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LRS Bianchi Type-II Cosmological Model with String Bulk Viscous Fluid and Magnetic Field in Barber’s Second Self Creation Theory

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Abstract

We present a solution of LRS Bianchi type-II space-time with string viscous fluid and magnetic field by solving the Barber’s field equations of self-creation theory of gravitation and it is seen that Barber scalar function $\phi$ affects the other physical parameters of the model and contributes a very high matter density, with high particle density at early stage of the universe and also it contributes the negative role of bulk viscous fluid in the evolution. Other geometrical and physical aspects of the model are also studied.

Keywords: Bianchi type-II; self-creation gravitational theory; electromagnetic fields; cosmology; hyperbolic geometric; isotropize; geometry

MSC (2010): 83D05, 83C50, 83F05

1. Introduction

General Relativity (GR) is modified by incorporating dark effects in two ways. The first category is that in which some exotic matter components are added in the energy-momentum tensor part of the action like the cosmological constant or quintessence field etc., while the second category is such that the whole gravitational action of GR is modified by including some dark energy source terms like extra order quantities of the Ricci scalar, such as $f(R), f(R,T), f(G)$ gravity etc. In $f(R)$ gravity, there has been several interesting dark energy as well as dark matter models that can be used to explore cosmological models. For
example, quadratic Ricci corrections can be used to discuss dynamics of inflationary universe [Sharif and Yousaf (2014)], inverse Ricci scalar invariant corrections may be used to discuss late time cosmic acceleration; see for instance, [Sharif and Yousaf (2014)]. There is a cubic three parametric \( f(R) \) corrections [Sharif and Yousaf (2015)], whose results are viable with lambda CDM model. It is interesting to notice that polynomial \( f(R) \) model can also be used to discuss inflationary universe [Sharif and Yousaf (2014)]. The late time acceleration as well as inflationary universe can be well discussed through generalized CDTT model [Sharif and Yousaf (2014)].

To expose the universe and to know more and more about it is one of the great issue of science and therefore we are taking-up and formulating the various models of the universe by involving various constraints on physical parameters like pressure, density, electromagnetic field etc. in our research. This is the motivation of this work to expose the universe by considering Bianchi type-II model with string bulk viscous fluid and with magnetic field.

Mach’s principle is not substantially accommodated i.e., inertial properties of matter is not taken satisfactorily into consideration in Einstein’s (1915) GR and therefore there have been many interesting attempts to generalize GR by incorporating the Mach’s principle and other desired features. One of them is Barber’s (1982) second self-creation theory of gravitation, developed by coupling the scalar field \( \phi(t) \) with trace of energy momentum tensor \( T_{ij} \) in order to accommodate the Mach’s principle substantially by the theory. This theory is also called self-creation cosmology (SCC). Thus, Barber (1982) modified the Einstein field equations by attaching the scalar field \( \phi(t) \) with the trace of energy momentum tensor \( T_{ij} \) as

\[ G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -\frac{8\pi T_{ij}}{\phi}, \]  

and the scalar field \( \phi \) satisfies the equation

\[ \Box \phi = \frac{8}{3} \pi \lambda_1 T. \]  

Here and hereafter, \( \Box \) stands for the invariant d’Alembertian in curved manifold, \( \Box \phi = \phi_{,kk} \), the vertical stroke ( | ) stands for covariant derivatives, \( \phi \) is the Barber scalar function of \( t \) and \( \frac{1}{\phi} = G \), in GR, \( T_{ij} \) is the stress energy momentum tensor, \( G_{ij} \) is an Einstein tensor, \( \lambda_1 \) is the coupling constant and \( 0 \leq |\lambda_1| \leq 10^{-1} \) and this restricted value of \( \lambda_1 \) is justified by the measurement of deflection of light.

The most famous scalar-tensor theory of gravitation is Brans-Dicke theory of gravitation. The Barber’s field Equations (1) and (2) are different from Brans-Dicke field equations

\[ G_{ij} = -8\pi \phi^{-1} T_{ij} - \omega \phi^{-2} \left( \phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_k \phi_k \right), \]

and

\[ \Box \phi = \phi_{,kk} = \frac{8\pi \phi^{-1} T}{(3+2\omega)}, \]
in the sense that, the scalar field $\phi(t)$ attached with energy-momentum tensor $T_{ij}$ in Barber’s field equations whereas this scalar field $\phi(t)$ is attached with both the energy-momentum tensor $T_{ij}$ and metric potential $g_{ij}$ in Brans-Dicke field equations. The second difference between these two theories is that when coupling constant tends to zero then Barber’s field equations approach Einstein’s field equations, whereas Brans-Dicke field equations approach Einstein’s field equations when coupling constant tends to infinity. These are the differences taking into consideration while studying Barber theory and Brans-Dicke theory.

In GR, the concept of Mach’s principle and local conservation of energy are not fully included and they been considered independently whereas these two concepts are considered together in Barber’s second self-creation theory of gravitation OR in self-creation cosmology (SCC), which may resolve the confusing and difficult problems like the dark energy problem, the lambda problem, accelerating cosmological expansion, the anomalous Pioneer spacecraft acceleration, a spin-up of the Earth and an apparent variation of $G$ observed from analysis of the evolution of planetary longitudes. This SCC is equivalent to GR in a vacuum with the consequence that the predictions of the theory are identical with GR in the standard solar system experiments. The model in SCC expands linearly in the framework of Einstein GR and they are static in Jordan frame. This SCC determine the total energy parameter to be one third and the cold dark matter density parameter to be two ninths, and yet in the Jordan frame the universe is similar to Einstein’s original static cylindrical model and spatially flat. Therefore, there is no need for a ‘Dark Energy’ hypothesis. Also there is no lambda problem, as the field equations determine the false vacuum energy density to be specific and feasibly small (Barber 2005).

Barber’s (1982) theory is a variable $G$-theory and predicts local effects, which are within the observational limits. In it, the Newtonian gravitational parameter $G$ is not a constant but a function of time parameter $t$. Also, the scalar field $\phi$ does not gravitate directly but simply divides the matter tensor acting as a reciprocal gravitational constant, which is not the case in Einstein’s GR. This theory is capable of verification or falsification. It can be done by observing the behavior of both bodies of degenerate matter and photons. An observation of anomalous precessions in the orbits of pulsars about central masses and an accurate determination of the deflection of light and radio waves passing close to the sun would verify or falsify such theory and determine $\lambda$. The theory predicts the same precession of the perihelia of the planets as general relativity and in that respect agrees with observation to within 1%.

The geometry of the universe is determined by the matter distribution contained in the universe. The matter means any type of quantity contains perfect fluid, viscous fluid, electromagnetic field etc. and it is represented by energy-momentum tensor. The energy-momentum tensor of a cloud of a string with viscous fluid and electromagnetic field is playing the important role in the evolution of the universe. Cosmic string played a significant role during an early stage of the evolution of the universe and gave rise to density perturbation which led to formation of galaxies (Zel’dovich, 1980). This cosmic string has stress energy and coupled to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from string. Also, cosmological models of a fluid with viscosity plays a significant role in the evolution of the universe. In the formation of galaxies and clusters in the universe, the matter distribution is satisfactorily described by a perfect fluid. However, when neutrino decoupling occurs, then the matters behave like a viscous fluid at early stage of the universe (Misner, 1968). The magnetic field is present in the galactic and intergalactic spaces and plays a significant role at the cosmological scale. Melvin
(1975) suggested that during the evolution of the universe, the matter was in highly ionized state and again this matter is smoothly coupled with the magnetic field and forms neutral matter as a result of universe expansion and therefore the presence of magnetic field in the universe is not unrealistic. Taking into consideration all this fact, it is important to consider an energy-momentum tensor of a cloud of a string with viscous fluid and electromagnetic field.

Right from Barber (1982), following researchers studied the theory and developed the models of the universe and investigated geometrical and physical aspects of the universe in this SCC theory. Pimental (1985) and Soleng (1987) have presented the Robertson Walker solutions in self-creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field. Reddy and Venkateswarlu (1989) have obtained spatially homogeneous and anisotropic Bianchi type-VI$_0$ cosmological models in Barber’s self-creation theory of gravitation both in vacuum and in the presence of perfect fluid with pressure equal to energy density. Venkateswarlu and Reddy (1990) have also got spatially homogeneous and anisotropic Bianchi type-I cosmological macro models when the source of gravitational field is a perfect fluid. Bianchi type-II and III models in self-creation cosmology have been deduced by Shanti and Rao (1991), Rao and Raju (1992) have discussed Bianchi type VIII and IX models in zero mass scalar fields and self-creation cosmology. Shri Ram and Sigh (1997) have obtained spatially homogeneous and isotropic R-W model of the universe in the presence of perfect fluid by using ‘gamma law’ equation of state. Recently, Panigrahi and Sahu (2003), Pradhan and Pandey (2004), Reddy (2006), Reddy et al. (2006), Venkateswarlu et al. (2008), Rao et al. (2008), Rao and Vinutha (2010), Rathore and Mandawat (2010), Borkar and Ashtankar (2011), Katore et al. (2011), Borkar and Ashtankar (2013) have deduced the plane and axially symmetric cosmological models and Bianchi type-I, II, VIII and IX string cosmological models in second self-creation theory of gravitation and they studied the geometrical and physical aspects of the models.

On a large scale structure the universe is homogeneous and isotropic in nature and Bianchi type models are generally nonhomogeneous and anisotropic. No one knows that the universe has particular and definite type of nature and therefore cosmologist have been exposing the universe right from many decades ago, by assuming an isotropic as well as anisotropic models in their research in order to get different types of secrets of the nature. Anisotropic cosmological models are studied in recent time with the advent of more precious data from measurement of cosmic microwave background (CMB) temperature anisotropy. Thus, the studies of anisotropic Bianchi models are important. In this note, we have taken-up the study of the locally rotationally symmetric (LRS) Bianchi type-II anisotropic model with string bulk viscous fluid along with magnetic field and have evaluated the important features of the model. It is realized that the Barber scalar function $\phi$ affects the behavior of physical parameters and contributes a very high density matter, with high particle density at the early stage of the universe and also it contributes the negative role of bulk viscous fluid in the evolution. Our results are generalizations, and match the results of Tyagi et al. (2011), in particular for the constant scalar function, $\phi$.

2. The Metric and Field Equations

We consider the Bianchi type-II space-time in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2,$$

(3)
where \((x, y, z)\) are space coordinates, \(t\) is time coordinate, and \(A\) and \(B\) are functions of \(t\) alone.

The energy-momentum tensor \(T^i_j\) for a cloud of strings with bulk viscous fluid and electromagnetic field \(E^i_j\) is given by

\[
T^i_j = \rho \, v^i v^j - \lambda \, x^i x^j - \xi \, v^i_{\parallel} \left( g^i_j + v^i v^j \right) + E^j_i.
\]  

Here, \(v^i\) is the four velocity vectors and \(x^i\) is the displacement vector and they satisfy the conditions

\[
v^i v^i = -1 = - x^i x^i, \quad (5)
\]

\[
v^i x^i = 0, \quad (6)
\]

and the electromagnetic field \(E^i_j\) [Lichnerowicz (1967)] is given by

\[
E^i_j = \bar{\mu} \left[ |h|^2 \left( v^i v^j + \frac{1}{2} g^i_j \right) - h_i h^j \right].
\]

The magnetic flux vector is given by

\[
h_i = \frac{\sqrt{-g}}{2b} \, \epsilon_{ijkl} F^{kl} \, v^l,
\]

where \(\bar{\mu}\) is the magnetic permeability, \(F^{kl}\) is the electromagnetic field tensor, \(\epsilon_{ijkl}\) is the Levi-Civita tensor, \(\xi\) is the coefficient of bulk viscosity, \(\rho\) is the proper density of a cloud strings. We are considering the string phases in our model and therefore its string tension density \(\lambda\) is appearing in the literature. Further, we loaded the particles on the string with density \(\rho_p\) and therefore the proper density \(\rho\) of a cloud string is to be related with string tension density \(\lambda\) and particle density \(\rho_p\) in the relation

\[
\rho = \rho_p + \lambda.
\]

The scalar expansion \(\theta\) is defined as \(v^i_{\parallel}\) and it has value

\[
\theta = \frac{2A_4}{A} + \frac{B_4}{B}.
\]

In a co-moving coordinate system, we assume

\[
v^i = (0, 0, 0, 1), \quad x^i = \left( \frac{1}{A}, 0, 0, 0 \right).
\]

The components of electromagnetic field \(F^{kl}\) appeared in the literature, due to the assumption of electric current produced along \(x\)-axis and the magnetic field is in \(y-z\) plane. The surviving component of \(F_{ij}\) is \(F_{23}\) only and \(F_{14} = F_{24} = F_{34} = 0\) due to assumption of infinite electrical conductivity. We have, \(h_1 \neq 0, \quad h_2 = h_3 = h_4 = 0\). Maxwell’s equations
\[ F_{ij[k} + F_{jk[l]} + F_{ki[l]} = 0, \quad (11) \]

and

\[ F_{ij}^{\mid ij} = 0, \quad (12) \]

are satisfied by

\[ F_{23} = H \text{ (constant)}. \quad (13) \]

Equation (8) leads to

\[ h_1 = \frac{H}{\mu B}. \quad (14) \]

Using the above Equation (14), from Equation (7), we have calculated the components of \( E_i^j \) as

\[ E_1^1 = \frac{-H^2}{2\mu A^2 B^2} = -E_2^2 = -E_3^3 = -E_4^4. \quad (15) \]

We write the components of energy momentum tensor \( T_i^j \) from Equation (4) (using Equation (15)) as

\[ T_1^1 = -\left( \lambda + \xi \theta + \frac{H^2}{2\mu A^2 B^2} \right), \quad T_2^2 = T_3^3 = -\xi \theta + \frac{H^2}{2\mu A^2 B^2}, \quad T_4^4 = -\rho - \frac{H^2}{2\mu A^2 B^2}. \quad (16) \]

The Barber’s field Equations (1) and (2), for the metric (3) with the components of energy momentum tensor from the Equations (16), take the form

\[
\begin{align*}
\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_{4B}}{AB} + \frac{B^2}{4A^4} &= \left[ \lambda + \xi \theta + \frac{H^2}{2\mu A^2 B^2} \right] 8\pi \phi^{-1}, \\
2 \frac{A_{44}}{A} + \frac{A^2}{A^2} - 3 \frac{B^2}{4A^4} &= \left[ \xi \theta - \frac{H^2}{2\mu A^2 B^2} \right] 8\pi \phi^{-1}, \\
\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_{4B}}{AB} + \frac{B^2}{4A^4} &= \left[ \xi \theta - \frac{H^2}{2\mu A^2 B^2} \right] 8\pi \phi^{-1}, \\
\frac{A_{4}^2}{A^2} + 2 \frac{A_{4B}}{AB} - \frac{B^2}{4A^4} &= \left[ \rho + \frac{H^2}{2\mu A^2 B^2} \right] 8\pi \phi^{-1}, \\
\phi_{44} + \phi_4 \left[ 2 \frac{A_{4}}{A} + \frac{B_4}{B} \right] &= d_1 (\lambda + 3\xi \theta + \rho),
\end{align*}
\]

where \( A \) and \( B \) are scale factors and

\[ d_1 = \frac{8\pi \lambda_1}{3}, \quad A_4 = \frac{\partial A}{\partial t}, \quad B_4 = \frac{\partial B}{\partial t}, \quad A_{44} = \frac{\partial^2 A}{\partial t^2} \text{ etc.} \]

3. Solution of the Field Equations
We are trying to find the solutions of a set of five differential Equations (17-21) and interpreting them with the barber scalar function, $\phi$. These five differential Equations (17-21) are in six unknowns $A, B, \rho, \lambda, \xi$, and $\phi$. In order to have a solution, we must assume one extra condition that shear $\sigma$ is proportional to the scalar expansion $\theta$, [Collins et al. (1980), Bali (1986)]. The motive behind assuming this condition is explained as follows:

Referring to Thorne (1967), the observations of velocity-redshift relation for extra-galactic sources suggest that the Hubble expansion of the universe is isotropic to within 30% (Kantowski and Sachs (1966) and Kristian and Sachs (1966)). More precisely, the redshift studies place the limit $\frac{\sigma}{H} \leq 0.30$, where $\sigma$ is shear and $H$ is Hubble constant. Collins et al. (1980) have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hypersurface satisfies the condition $\frac{\sigma}{\theta} = \text{constant}$. The condition $\frac{\sigma_1^4}{\theta} = \text{constant}$ leads to $B = lA^n$, when the tilt angle is zero. Here, $\sigma_1^4$ is the eigenvalue of shear tensor $\sigma_j^i$.

This condition leads to the relation between the metric potential $A$ and $B$ as

$$B = lA^n, \quad (22)$$

where $l \geq 1$ and $n \neq 1$ are constants.

From Equations (18) and (19), we have

$$A_{44} + (1 + n)\frac{A_4^2}{A} = \frac{l^2}{(1-n)}A^{2n-3}. \quad (23)$$

We denote $A_4 = \eta$ then $A_{44} = A_4 \frac{d\eta}{dA}$ and Equation (23) can be reduced to the first order differential equation as

$$\eta \frac{d\eta}{dA} + (1 + n)\frac{\eta^2}{A} = \frac{l^2}{(1-n)}A^{2n-3},$$

which is a Bernoulli equation and its solution is

$$\frac{A^{n+1}}{[c_1 + c_2 A^{4n}]^2} dA = dt, \quad (24)$$

where $c_1$ is the constant of integration and $c_2 = \frac{l^2}{2n(1-n)}$, a constant.

Equation (24) is very difficult to integrate and therefore for simplicity, we are assuming the particular value of constants, $c_1 = 0$, $l = 1$, $n = \frac{1}{2}$ and $c_2 = 2$ to solve Equation (24). With these particular values, the differential Equation (24) has the solution

$$\frac{\sqrt{2}}{3} A^3 = t + c_3,$$

where $c_3$ is the constant of integration taken to be zero. So that, we have,
\[ A = \left(\frac{3t}{\sqrt{2}}\right)^{\frac{2}{3}}. \]  

(25)

Using Equation (25) in Equation (22), we write
\[ B = \left(\frac{3t}{\sqrt{2}}\right)^{\frac{1}{3}}. \]  

(26)

Thus, the solutions of Barber’s field equations for Bianchi type-II metric with string bulk viscous fluid and with magnetic field are given by Equation (25) and (26) and the required metric is
\[ ds^2 = -dt^2 + \left(\frac{3t}{\sqrt{2}}\right)^{\frac{4}{3}} (dx^2 + dz^2) + \left(\frac{3t}{\sqrt{2}}\right)^{\frac{2}{3}} (dy - xdz)^2. \]  

(27)

4. Geometrical and Physical Significance

Using the values of \( A \) and \( B \) from Equations (25) and (26) in Equations (18) and (20), we obtained
\[ 8\pi \phi^{-1} \xi = \frac{8\pi H^2}{9 \mu t^2} \phi^{-1} - \frac{1}{6t^2}, \]  

(28)

and
\[ 8\pi \phi^{-1} \rho = \frac{5}{6t^2} - \frac{8\pi H^2}{9 \mu t^2} \phi^{-1}, \]  

(29)

respectively. Using Equations (25), (26) and (28), from Equation (17), we have calculated the string tension density \( \lambda \) as
\[ \lambda = \frac{-2H^2}{9 \mu t^2}. \]  

(30)

The expansion \( \theta \) is given by Equation (10) and it has value
\[ \theta = \frac{5}{3t}. \]  

(31)

Using Equation (31) in Equation (28), we obtained
\[ \xi = \left(\frac{H^2}{15 \mu t} - \frac{1}{80 \pi t} \phi\right). \]  

(32)

Using Equations (28)-(30), we write the Equation (21) for \( \phi \) as
\[ t^2 \phi_{44} + \frac{5}{3} t \phi_4 = \alpha \phi, \]  

(33)
where \( \alpha = \frac{\pi \lambda_1}{9} \) is constant. After solving the above differential Equation (33), we arrived at its zero particular integral and its complementary function (C. F.) is given by

\[
(D_1 t^{m_1} + D_2 t^{m_2}).
\]

So that the solution of above differential Equation (33) will be

\[
\phi = D_1 t^{m_1} + D_2 t^{m_2}.
\]  
(34)

We write

\[
\phi = \phi_1 + \phi_2,
\]  
(35)

where,

\[
\phi_1 = D_1 t^{m_1},
\]  
(36)

and

\[
\phi_2 = D_2 t^{m_2},
\]  
(37)

in which

\[
m_1 = \frac{1}{3}(-1 + \sqrt{1 + \pi \lambda_1}) \quad \text{and} \quad m_2 = \frac{1}{3}(-1 - \sqrt{1 + \pi \lambda_1}),
\]  
(38)

and \( D_1, D_2 \) are constants.

Both the scalar function \( \phi_1 \) and \( \phi_2 \) describe the behavior of scalar function \( \phi \). Initially, at \( t = 0 \), we have \( \phi_1 = 0 \) and \( \phi_2 = 0 \) and then the scalar function \( \phi_1 \) is growing, while \( \phi_2 \) is decaying, when \( t \) increases, for the coupling constant \( \lambda_1 > 0 \). Both the scalar field \( \phi_1 \) and \( \phi_2 \) are imaginary for the coupling constant \( \lambda_1 < 0 \).

When coupling constant \( \lambda_1 \to 0 \) then the scalar function \( \phi_1 \) approaches a constant value whereas the scalar function \( \phi_2 \) is a variable of time \( t \). In order to match with Einstein general relativity, the scalar function \( \phi \) should be constant, when \( \lambda_1 \to 0 \). Hence we are neglecting the time variable function of \( t \) in our further discussion. Therefore Equation (35) leads to

\[
\phi = \phi_1 = t^{m_1}, \quad \text{with} \quad D_1 = 1.
\]  
(39)

The scalar function \( \phi \) is an increasing function of time \( t \) and affects the behavior of physical parameters in the model.

The spatial volume \( V \) of the model is given by

\[
V = R^3 = A^2B = \left[ \frac{3t}{\sqrt{2}} \right]^5.
\]  
(40)

The scale factors \( A, B \) and volume \( V \) are increasing functions of time, \( t \). Initially, when \( t \to 0 \), the scale factors \( A, B \) and volume \( V \) attain zero value and finally, when \( t \to \infty \), they attain
infinite values. This shows that the model starts evolving with zero volume and zero scale factor and expanding with infinite volume at final stage.

We have calculated the energy density $\rho$, from Equation (29) as

$$\rho = \left( \frac{5}{48\pi} \frac{m_1}{t^2} - \frac{H^2}{9\mu t^2} \right) = \left( \frac{5}{48\pi} \frac{\phi}{t^2} - \frac{H^2}{9\mu t^2} \right).$$  \hspace{1cm} (41)

Lambda-CDM is an abbreviation for Lambda - Cold Dark Matter. It is frequently referred to as the standard model of big bang cosmology, since it attempts to explain the existence and structure of the cosmic microwave background, the large scale structure of galaxy clusters and distribution of hydrogen, helium, oxygen and also the accelerating expansion of the universe observed in light from distant galaxies and supernovae. It is the simplest model that is in general agreement with observed phenomena. The energy density $\rho$ in our model is a function of time, $t$. It attained a very large value in the beginning of the big-bang cosmology and it is continuously decreasing, with increased time $t$ and approaches zero at the final stage. Thus, the evolution of the energy density of the universe according to the model represents the same time proportionality as in lambda CDM model of big bang cosmology.

Using this value (Equation (41)) of energy density $\rho$ and the value of $\lambda$ from Equation (30), Equation (9) yields the particle density $\rho_p$ as

$$\rho_p = \left( \frac{5}{48\pi} \frac{m_1}{t^2} + \frac{H^2}{9\mu t^2} \right) = \left( \frac{5}{48\pi} \frac{\phi}{t^2} + \frac{H^2}{9\mu t^2} \right).$$  \hspace{1cm} (42)

The conservation (local) of energy momentum tensor and Mach’s principle are not fully accommodated in general relativity and therefore they are taken into consideration together in Barber self-creation theory of gravitation by attaching the scalar function $\phi$ with $T_{ij}$ and hence we are evaluating the model in view of the physical significance of scalar function $\phi$ in other physical parameters.

The energy density $\rho$ is a function of time, $t$. Initially, when $t \to 0$, it attains a very large value and it is decreasing continuously with increasing time $t$ and reaches to zero, when $t \to \infty$. We observed the similar nature of particle density $\rho_p$ as that of energy density $\rho$ in the model. In these physical parameters $\rho$ and $\rho_p$, the barber scalar function $\phi$ is playing the important role and affect the behavior of these parameters $\rho$ and $\rho_p$. Due to its presence, both $\rho$ and $\rho_p$ attain very very large values, i.e., they diverges to infinity at initial stage and the models occupied a very highly compact matter. Our results go over the results of Atul Tyagi et al. (2011), in regards to energy density and particle density, and in particular for constant $\phi$. Thus, the scalar function $\phi$ contributes highly compact matter in early universe. One can relate the scalar function $\phi$ with magnetic field and matter density as $\rho \geq 0$ if $\frac{H^2}{\mu} \leq \frac{15}{16\pi} \phi$ and when $\frac{H^2}{\mu} > \frac{15}{16\pi} \phi$ then $\rho < 0$. Thus, the model exists if $\frac{H^2}{\mu} \leq \frac{15}{16\pi} \phi$ and the non-existence of the model is shown for $\frac{H^2}{\mu} > \frac{15}{16\pi} \phi$. 
Equation (30) shows the nature of the string tension density $\lambda$ which appeared with negative value. This shows that the string phases in the universe switched-off or disappeared without affecting the time factor (see Latelier (1983)).

Using the expression of $\theta$, from Equation (10) and $\phi$ from Equation (39), from Equation (28), we have calculated the coefficient of bulk viscosity $\xi$ as

$$\xi = \left( \frac{H^2}{15 \mu t} - \frac{m_1}{80 \pi t} \right) = \left( \frac{H^2}{15 \mu t} - \frac{\phi}{80 \pi t} \right),$$

and shear $\sigma$ have the value

$$\sigma = \frac{1}{3\sqrt{3} t}. \qquad (44)$$

The role of barber scalar function $\phi$ appeared in the coefficient of bulk viscosity $\xi$. Initially, when $t \to 0, \xi \to -\infty$. Thus, $\xi$ is always negative; its value lies in forth quadrant. Due to the presence of barber scalar function $\phi$, the coefficient of bulk viscosity appeared with negative value for the whole range of time $t$ and therefore there is no role of viscous fluid in the model. Thus, the barber scalar function $\phi$ contributes the negative role of viscous fluid or abolished the viscosity. Further, it is seen that, when $\phi < \frac{16\pi H^2}{3\mu}$, then the coefficient of bulk viscosity $\xi$ is positive. Thus, the Barber scalar function $\phi$ with value less than $\left(\frac{16\pi H^2}{3\mu}\right)$ leads to viscous fluid model and $\phi \geq \frac{16\pi H^2}{3\mu}$ gives rise to non-viscous fluid model.

The scalar expansion $\theta$ is a decreasing function of time, $t$. It attains a very large value in the beginning and a small value in the end of the model. Thus, the model starts evolving with highest expansion and expansion the continuously decreases and approaches zero at the end and is not affected by scalar function $\phi$. With regard to the nature of shear $\sigma$, we can put the similar argument as that of scalar expansion $\theta$ that, it attains the highest value in the beginning and very small value in the end. The ratio $\frac{\sigma}{\theta} \neq 0$, as $t \to \infty$ shows an anisotropic model.

We summarized our results with the results of Tyagi et al. (2011) and realized that our results are generalizations and Barber’s scalar function $\phi(t)$ contributes the very high density matter with highest particle density and negative role of bulk viscosity in early universe.

5. Conclusion

1. We have presented the solution of LRS Bianchi type-II space-time with string viscous fluid and magnetic field by solving the field equations of Barber’s self-creation theory of gravitation and it is observed that our results are the generalization of Tyagi et al. (2011) in regards to energy density $\rho$, particle density $\rho_p$ and coefficient of bulk viscosity $\xi$, that the universe occupied a very high density matter, with high particle density and with negative role of bulk viscosity in early stage.

2. The scale factors $A, B$ and volume $V$ are increasing functions of time, $t$. Initially, when $t \to 0$, the scale factors $A, B$ and volume $V$ attain zero value and finally when $t \to \infty$,
they attain infinite value. This shows that the model starts evolving with zero volume and zero scale factor and expanding with infinite volume at the final stage.

3. The barber scalar function φ affects the behavior of physical parameters. Also, the energy density ρ, and particle density ρp occupied the highest values at the initial stage of the model. This shows that the barber scalar function φ contributes a very high density matter with high particle density in early universe.

4. Positive string tension density λ reflects the existence of string phases in the universe. In our model, string tension density λ always appeared with negative value. The scalar function φ does not affect the behavior of string tension density λ. Because of the negative value of string tension density λ we would like to say that the string phases switched off or disappeared.

5. The coefficient of bulk viscosity is always negative due to the presence of scalar function φ, but the value of φ \(< \frac{16\pi H^2}{3\mu}\) can lead viscosity in the model.

6. The scalar expansion θ in the model is found to be inversely proportional to cosmic time t in the evolution. This suggested that when t → 0, θ admits a very high range and when t → ∞, θ goes to zero. Thus, our model starts evolving with high range of expansion; it is continuously decreasing and approaches zero at later stage. We would like to mention here that this cosmic expansion would be in conflict with observation but the universe has mysterious nature and no one can exactly observe the exact nature and therefore there may be chance to such type of nature.

7. The shear σ in the model behave in a similar way as that of expansion and the model starts with highest shear and ends with shear less.

8. The ratio \(\frac{σ}{θ} \neq 0\), when t → ∞ shows that the model does not isopropize.

We have evaluated LRS Bianchi type-II space-time with string viscous fluid and magnetic field by solving the Barber’s field equations of self-creation theory of gravitation. It is realized that the Barber scalar function φ affects the behavior of physical parameters and contributes a very high density matter with high particle density at early stage of the universe and also it contributes the negative role of bulk viscous fluid in the evolution. Our results are generalizations and matched the results of Tyagi et al. (2011), in particular for constant scalar function φ.

This work can be extended further for spherical, cylindrical, planer space-time as well as restricted class of axial a stellar system in our future plan.

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