \sum_{j} \sum_{t=0}^{T-1} \sum_{t':t'+\lambda_{ji}(t')=t} x_{ji}(t-\lambda_{ji}(t')) = b(i), \qquad \forall i \in N, \tag{4}$$

$$0 \le x_{ij}(t) \le U_{ij}(t), \qquad \forall (i,j) \in A, \forall t \in \tau, \tag{5}$$

Constraint (2) shows the objective function which deals with network cost minimization on all time-steps and arcs. Constraints (3) indicate that storage in middle nodes is not allowed. Constraints (4) show that the discrete-time flow should be valid during the time-steps $\tau = \{0,1,2,...,T-1\}$ for supply and demand nodes. In other words, after the final solution is obtained, supply nodes should consume all their supplies and demand nodes should have all their needs. The equality constraints (4) are called balance constraints or flow conservation constraints. A discrete dynamic flow with time horizon T is a feasible flow like x, which satisfies all constraints (3), (4) and (5) and we must have:

$$x_{ij}(t) = 0, \qquad \forall (i,j) \in A, \forall t > T-1. \tag{6}$$

2.2. Continuous minimum cost flow in dynamic networks

Suppose that $\tau \in [0,T]$ and also all the other definitions in Section 2.1 are valid. The continuous minimum cost flow in dynamic networks is modeled as follows:

$$\min \sum_{(i,j)\in A} \int_{0}^{T} C_{ij}(t)\, x_{ij}(t)\, dt, \tag{7}$$

$$\sum_{j} \int_{0}^{\theta} x_{ij}(t) - \sum_{j} \sum_{t':t'+\lambda_{ji}(t')=t}^{\theta} \int_{0}^{\theta} x_{ji}(t-\lambda_{ji}(t')) = 0, \qquad \forall i \in N, \theta \in [0,T], \tag{8}$$

$$\sum_{j} \int_{0}^{T} x_{ij}(t) - \sum_{j} \sum_{t':t'+\lambda_{ji}(t')=t}^{T} \int_{0}^{T} x_{ji}(t-\lambda_{ji}(t')) = b(i), \qquad \forall i \in N, \tag{9}$$

$$0 \le x_{ij}(t) \le U_{ij}(t), \qquad \forall (i,j) \in A, \forall t \in [0,T], \tag{10}$$

Constraints (8) and (9) indicate the storage and the flow constraints, respectively.

2.3. Discrete and Continuous models relationship

In this section, the relationship between dynamic discrete and continuous time network flow problem is detected. Every discrete-time flow x in the dynamic network G corresponds to the continuous-time flow $\overline{x}$ in G and vice versa [Ford and Fulkerson (1958), Hoppe (1995), Klinz and Woeginger (2004), Salehi and Hosseini (2011)].

Lemma 2.1. [Salehi and Hosseini (2010, 2011)]

Every feasible flow of continuous time like x on dynamic network flow of G corresponds with a feasible flow of continuous time like $\overline{x}$ on G and vice versa.

In Lemma 2.1, the cost of the flow is fixed. Therefore, for dynamic network flow, each problem of continuous time could be turned into a discrete one. Fleischer and Tardos (1998) showed that many of the applied algorithms to solve discrete time problems could be expanded to solve continuous time problems even if T is not an integer.

Theorem 2.2. [Klinz and Woeginger (2004)]

The minimum cost problem on dynamic network flows is NP-hard.

Theorem 2.3. [Klinz and Woeginger (2004)]

The complexity time of the minimum cost problem on dynamic network flows is pseudo polynomial.

3. Dynamic minimum cost flow with uncertain costs

In this section, the dynamic minimum cost flow problem is discussed in which the arc costs are uncertain and belong to the intervals.

3.1. Problem formulation

Consider the formulations in the previous section. In a network flow problem, the cost values on the arcs can be subject to uncertainty, where not necessarily all arcs must be concerned. In this case, discrete dynamic network flow formulation with uncertain cost is, as follows:

$$\begin{aligned}
&\min \sum_{(i,j)\in A} \sum_{t=0}^{T-1} \tilde{C}_{ij}(t)\, x_{ij}(t), \\
&\sum_{j} \sum_{t=0}^{\theta} x_{ij}(t) - \sum_{j} \sum_{t=0}^{\theta} \sum_{t':t'+\lambda_{ji}(t')=t} x_{ji}(t-\lambda_{ji}(t')) = 0, \qquad \forall i \in N, \theta \in \tau, \\
&\sum_{j} \sum_{t=0}^{T-1} x_{ij}(t) - \sum_{j} \sum_{t=0}^{T-1} \sum_{t':t'+\lambda_{ji}(t')=t} x_{ji}(t-\lambda_{ji}(t')) = b(i), \quad \forall i \in N, \\
&0 \le x_{ij}(t) \le U_{ij}(t), \qquad \forall (i,j) \in A, \forall t \in \tau,
\end{aligned} \tag{11}$$

where

$$\tilde{C}_{ij}(t) \in [C_{ij}(t),\, C_{ij}(t) + C'_{ij}(t)].$$

For each arc $(i, j) \in A$, $C_{ij}(t)$ is the nominal cost on arc (i, j), $C'_{ij}(t)$ describes the possible uncertainty in the cost value on arc (i, j). Now we put:

$$A'(t) = \left\{ (i, j, t) : \; C'_{ij}(t) \geq 0 \right\}. \tag{12}$$

So the purpose of the robust optimization is the minimization of total nominal costs plus most value of uncertainty set for all time-steps. Then the robust dynamic minimum cost flow problem formulation is presented, as follows:

$$\begin{aligned}
&\min \sum_{(i,j)\in A} \sum_{t=0}^{T-1} C_{ij}(t)\, x_{ij}(t) + \max_{A'(t)} \sum_{(i,j,t)\in A'(t)} C'_{ij}(t)\, x_{ij}(t), \\
&\sum_{j} \sum_{t=0}^{\theta} x_{ij}(t) - \sum_{j} \sum_{t=0}^{\theta} \sum_{t':t'+\lambda_{ji}(t')=t} x_{ji}(t-\lambda_{ji}(t')) = 0, && \forall i \in N, \theta \in \tau, \\
&\sum_{j} \sum_{t=0}^{T-1} x_{ij}(t) - \sum_{j} \sum_{t=0}^{T-1} \sum_{t':t'+\lambda_{ji}(t')=t} x_{ji}(t-\lambda_{ji}(t')) = b(i), && \forall i \in N, \\
&0 \leq x_{ij}(t) \leq U_{ij}(t), && \forall (i,j) \in A, \forall t \in \tau, \\
&A'(t) \subseteq A(t), \left|A'(t)\right| \leq \Gamma, \Gamma \in [0, |A|T],
\end{aligned} \tag{13}$$

Assume that X is the feasible region of formulation (13), then formulation (13) can be converted, as follows (for more details see Bertsimas and Sim (2003)):

$$\begin{aligned}
&\min_{x\in X} \sum_{(i,j)\in A} \sum_{t=0}^{T-1} C_{ij}(t)\, x_{ij}(t) + \max \sum_{(i,j,t)\in A'(t)} C'_{ij}(t)\, x_{ij}(t)\, \gamma_{ij}(t), \\
&s.t \\
&0 \leq \gamma_{ij}(t) \leq 1, && \forall (i,j) \in A(t), \forall t \in \tau, \\
&\sum_{(i,j)\in A(t)} \gamma_{ij}(t) \leq \Gamma(t), && t = 0,1,\ldots,T-1,
\end{aligned} \tag{14}$$

For a fixed $x \in X$, the dual of the inner maximization problem of (14) is:

$$\begin{aligned}
&\min \Gamma V + \sum_{(i,j,t)\in A(t)} W_{ij}(t), \\
&\text{s.t.} \\
&W_{ij}(t) + V \geq C'_{ij}(t)\, x_{ij}(t), && \forall (i,j,t) \in A'(t), \forall t \in \tau, \\
&W_{ij}(t) \geq 0, && \forall (i,j,t) \in A'(t), \forall t \in \tau, \\
&V \geq 0,
\end{aligned} \tag{15}$$

By applying the strong Duality Theorem and formulation (15), formulation (14) can be rewritten as follows:

$$\min\left[\sum_{(i,j)\in A}\sum_{t=0}^{T-1} C_{ij}(t)\,x_{ij}(t) + \underset{W,V}{Min}\left(\Gamma V + \sum_{(i,j,t)\in A'(t)} W_{ij}(t)\right)\right],$$

$$\begin{aligned}
&\text{s.t.}\\
&W_{ij}(t)+V \ge C'_{ij}(t)\,x_{ij}(t), && \forall (i,j,t)\in A'(t),\ \forall t\in\tau,\\
&W_{ij}(t)\ge 0, && \forall (i,j,t)\in A'(t),\ \forall t\in\tau,\\
&V\ge 0,\\
&x\in X,
\end{aligned} \tag{16}$$

The variables X, V and W are minimization variables in formulation (16). Moreover, both types of optimization are minimization in objective function (16). So problem (16) can be rewritten as follows:

$$\min_{V\ge\circ}\ Z(V), \tag{17}$$

where

$$Z(V) = \Gamma V + \underset{x,w}{Min}\left(\sum_{t=0}^{T-1}\sum_{(i,j)\in A} C_{ij}(t)\,x_{ij}(t) + \sum_{(i,j)\in A'(t)} W_{ij}(t)\right),$$

$$\begin{aligned}
&s.t\\
&W_{ij}(t)\ge C'_{ij}(t)\,x_{ij}(t) - V(t), && \forall (i,j,t)\in A'(t)\ ,\ \forall t\in\tau,\\
&W_{ij}(t)\ge 0, && \forall (i,j,t)\in A'(t)\ ,\forall t\in\tau,\\
&V\ge 0\ ,\forall x\in X,
\end{aligned} \tag{18}$$

Theorem 3.1.

$Z(V)$ is convex function of V .

***Proof*:**

Assume that $V_1\ge 0$, $V_2\ge 0$, (x_1,w_1) and (x_2,w_2) are optimal solutions for formulation (18), corresponding to V_1, V_2, respectively. The feasible region formulation (18) is convex; so for each $\alpha\in[0,1]$:

$$\begin{aligned}
&\alpha Z(V_1) + (1-\alpha) Z(V_2) \\
&\quad = \alpha \left[\Gamma V_1 + \sum_{t=0}^{T-1} \sum_{(i,j)\in A} C_{ij}(t)(x_1)_{ij}(t) + \sum_{(i,j,t)\in A'(t)} (W_1)_{ij}(t) \right] \\
&\qquad + (1-\alpha) \left[\Gamma V_2 + \sum_{t=0}^{T-1} \sum_{(i,j)\in A} C_{ij}(t)(x_2)_{ij}(t) + \sum_{(i,j,t)\in A'(t)} (W_2)_{ij}(t) \right] \\
&\quad = \Gamma\left(\alpha V_1 + (1-\alpha)V_2\right) + \sum_{t=0}^{T-1} \sum_{(i,j)\in A} C_{ij}(t) \left[\alpha (x_1)_{ij}(t) + (1-\alpha)(x_2)_{ij}(t) \right] \\
&\qquad + \sum_{(i,j,t)\in A'(t)} \left[\alpha (W_1)_{ij}(t) + (1-\alpha)(W_2)_{ij}(t) \right] \geq Z(\alpha V_1 + (1-\alpha)V_2).
\end{aligned}$$

Our purpose is to solve the discrete dynamic minimum cost flow problem with uncertain arc cost. For this purpose, by eliminating $w_{ij}(t)$ from formulation (18), $\mathrm{Z(V)}$ is equal to:

$$\Gamma V + \min_{x} \left[\sum_{t=0}^{T-1} \sum_{(i,j)\in A} C_{ij}(x) x_{ij}(t) + \sum_{(i,j,t)\in A'(t)} C'_{ij}(t) Max\left\{ x_{ij}(t) - \frac{\mathrm{V}}{\mathrm{C}'_{\mathrm{ij}}(t)}, 0 \right\} \right] \tag{19}$$

such that $V \geq 0$ and $x \in X$.

Considering the above problem for fixed V , then we have:

$$Z(V) = \min_{x\in X} \left(\sum_{t=0}^{T-1} \sum_{(i,j)\in A} C_{ij}(x) x_{ij}(t) + \sum_{(i,j,t)\in A'(t)} C'_{ij}(t) Max\left\{ x_{ij}(t) - \frac{\mathrm{V}}{\mathrm{C}'_{\mathrm{ij}}(t)}, 0 \right\} \right) \tag{20}$$

We insert two new nodes i' and j', which would be introduced for each arc (i, j). Meanwhile, four arcs (i, i'), (i', j'), (j', i) and (i', j) with the new capacity and costs introduced in Figure 1, would be added to the network and arc (i, j) would be eliminated finally.

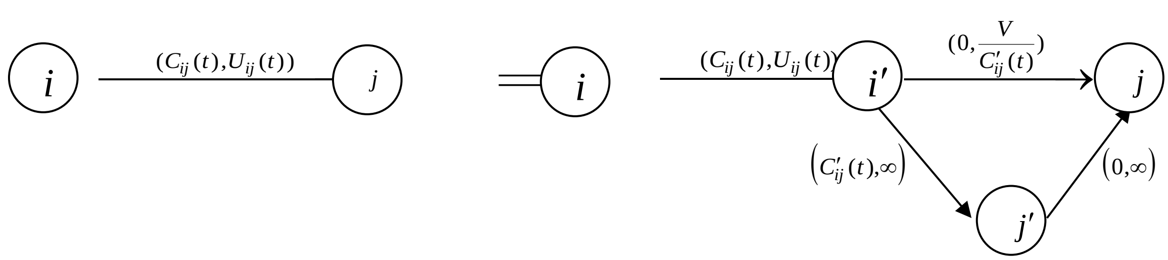

Figure 1. Improved network

3.2. How to create an improved network

An improved network can be created where it can convert a robust dynamic network to deterministic one. It is clear that an improved network can be considered with infinite capacities.

Theorem 3.2.

For fixed V, formulation (20) can be considered as a discrete dynamic network flow problem by using the mentioned improved network.

Proof:

Suppose that x is optimal solution of formulation (20), in which it is transferred to the improved network $G' = (N', A')$. In G', if $x_{ij}(t) \le \dfrac{V}{c'_{ij}(t)}$, then by passing from arcs (i, i') and (i', j), the cost of flow sending along path $i - i' - j$ is:

$$C_{ii'}(t)\, x_{ij}(t) + C_{i'j}(t)\, x_{ij}(t) = C_{ij}(t)\, x_{ij}(t).$$

If $x_{ij}(t) \ge \dfrac{V}{c'_{ij}(t)}$, for a given time t and a given arc (i, j) with $(i, j, t) \in \mathrm{A}'(\mathrm{t})$, then in G' the flow would be first passed along the arc (i, i'). Then, an amount of $\dfrac{V}{c'_{ij}(t)}$ would be passed from i' to j along arc (i', j) and the excess amount $x_{ij}(t) - \dfrac{V}{c'_{ij}(t)}$ would be passed along the arcs (i', j') and (j', j). Then the cost of flow sending from i' to j under condition that $x_{ij}(t) \ge \dfrac{V}{c'_{ij}(t)}$ is

$$C_{ii'}(t)\, x_{ij}(t) + C_{i'j}(t)\frac{V}{C'_{ij}(t)} + C_{i'j'}\left(x_{ij}(t) - \frac{V}{C'_{ij}(t)}\right) + C_{j'j}\left(x_{ij}(t) - \frac{V}{C'_{ij}(t)}\right)$$
$$= C_{ij}(t)\, x_{ij}(t) + C'_{ij}(t)\left(x_{ij}(t) - \frac{V}{C'_{ij}(t)}\right). \square$$

4. Algorithm

Definition 4.1.

A function $f(x)$ is unimodal:

I. If $x_1 < x_2 < x^*$ then $f(x_1) > f(x_2)$, or
II. If $x^* < x_1 < x_2$ then $f(x_1) < f(x_2)$.

Here, x^* is the minimizer of $f(x)$.

Remark 4.2.

Some instances of unimodal functions are illustrated in Figure 2.

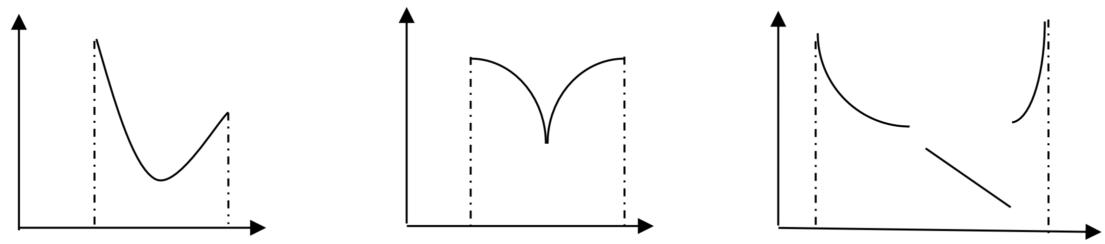

Figure 2. Some instances of unimodal functions

A unimodal function can be a non-differentiable or a discontinues function.

The optimum point V^* is obtained by using the outcomes of Theorem 3.1 and 3.2. In other words, if $Z(V)$ is convex, it is also unimodal. We propose an algorithm by combining the Interval Halving Algorithm and Dynamic Network Flow to find the minimum of problem (20). We can search the minimum $Z(V)$ in $[\bar{V}_0 = 0, \bar{V}_1]$ such that $\bar{V}_1 := \max_{t \in T, (i,j) \in A} U_{ij}(t) C'_{ij}(t)$, by using the Interval Halving Algorithm. In this method, exactly one-half of the existing interval is discarded in every phase. Finally, interval halving method gives $V^* \in [a,b]$ as minimum such that $b - a \leq \varepsilon$, where ε is given.

This algorithm is described by the following steps:

1. Let $G = (N, A)$ be a given dynamic network flow with uncertain costs belonging to interval.

2. Consider $\Gamma \in [0, |A|T]$ and ε given as robustness parameter and stopping criterion, respectively.

3. Let L_0 be length of interval $[\bar{V}_0, \bar{V}_1]$, where $\bar{V}_0 := 0$ and $\bar{V}_1 := \max_{t \in T, (i,j) \in A} U_{ij}(t) C'_{ij}(t)$. Divide the L_0 into four equal parts. Then label the middle point as V_0, the quarter-points as V_1 and V_2, respectively.

4. Evaluate $Z(V_0)$, $Z(V_1)$ and $Z(V_2)$ for problem (18) using convexity Z, Theorem 3.2. and dynamic network solving algorithm. Then let $Z_0 = Z(V_0)$, $Z_1 = Z(V_1)$ and $Z_2 = Z(V_2)$.

5. (a) If $Z_2 > Z_0 \geq Z_1$ (Figure 3), discard the subinterval $(V_0, \bar{V}_1)$ and let $\bar{V}_1 := V_0$, $V_0 := V_1$ and go to step 6.

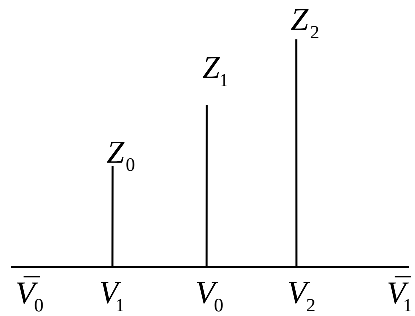

Figure 3. The step 5, (a)

(b) If $Z_1 > Z_0 \geq Z_2$ as shown in Figure 4, remove the subinterval $(\bar{V}_0, V_0)$ and let $\bar{V}_0 := V_0$, $V_0 := V_2$ and go to step 6.

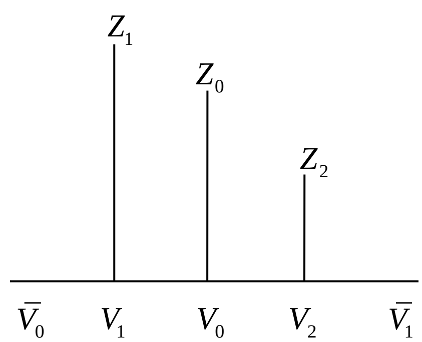

Figure 4. The step 5, (b)

(c) If $Z_1 > Z_0$, $Z_2 > Z_0$ (Figure 5) , discard both the subintervals $(\bar{V}_0, V_1)$ and $(V_2, \bar{V}_1)$, let $\bar{V}_0 := V_1$ and $\bar{V}_1 := V_2$, and go to step 6.

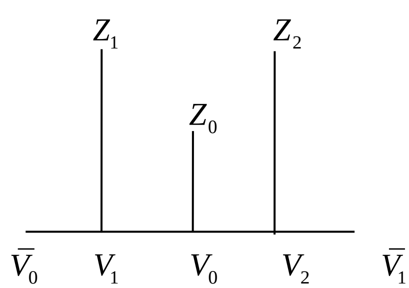

Figure 5. The step 5, (c)

6. Let $L := \bar{V}_1 - \bar{V}_0$. Test whether the new L satisfies the stopping criterion $L \le \varepsilon$. If the stopping criterion is satisfied set $V^* := V_0$ as the optimum solution with objective function value Z^*, where $Z^*(V^*) := Z(V_0)$. Otherwise, let the new $L_0 := L$ and go to step 3.

5. Example

Assume that there is a network G as shown in Figure 6, where the required information is given, as follows:

For arc $(1,3)$, $\forall t \in \{0,1,2,3\}$: $(C_{13}(t), U_{13}(t)) = (3,3)$,

For arc $(2,4)$, $\forall t \in \{0,1,2,3\}$: $(C_{24}(t), U_{24}(t)) = (2,4)$.

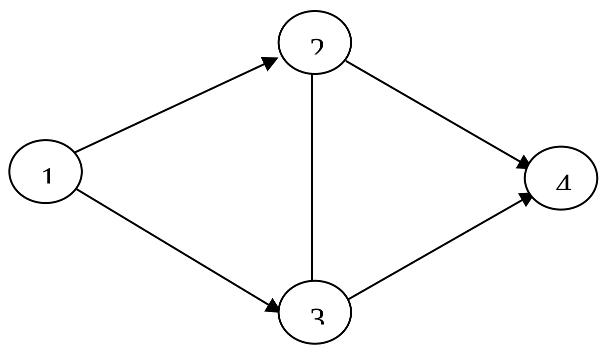

Figure 6. A numerical example

Moreover, $T = 3$ and $\forall t \in \{0,1,2,3\}, \forall (i.j) \in A : \lambda_{ij}(t) = 1$, and more information is given in Table 1.

Table 1. Information about example

Arc	$(1,2)$		$(2,3)$		$(3,4)$	
t \ C,U	$[C_{ij}(t), C_{ij}(t)+C'_{ij}(t)]$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$[C_{ij}(t), C_{ij}(t)+C'_{ij}(t)]$	$U_{ij}(t)$
0	[2,4]	3	2	4	[2,3]	3
1	[1,3]	2	3	3	[3,4]	2
2	[2,4]	3	2	2	[2,3]	4
3	[3,5]	4	3	3	[4,5]	3

Improved network for Figure 6 is shown in Figure 7. Moreover, information about improved network is given, as follows:

For arc $(1,3)$, $\forall t \in \{0,1,2,3\}$: $(C_{13}(t), U_{13}(t)) = (3,3)$,
For arc $(2,4)$, $\forall t \in \{0,1,2,3\}$: $(C_{24}(t), U_{24}(t)) = (2,4)$.

Moreover, $\forall t \in \{0,1,2,3\}$: $\lambda_{1'2'}(t) = \lambda_{1'2}(t) = \lambda_{2'2}(t) = \lambda_{3'4'}(t) = \lambda_{3'4}(t) = \lambda_{4'4}(t) = 0$ and another $\lambda_{ij}(t)$ is equal to 1.

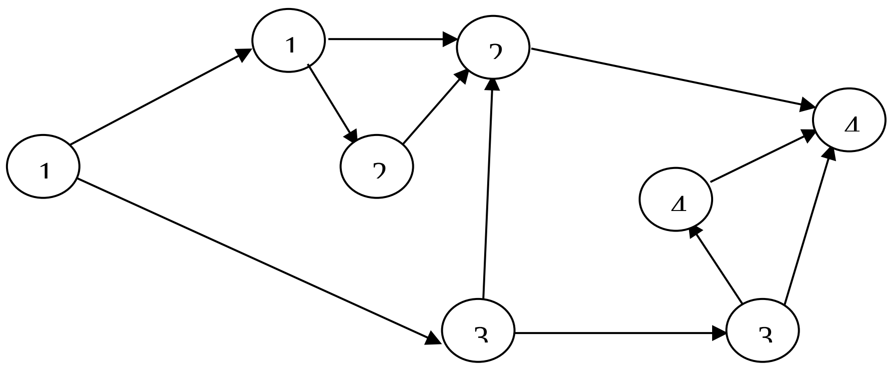

Figure 7. An improved network for G

With changes shown in Fig 1, we have Table 2, as follows:

Table 2. Information of improved network

arc	$(1,1')$		$(1',2)$		$(1',2')$		$(2',2)$		$(3,3')$		$(3',4)$		$(3',4')$		$(4',4)$	
C,U	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$	$C_{ij}(t)$	$U_{ij}(t)$
t = 0	2	3	0	$V/2$	2	∞	0	∞	2	3	0	V	1	∞	0	∞
t = 1	1	2	0	$V/2$	2	∞	0	∞	3	2	0	V	1	∞	0	∞
t = 2	2	3	0	$V/2$	2	∞	0	∞	2	4	0	V	1	∞	0	∞
t = 3	3	4	0	$V/2$	2	∞	0	∞	4	3	0	V	1	∞	0	∞

An improved dynamic network flow G , was solved by expanded network for $\Gamma = 0,1$ and 2 . We considered $b(1) = 2$, $b(4) = -2$ and $b(2) = b(3) = 0$ and minimum cost in dynamic network flow with uncertain cost belonging to interval in Figure 2, for $\Gamma = 0,1$ and 2 were obtained for formulations 14, 18 and 20, respectively by the described algorithm in the previous section.

6. Conclusion

In this paper, we considered minimum cost flow problem in dynamic network. We surveyed dynamic minimum cost flow in discrete and continuous networks. We focused on the problem with uncertain arc costs. Moreover, a short introduction into the problem formulation and objectives were given. A robust formulation was presented to take care of uncertainty and an algorithm was proposed to find the solution. In this algorithm, the concept of unimodal function and interval halving algorithm were applied.

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REFERENCES

Ahuja, R. K., Magnanti, T. L. and Orlin J. B. (1993). Network Flows: Theory, Algorithms, and Applications, Englewood Criffs, Prentice Hall.

Ahuja, R. K., Orlin, J. B., Pallottino, S. and Scutella, M. G. (2003). Dynamic Shortest Paths Minimizing Travel Times and Costs, Networks. Vol. 41, pp. 197-205.

Aronson, J. (1989). A survey of dynamic network flows, Ann. Oper. Res. Vol. 20, pp. 1-66.

Bazaraa, M. and Jarvis, J. (1977). Linear Programming and Network Flows, John Wiley, New York.

Ben Tal, A. and Nemirovski, A. (1998). Robust solutions of uncertain linear programs, Oper. Res. Lett. Vol. 25, pp. 1-13.

Ben Tal, A. and Nemirovski, A. (1999). Robust Convex Optimization, Oper. Res. Vol. 23, pp. 769-805.

Ben Tal, A. and Nemirovski, A. (2000). Robust solutions of Linear Programming problems contaminated with uncertain data, Math. Program. A. Vol. 88, pp. 411-424.

Bertsimas, D. and Sim, M. (2003). Robust discrete optimization and network flows, Math. Program. B. Vol. 98, pp. 49-71.

Fleischer, L. and Tardos, M. E. (1998). Efficient Continuous-time dynamic network flow algorithms, Oper. Res. Lett. Vol. 23, pp. 71-80.

Fonoberova, M. (2007). Optimal Flows in Dynamic Networks and Algorithm for their Finding, Ph.D. Thesis, Moldova University.

Ford, L. and Fulkerson, D. (1958). Constructing maximal dynamic flows from static flows, Oper. Res. Vol. 6 , pp. 419-433.

Ford, L. and Fulkerson, D. (1962). Flows in Networks Princeton University Press, Princeton, NJ.

Gast, S. (2010). Network Flow Problems with Uncertain Input Data in the Context of Supply Chain Management Applications Ph.D. Thesis, Department of Mathematics, University of Erlangen-Nurnberg.

Glockner, G. and Nemhauser, G. (2000). A dynamic network flow problem with uncertain arc capacities: formulation and problem structure, Oper. Res. Vol. 48, pp. 33242.

Hashemi, S. M., Mokarami, Sh. and Nasrabadi, E. (2010). Dynamic shortest path problems with time-varying costs, Optim lett. Vol. 4, pp. 147-156.

Hashemi, S. M. and Nasrabadi, E. (2012). On solving continuous-time dynamic network flows, J. Glob. Optim. Vol. 53, pp. 497-524.

Hoppe, B. (1995). Efficient Dynamic Network Flow Algorithms, Ph.D. Thesis, Department of Computer Science Technical, Cornell University.

Klinz, B. and Woeginger, G. J. (2004). Minimum cost dynamic flows: the series-parallel case, Networks. Vol. 43, pp. 153-162.

Kouvelis, P. and Yu, G. (1997). Robust Discrete Optimization and its Applications, Kluwer Academic Publishers, Amsterdam.

Nasrabadi, E. and Hashemi, M. (2010). Minimum cost time-varying network flow problems, Optim. Method. Softw. Vol. 25, No.3, pp. 429-447.

Philpot, B.B. (1982). Algorithm for continuous network flow problem, Ph.D. Thesis, Engineering Department, University of Cambridge.

Salehi Fathabadi, H. and Hosseini, S. A. (2011). An introduction to dynamic generative networks: Minimum cost flow, Appl. Math. Model. Vol. 35, pp. 5017-5025.

Salehi Fathabadi, H. and Hosseini, S. A. (2010). Maximum flow problem on dynamic generative network flows with time-varying bounds, Appl. Math. Model. Vol. 34, pp. 2136-2147.

Skutella, M. (2009). An introduction to network flows over time, Research Trends in Combinatorial Optimization. pp. 451-482.

Soyster, A. L. (1973). Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming, Oper. Res. Vol. 21, pp. 1154-1157.

Tahmasbi, R., Nasrabadi, E. and Hashemi, S. M. (2013). The value of information in stochastic maximum flow problems, Com. Oper. Res. Vol. 40, pp. 1744-1751.