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F–Geometric Mean Graphs

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Abstract:

In a study of traffic, the labelling problems in graph theory can be used by considering the crowd at every junction as the weights of a vertex and expected average traffic in each street as the weight of the corresponding edge. If we assume the expected traffic at each street as the arithmetic mean of the weight of the end vertices, that causes mean labelling of the graph. When we consider a geometric mean instead of arithmetic mean in a large population of a city, the rate of growth of traffic in each street will be more accurate. The geometric mean labelling of graphs have been defined in which the edge labels may be assigned by either flooring function or ceiling function. In this, the readers will get some confusion in finding the edge labels which edge is assigned by flooring function and which edge is assigned by ceiling function. To avoid this confusion, we establish the *F*-geometric mean labelling on graphs by considering the edge labels obtained only from the flooring function. An *F*-Geometric mean labelling of a graph *G* with *q* edges, is an injective function from the vertex set of *G* to $\{1, 2, 3,..., q+1\}$ such that the edge labels obtained from the flooring function of geometric mean of the vertex labels of the end vertices of each edge, are all distinct and the set of edge labels is $\{1, 2, 3,..., q\}$. A graph is said to be an *F*–Geometric mean graph if it admits an *F*–Geometric mean labelling. In this paper, we study the *F*-geometric meanness of the graphs such as cycle, star graph, complete graph, comb, ladder, triangular ladder, middle graph of path, the graphs obtained from duplicating arbitrary vertex by a vertex as well as arbitrary edge by an edge in the cycle and subdivision of comb and star graph.

Keywords: Labelling, *F*-Geometric mean labelling, *F*-Geometric mean graph

MSC 2010 No.: 05C78

I. I. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let *G(V, E)* be a graph with *p* vertices and *q* edges. For notations and terminology, the readers are referred to the book of Harary (1972). For a detailed survey on graph labelling we refer the reader to the book of Gallian (2014).

A path on *n* vertices is denoted by P_n and a cycle on *n* vertices is denoted by C_n. $K_{1,n}$ is called a star graph and it is denoted by S_n. GoK_1 is the graph obtained from *G* by attaching a new pendant vertex to each vertex of *G*. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the *Cartesian product* $G_1 \times G_2$ has $p_1 p_2$ vertices which are $\{(u,v): u \in G_1, v \in G_2\}$. The edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The *middle graph* $M(G)$ of a graph *G* is the graph whose vertex set is $\{v : v \in V(G)\} \cup \{e : e \in E(G)\}$ and the edge set is

$$\{e_1 e_2 : e_1, e_2 \in E(G) \text{ and } e_1 \text{ and } e_2 \text{ are adjacent edges of } G\}$$
$$\cup \{ve : v \in V(G), e \in E(G) \text{ and e is incident with } v\}.$$

Let *G* be a graph and let v be a vertex of *G*. The duplicate graph $D(G, v')$ of *G* is the graph whose vertex set is $V(G) \cup \{v'\}$ and edge set is

$$E(G) \cup \{v'x : x \text{ is the vertex adjacent to } v \text{ in G}\}.$$

Let *G* be a graph and let $e = uv$ be an edge of *G*. The duplicate graph $D(G, e' = u'v')$ of *G* is the graph whose vertex set is $V(G) \cup \{u', v'\}$ and edge set is $E(G) \cup \{u'x, v'y : x$ and y are the vertices adjacent with u and v in G respectively$\}$. The *triangular ladder* TL_n, $n \geq 2$ is a graph obtained by completing the ladder $P_2 \times P_n$ by adding the edges $u_i v_{i+1}$ for $1 \leq i \leq n-1$. For a graph *G* the graph *S*(*G*) is obtained by subdividing each edge of *G* by a vertex. An arbitrary subdivision of a graph *G* is a graph obtained from *G* by a sequence of elementary subdivisions forming edges into paths through new vertices of degree 2.

The study of graceful graphs and graceful labelling methods was first introduced by Rosa (1967). The concept of mean labelling was first introduced and developed by Somasundaram and Ponraj (2003). Further, it was studied by Vasuki et al. (2009, 2010, 2011). Vaidya and Lekha Bijukumar (2010) discussed the mean labelling of some graph operations. Mohanaselvi and Hemalatha (2014) discussed the super geometric mean labelling of various classes of some graphs.

A function f is called an *F*-Geometric mean labelling of a graph *G(V,E)* if $f: V(G) \rightarrow \{1,2,3,...,q+1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,2,3,...,q\}$ defined as

$$f^*(uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor, \text{ for all } uv \in E(G),$$

is bijective. A graph that admits an *F*–Geometric mean labelling is called an *F*–Geometric mean graph.

Somasundaram et al. (2011) defined the geometric mean labelling as follows: A graph $G=(V,E)$ with p vertices and q edges is said to be a geometric mean graph if it possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2,...,q+1$ in such a way that when each edge $e = uv$ is labelled with $f(uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ or $\left\lceil \sqrt{f(u)f(v)} \right\rceil$, then the edge labels are distinct.

Somasundaram et al. (2012) have given the geometric mean labelling of the graph $C_5 \cup C_7$ as in the Figure 1.

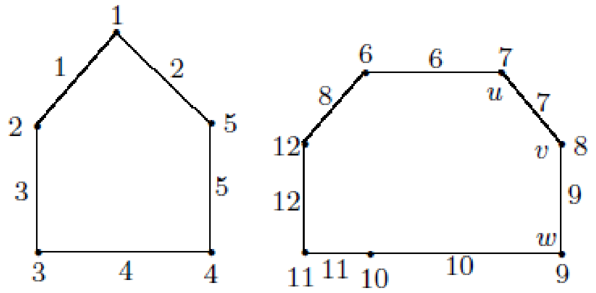

Figure 1. A Geometric mean labelling of $C_5 \cup C_7$

From the above figure, for the edge uv, they have used flooring function $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ and for the edge vw, they have used ceiling function $\left\lceil \sqrt{f(v)f(w)} \right\rceil$ for fulfilling their requirement. To avoid the confusion of assigning the edge labels in their definition, we just consider the flooring function $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ for our discussion. Based on our definition, the *F*-Geometric mean labelling of the same graph $C_5 \cup C_7$ is given in Figure 2.

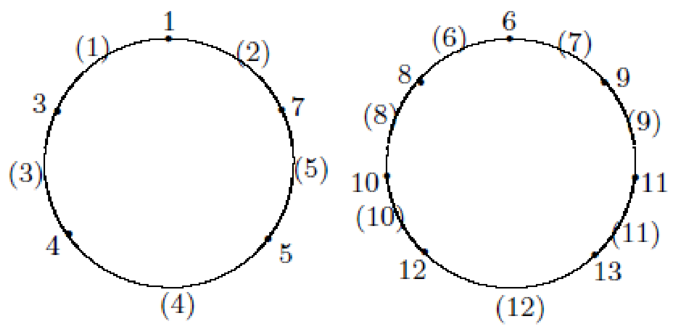

Figure 2. An *F*-Geometric mean labelling of $C_5 \cup C_7$ and its edge labelling

In this paper, we study the *F*-Geometric meanness of the graphs, namely, cycle C_n for $n \geq 3$, the star graph S_n for $n \leq 3$, the complete graph K_n for $n \leq 3$, the comb $P_n o K_1$ for any positive integer *n*, the ladder $P_2 \times P_n$ for any positive integer *n*, the middle graph $M(P_n)$, the graphs obtained by duplicating an arbitrary vertex as well as arbitrary edge in the cycle C_n, the triangular ladder TL_n for $n \geq 2$, the graph $S(P_n o K_1)$ and the arbitrary subdivision of S_3.

2. Main Results

To study the *F*-geometric meanness, some of the standard graphs, and graphs obtained from some graph operations are taken for discussion.

Lemma 2.1.

Let G be a graph. If $|V(G)| > |E(G)| + 1$, then G does not admit an *F*-Geometric mean labelling.

Proof:

If $|V(G)| > |E(G)| + 1$, then the vertex labelling will not be injective and hence the result follows.

Theorem 2.2.

The union of any two trees is not an *F*-Geometric mean graph.

Proof:

Let G be the union of two trees S and T. Then If $|V(G)| = |V(S)| + |V(T)|$ and $|E(G)| = |E(S)| + |E(T)| = |V(S)| + |V(T)| - 2$ then by Lemma 2.1, the result follows.

Corollary 2.3.

Any forest is not an *F*-Geometric mean graph.

Theorem 2.4.

Every cycle is an *F*-Geometric mean graph.

Proof:

Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n.

We define

$$f: V(C_n) \to \{1, 2, 3, \ldots, n+1\}$$

as follows:

$$f(v_i) = \begin{cases} i, & 1 \le i \le \left\lfloor \sqrt{n+1} \right\rfloor - 1, \\ i+1, & \left\lfloor \sqrt{n+1} \right\rfloor \le i \le n\ . \end{cases}$$

The induced edges labelling is as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} i, & 1 \le i \le \left\lfloor \sqrt{n+1} \right\rfloor - 1, \\ i+1, & \left\lfloor \sqrt{n+1} \right\rfloor \le i \le n-1, \end{cases}$$

and

$$f^*(v_1 v_n) = \left\lfloor \sqrt{n+1} \right\rfloor .$$

Hence, f is an F-Geometric mean labelling of the cycle C_n. Thus the cycle C_n is an F-Geometric mean graph.

An F-Geometric mean labelling of C_6 is shown in Figure 3.

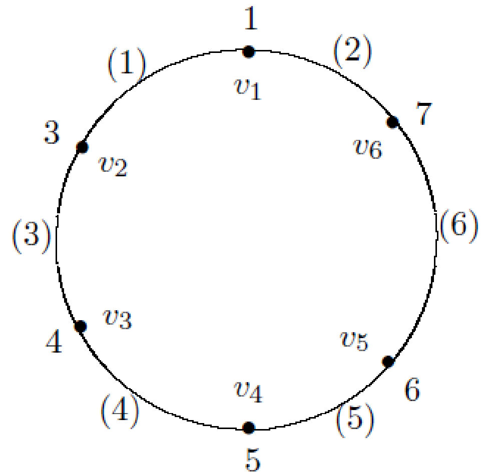

Figure 3. An F-Geometric mean labelling of C_6 and its edge labelling

Theorem 2.5.

The star graph S_n is an F-Geometric mean graph if and only if $n \le 3$.

Proof:

The number of vertices and edges of S_n are $n+1$ and n respectively. If f is an F-geometric mean labelling of S_n, then it is a bijective function from $V(S_n)$ to $\{1,2,3,\ldots,n+1\}$. As we have to label 1 for an edge, the vertex labels of its pair of adjacent vertices are either 1 and 2 or 1 and 3. So, the central vertex of S_n is labelled as either 1 or 2 or 3. 1 cannot be a label for the central vertex in the case of $n \geq 2$, since two of the pendant vertices of S_n are to be labelled as 2 and 3. When $n \geq 3$, 2 cannot be the label for the central vertex, since two of its pendant vertices having the labels 3 and 4. When $n \geq 4$, the pendant vertices are labelled to be 4 and 5 if the label of central vertex is 3.

The F-Geometric mean labelling of S_n, $n \leq 3$ are given in Figure 4.

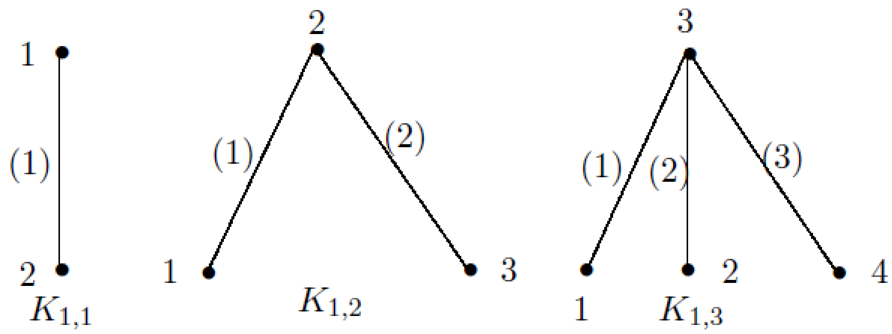

Figure 4. The F-Geometric mean labelling of S_n, $n \leq 3$ and its edge labelling

Theorem 2.6.

Every complete graph $K_n, n \geq 4$ is not an F-Geometric mean graph.

Proof:

To get the edge label q, q and q+1 should be the vertex labels for two of the vertices of K_n, say x and y. Also to obtain the edge label 1, 1 is to be a vertex label of a vertex of K_n, say v. Since $q = nC_2$ in K_n and $q+1 < (n-1)^2$ for $n \geq 4$, the edge labels of the edges vx and vy are one and the same. Hence K_n is not an F-Geometric mean graph. While $n = 2$ and 3, the F-geometric mean labelling of K_n are given in Figure 5.

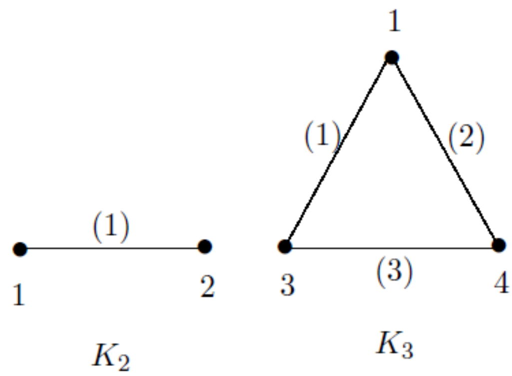

Figure 5. The F-Geometric mean labelling of K_2 and K_3 and its edge labelling

Theorem 2.7.

Every comb graph is an F-Geometric mean graph.

Proof:

Let $G = P_n o K_1$ be a comb graph for any positive integer n having $2n$ vertices and $2n-1$ edges. Let $u_1, u_2, ..., u_n$ be the vertices of the path P_n and v_i be the pendant vertices attached at each u_i, for $1 \le i \le n$. Then, the edge set of G is $\{u_i u_{i+1}; 1 \le i \le n-1\} \cup \{u_i v_i; 1 \le i \le n\}$.

We define $f : V(G) \to \{1,2,3,...,2n\}$ as follows:

$$f(u_i) = 2i, \text{ for } 1 \le i \le n \text{ and } f(v_i) = 2i - 1, \text{ for } 1 \le i \le n.$$

The induced edge labelling is as follows:

$$f^*(u_i u_{i+1}) = 2i, \text{ for } 1 \le i \le n-1 \text{ and } f^*(u_i v_i) = 2i - 1, \text{ for } 1 \le i \le n.$$

Hence, f is an F-Geometric mean labelling of the comb $P_n o K_1$. Thus, the comb $P_n o K_1$ is an F-Geometric mean graph for any positive integer n.

An F-Geometric mean labelling of $P_8 o K_1$ is shown in Figure 6.

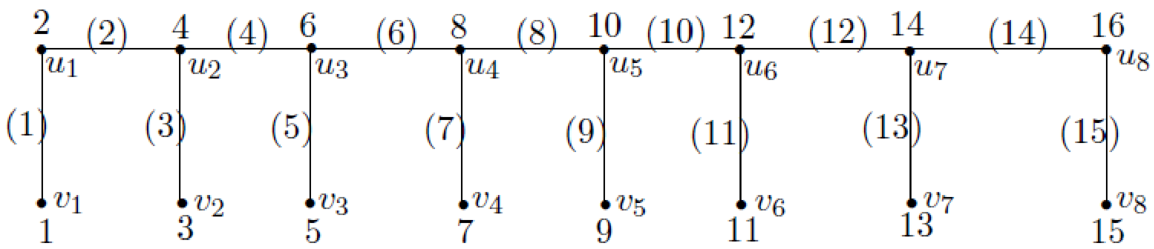

Figure 6. An F-Geometric mean labelling of $P_8 o K_1$ and its edge labelling

Theorem 2.8.

Every ladder graph is an F-Geometric mean graph.

Proof:

Let $G = P_2 \times P_n$ be a ladder graph for any positive integer n having $2n$ vertices and $3n-2$ edges. Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the vertices of G. Then the edge set of G is

$$\{u_i u_{i+1}, v_i v_{i+1}; 1 \le i \le n-1\} \cup \{u_i v_i; 1 \le i \le n\}.$$

We define $f : V(G) \to \{1,2,3,...,3n\text{-}1\}$ as follows:

$$f(u_i) = 3i - 1, \text{ for } 1 \le i \le n \text{ and } f(v_i) = 3i - 2, \text{ for } 1 \le i \le n.$$

The induced edge labelling is as follows:

$$f^*(u_iu_{i+1}) = 3i, \text{ for } 1 \le i \le n-1, \ f^*(v_iv_{i+1}) = 3i-1, \text{ for } 1 \le i \le n-1, \text{ and}$$
$$f^*(u_iv_i) = 3i-2, \text{ for } 1 \le i \le n.$$

Hence, *f* is an *F*-Geometric mean labelling of the ladder $P_2 \times P_n$. Thus, the ladder $P_2 \times P_n$ is an *F*-Geometric mean graph for any positive integer *n*.

An *F*-Geometric mean labelling of $P_2 \times P_6$ is shown in Figure 7.

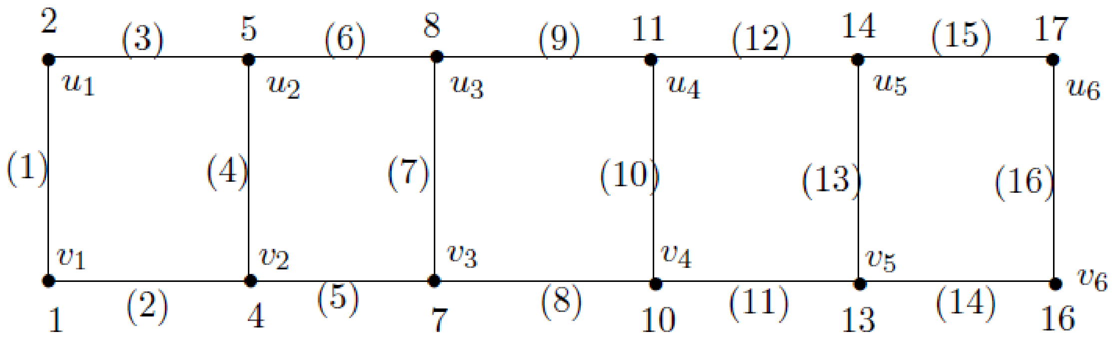

Figure 7. An *F*-Geometric mean labelling of $P_2 \times P_6$ and its edge labelling

Theorem 2.9.

The middle graph of a path is an *F*-Geometric mean graph.

Proof:

Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(P_n) = \{e_i = v_iv_{i+1}; 1 \le i \le n-1\}$ be the vertex set and edge set of the path P_n. Then,

$$V(M(P_n)) = \{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1}\} \text{ and}$$
$$E(M(P_n)) = \{v_ie_i, e_iv_{i+1}; 1 \le i \le n-1\} \cup \{e_ie_{i+1}; 1 \le i \le n-2\}.$$

We define $f : V(M(P_n)) \to \{1, 2, 3, \ldots, 3n-3\}$ as follows:

$$f(v_i) = 3i-2, \text{ for } 1 \le i \le n-1, \ f(v_n) = 3n-3 \text{ and } f(e_i) = 3i-1, \text{ for } 1 \le i \le n-1.$$

The induced edge labelling is as follows:

$$f^*(v_ie_i) = 3i-2, \text{ for } 1 \le i \le n-1, \ f^*(e_iv_{i+1}) = 3i-1, \text{ for } 1 \le i \le n-1$$
$$\text{and} \quad f^*(e_ie_{i+1}) = 3i, \text{ for } 1 \le i \le n-2.$$

Hence, f is an F-Geometric mean labelling of the middle graph $M(P_n)$. Thus, the middle graph $M(P_n)$ is an F-Geometric mean graph.

An F-Geometric mean labelling of $M(P_6)$ is shown in Figure 8.

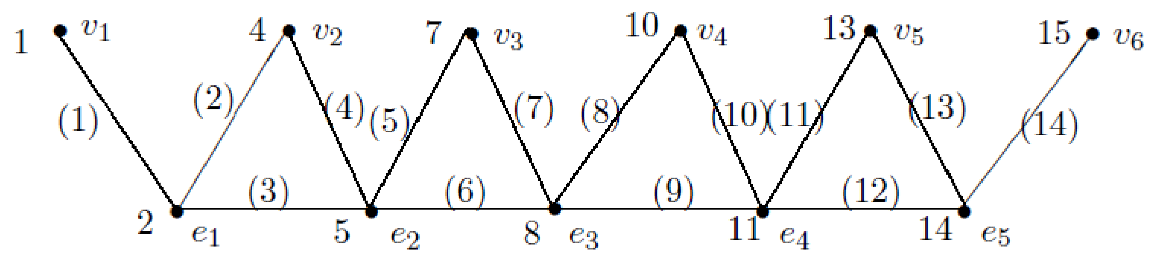

Figure 8. An F-Geometric mean labelling of $M(P_6)$ and its edge labelling

Theorem 2.10.

For any vertex v of the cycle C_n, the duplicate graph $D(C_n, v')$ is an F-Geometric mean graph, for $n \geq 3$.

Proof:
Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n and let $v = v_1$ and its duplicated vertex is v_1'.

Case (i). $n \geq 5$

We define $f : V(D(C_n, v')) \to \{1,2,3,...,n+3\}$ as follows:

$$f(v_1) = n-1,\ f(v_1') = n+1,\ f(v_2) = n+2,\ f(v_3) = n+3, \text{ and}$$
$$f(v_i) = \begin{cases} i-3, & 4 \leq i \leq \left\lfloor \sqrt{n+3} \right\rfloor + 2, \\ i-2, & \left\lfloor \sqrt{n+3} \right\rfloor + 3 \leq i \leq n. \end{cases}$$

The induced edge labelling is as follows:

$$f^*(v_1v_2) = n,\ f^*(v_1v_n) = n-2,\ f^*(v_1'v_2) = n+1,\ f^*(v_1'v_n) = n-1,\ f^*(v_2v_3) = n+2,$$
$$f^*(v_3v_4) = \left\lfloor \sqrt{n+3} \right\rfloor \text{ and } f^*(v_iv_{i+1}) = \begin{cases} i-3, & 4 \leq i \leq \left\lfloor \sqrt{n+3} \right\rfloor + 2, \\ i-2, & \left\lfloor \sqrt{n+3} \right\rfloor + 3 \leq i \leq n-1. \end{cases}$$

Hence, f is an F-Geometric mean labelling of the graph $D(C_n, v')$.

Case (ii). $n = 3,4$

The F-Geometric mean labelling of $D(C_3, v_1')$ and $D(C_4, v_1')$ are given in Figure 9.

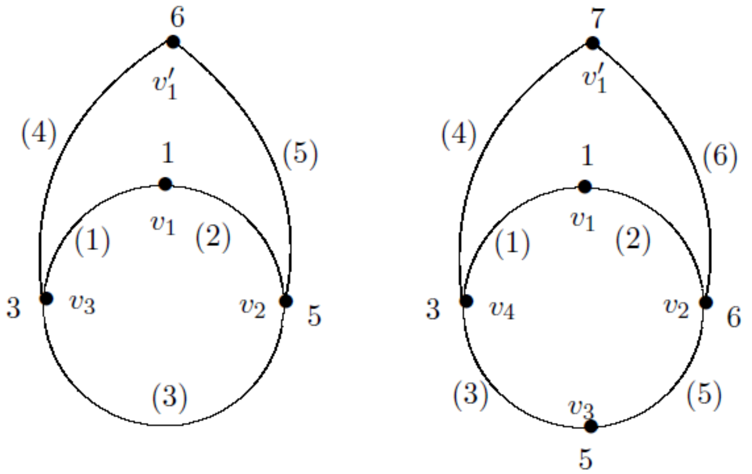

Figure 9. The *F*-Geometric mean labelling of $D(C_3, v_1')$ and $D(C_4, v_1')$ and its edge labelling

An *F*-geometric mean labelling of the graph G obtained by duplicating the vertex v_1 of the cycle C_8 is shown in Figure 10.

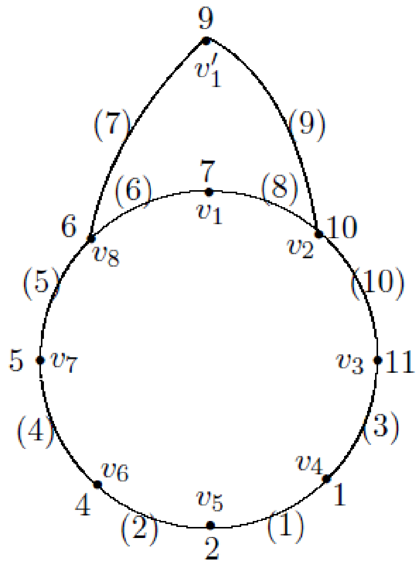

Figure 10. An *F*-Geometric mean labelling of $D(C_8, v_1')$ and its edge labelling

Theorem 2.11.

For any edge e of the cycle C_n, the duplicate graph $D(C_n, e')$ is an *F*-Geometric mean graph, for $n \geq 3$.

Proof:

Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n and let $e = v_1v_2$ and its duplicated edge is $e' = v_1'v_2'$.

Case (i). $n \geq 6$

We define $f : V(D(C_n, e')) \rightarrow \{1,2,3,\ldots,n+4\}$ as follows:

$$f(v_1) = n-1,\ f(v_1') = n+1,\ f(v_2) = n+2,\ f(v_2') = n+3,$$
$$f(v_3) = n+4 \ \text{ and } \ f(v_i) = \begin{cases} i-3, & 4 \leq i \leq \left\lfloor \sqrt{n+4} \right\rfloor + 2, \\ i-2, & \left\lfloor \sqrt{n+4} \right\rfloor + 3 \leq i \leq n. \end{cases}$$

The induced edge labelling is as follows:

$$f^*(v_1v_2) = n,\ f^*(v_1v_n) = n-2,\ f^*(v_1'v_n) = n-1,\ f^*(v_1'v_2') = n+1,$$
$$f^*(v_2'v_3) = n+3,\ f^*(v_2v_3) = n+2,\ f^*(v_3v_4) = \left\lfloor \sqrt{n+4} \right\rfloor$$
$$\text{and } \ f^*(v_iv_{i+1}) = \begin{cases} i-3, & 4 \leq i \leq \left\lfloor \sqrt{n+4} \right\rfloor + 2, \\ i-2, & \left\lfloor \sqrt{n+4} \right\rfloor + 3 \leq i \leq n-1. \end{cases}$$

Hence, f is an F-Geometric mean labelling of the graph $D(C_n, e')$.

Case (ii). $n = 3,4,5$

The F-Geometric mean labelling of $D(C_3, v_1'v_2')$, $D(C_4, v_1'v_2')$ and $D(C_5, v_1'v_2')$ are given in Figure 11.

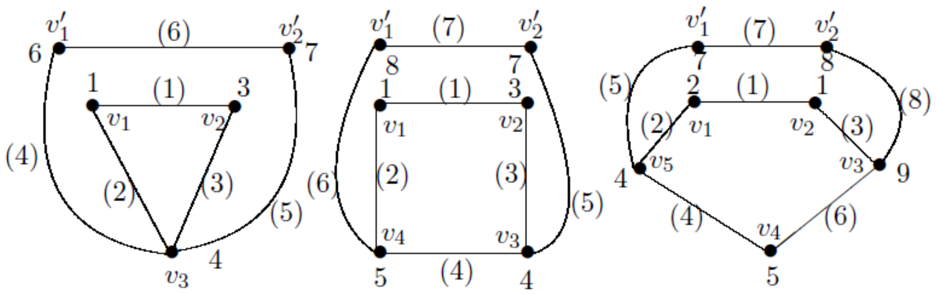

Figure 11. An F-Geometric mean labelling of $D(C_3, v_1'v_2')$, $D(C_4, v_1'v_2')$ and $D(C_5, v_1'v_2')$ and its edge labelling

An F-geometric mean labelling of the graph G obtained by duplicating an edge v_1v_2 of the cycle C_9 is shown in Figure 12.

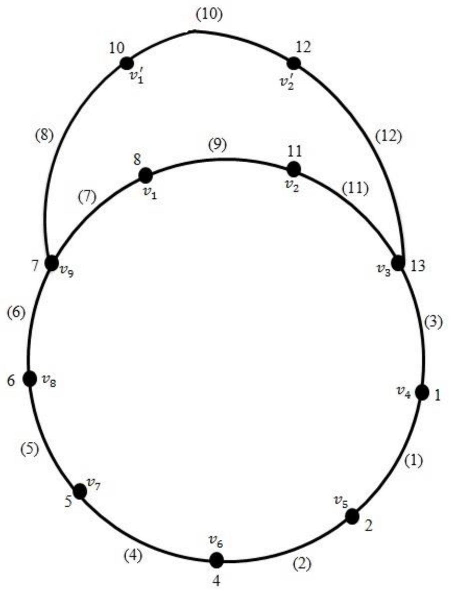

Figure 12. An *F*-Geometric mean labelling of $D\left(C_9, v_1'v_2'\right)$ and its edge labelling

Theorem 2.12.

The triangular ladder TL_n is an *F*-Geometric mean graph, for $n \geq 2$.

Proof:

Let $\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$ be the vertex set of TL_n and let $\{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1};\ 1 \leq i \leq n-1\} \cup \{u_i v_i;\ 1 \leq i \leq n\}$ be the edge set of TL_n. Then TL_n have $2n$ vertices and $4n-3$ edges.

We define $f: V(TL_n) \rightarrow \{1,2,3,...,4n-2\}$ as follows:

$$f(u_i) = 4i-1, \text{ for } 1 \leq i \leq n-1,\ f(u_n) = 4n-2 \text{ and } f(v_i) = 4i-3, \text{ for } 1 \leq i \leq n.$$

The induced edge labelling is as follows:

$$f^*(u_i u_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-1,\ f^*(u_i v_i) = 4i-3, \text{ for } 1 \leq i \leq n,$$
$$f^*(u_i v_{i+1}) = 4i-1, \text{ for } 1 \leq i \leq n-1 \text{ and } f^*(v_i v_{i+1}) = 4i-2, \text{ for } 1 \leq i \leq n-1.$$

Hence, *f* is an *F*-Geometric mean labelling of the TL_n. Thus the triangular ladder TL_n is an *F*-Geometric mean graph, for $n \geq 2$.

An *F*-Geometric mean labelling of TL_8 is shown in Figure 13.

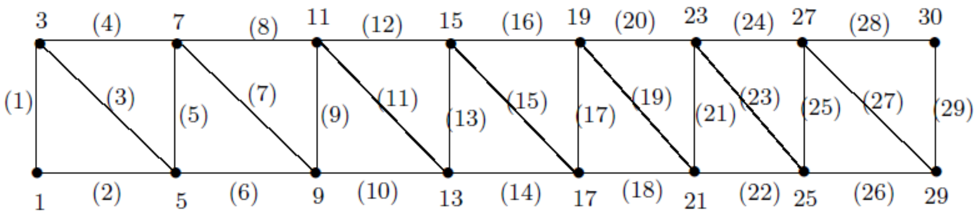

Figure 13. An F-Geometric mean labelling of TL_8 and its edge labelling

Theorem 2.13.

$S(P_n o K_1)$ is an F-Geometric mean graph, for $n \geq 2$.

Proof:

Let $V(P_n o K_1) = \{u_i, v_i \,; 1 \leq i \leq n\}$ and $E(P_n o K_1) = \{u_i u_{i+1}; 1 \leq i \leq n-1\} \cup \{u_i v_i \,; 1 \leq i \leq n\}$. Let x_i be the vertex which divides the edge $u_i v_i$, for $1 \leq i \leq n$ and y_i be the vertex which divides the edge $u_i u_{i+1}$, for $1 \leq i \leq n-1$. Then $V(S(P_n o K_1)) = \{u_i, v_i, x_i, y_j \,; 1 \leq i \leq n, 1 \leq j \leq n-1\}$ and $E(S(P_n o K_1)) = \{u_i x_i, v_i x_i \,; 1 \leq i \leq n\} \cup \{u_i y_i, y_i u_{i+1}; 1 \leq i \leq n-1\}$.

We define $f: V(S(P_n o K_1)) \to \{1,2,3 \ldots, 4n-1\}$ as follows:

$$f(u_i) = 4i-1, \text{ for } 1 \leq i \leq n, \; f(y_i) = 4i+1, \text{ for } 1 \leq i \leq n-1,$$

$$f(x_i) = 4i-2, \text{ for } 1 \leq i \leq n \quad \text{and} \quad f(v_i) = \begin{cases} 1, & i = 1, \\ 4i-4, & 2 \leq i \leq n. \end{cases}$$

The induced edge labelling is as follows:

$$f^*(u_i y_i) = 4i-1, \text{ for } 1 \leq i \leq n-1, \; f^*(y_i u_{i+1}) = 4i+1, \text{ for } 1 \leq i \leq n-1,$$

$$f^*(u_i x_i) = 4i-2, \text{ for } 1 \leq i \leq n \text{ and } f^*(v_i x_i) = \begin{cases} 1, & i = 1, \\ 4i-4, & 2 \leq i \leq n. \end{cases}$$

Hence, f is an F-Geometric mean labelling of $S(P_n o K_1)$. Thus, the graph $S(P_n o K_1)$ is an F-Geometric mean graph, for $n \geq 2$.

An F-Geometric mean labelling of $S(P_5 o K_1)$ is shown in Figure 14.

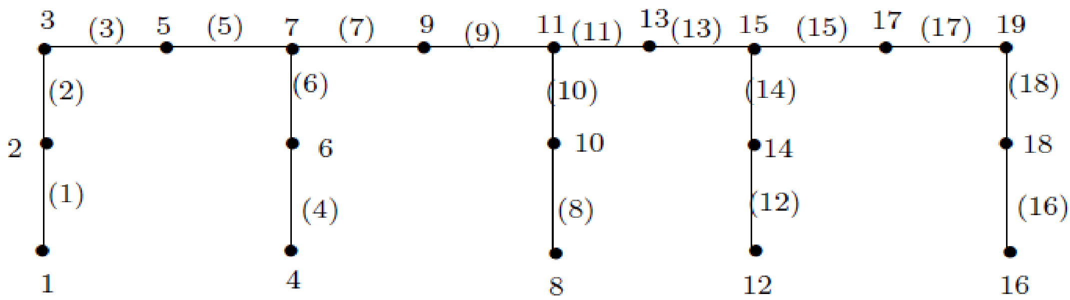

Figure 14. An F-Geometric mean labelling of $S(P_5 o K_1)$ and its edge labelling

Theorem 2.14.

Any arbitrary subdivision of S_3 is an *F*-Geometric mean graph.

Proof:

Let v_0, v_1, v_2, v_3 be the vertices of S_3 in which v_0 is the central vertex and v_1, v_2 and v_3 are the pendant vertices of S_3. Let the edges v_0v_1, v_0v_2 and v_0v_3 of S_3 be subdivided by p_1, p_2 and p_3 number of new vertices respectively. Let *G* be a graph of arbitrary subdivision of S_3.

Let $v_0, v_1^{(1)}, v_2^{(1)}, v_3^{(1)},..., v_{p_1}^{(1)}+1(=v_1)$, $v_0, v_1^{(2)}, v_2^{(2)}, v_3^{(2)},..., v_{p_2}^{(2)}+1(=v_2)$ and $v_0, v_1^{(3)}, v_2^{(3)}, v_3^{(3)},..., v_{p_3}^{(3)}+1(=v_3)$ be the vertices of *G* and $v_0 = v_0^{(i)}$, for $1 \le i \le 3$.

Let $e_j^{(i)} = v_{j-1}^{(i)} v_j^{(i)}$, $1 \le j \le p_i + 1$ and $1 \le i \le 3$ be the edges of *G* and *G* has $p_1 + p_2 + p_3 + 4$ vertices and $p_1 + p_2 + p_3 + 3$ edges with $p_1 \le p_2 \le p_3$.

We define $f : V(G) \to \{1,2,3,..., p_1 + p_2 + p_3 + 4\}$ as follows:

$$f(v_0) = p_1 + p_2 + 3,\ f\left(v_i^{(1)}\right) = p_1 + p_2 + 4 - 2i, \text{ for } 1 \le i \le p_1 + 1,$$

$$f\left(v_i^{(2)}\right) = \begin{cases} p_1 + p_2 + 3 - 2i, & 1 \le i \le p_1 + 1, \\ p_2 + 2 - i, & p_1 + 2 \le i \le p_2 + 1 \text{ and } p_1 \ne p_2 \end{cases}$$

$$\text{and } f\left(v_i^{(3)}\right) = p_1 + p_2 + 3 + i, \text{ for } 1 \le i \le p_3 + 1.$$

The induced edge labelling is as follows:

$$f^*(v_i^{(1)} v_{i+1}^{(1)}) = p_1 + p_2 + 2 - 2i, \text{ for } 1 \le i \le p_1,$$

$$f^*\left(v_i^{(2)} v_{i+1}^{(2)}\right) = \begin{cases} p_1 + p_2 + 1 - 2i, & 1 \le i \le p_1, \\ p_2 + 1 - i, & p_1 + 1 \le i \le p_2 \text{ and } p_1 \ne p_2 \end{cases},$$

$$f^*(v_i^{(3)} v_{i+1}^{(3)}) = p_1 + p_2 + 3 + i, \text{ for } 1 \le i \le p_3,\ f^*(v_0 v_1^{(1)}) = p_1 + p_2 + 2,$$

$$f^*(v_0 v_2^{(1)}) = p_1 + p_2 + 1 \text{ and } f^*(v_0 v_3^{(1)}) = p_1 + p_2 + 3.$$

Hence, *f* is an *F*-Geometric mean labelling of *G*. Thus, the arbitrary subdivision of S_3 is an *F*-Geometric mean graph.

An *F*-Geometric mean labelling of *G* with $p_1 = 6, p_2 = 9$ and $p_3 = 10$ is as shown in Figure 15.

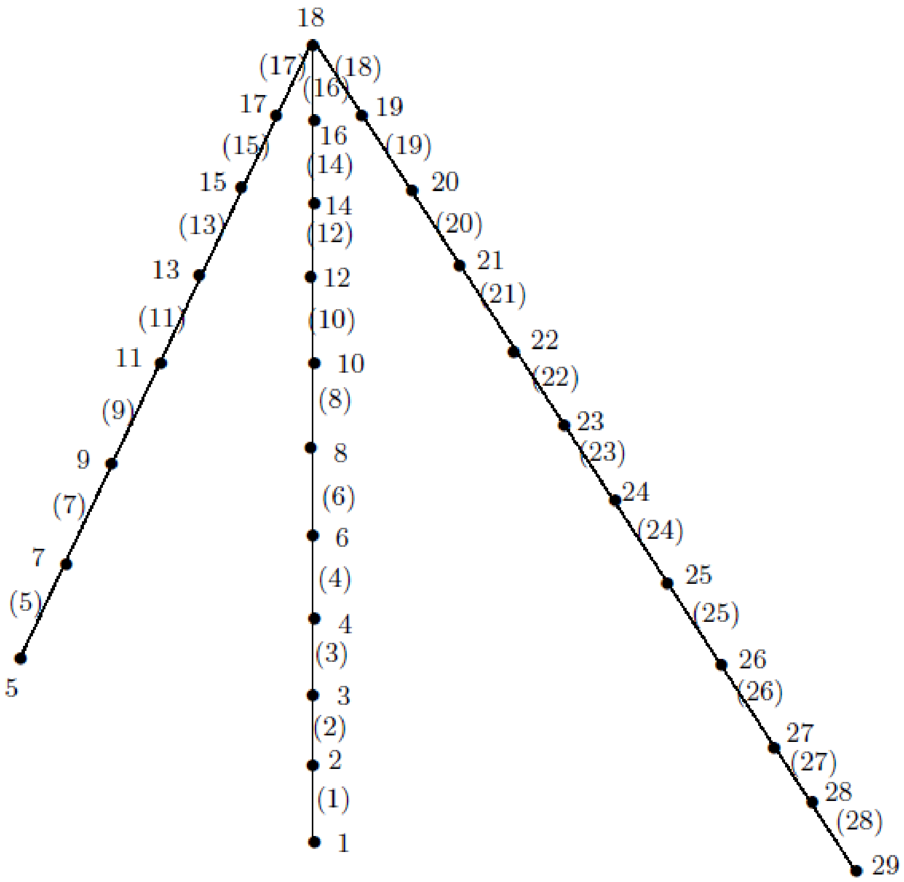

Figure 15. An *F*-Geometric mean labelling of arbitrary subdivision of S_3 and its edge labelling

3. Conclusion

In this paper, we analysed the F-Geometric meanness of some standard graphs. We propose the following open problems to the readers for further research work.

Open Problem 1.

Find a sub graph of a graph in which the graph is not an *F*-Geometric mean graph.

Open Problem 2.

Find a necessary condition for a graph to be an *F*-Geometric mean graph.

By Theorem 2.6, we observe that $G + e$ is not necessarily an *F*-geometric mean graph when G is an *F*-geometric mean graph and e is an additional edge. Also from Theorem 2.2, $G - e$ is not necessarily an *F*-geometric mean graph when e is a cut edge and G is an *F*-geometric mean graph. So it is possible to discuss the remaining case.

Open Problem 3.

For a non-cut edge e, characterize the F-geometric mean graph G in which G - e is also an F-geometric mean graph.

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