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Exact implicit Solution of Nonlinear Heat Transfer in Rectangular Straight Fin Using Symmetry Reduction Methods

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Abstract

In this paper, the exact implicit solution of the second order nonlinear ordinary differential equation which governing heat transfer in rectangular fin is obtained using symmetry reduction methods. General relationship among the temperature at the fin tip, the temperature gradient at the fin base, the mode of heat transfer, n and the fin parameters N and $\mathcal{E}$ is obtained. Some numerical examples are discussed and it is shown that the temperature of fin increases when approaching from the heat source. The relationship between the fin efficiency and the temperature of fin tip is obtained for any value of the mode of heat transfer n. The relationship between the fin efficiency and both the parameter N and the temperature gradient at the fin base is obtained. To our best knowledge, solutions obtained in this paper are new.

Keywords: Rectangular fin; Heat transfer; Lambda λ-symmetry; Lie point symmetry

MSC 2010 No.: 80A20, 35D99, 35Q79

1. Introduction

Fins are used to enhance convective heat transfer in a wide range of engineering applications, and they offer practical means for achieving a large total heat transfer surface area without the use of an excessive amount of primary surface area. Fins are commonly applied for heat management in electrical appliances such as computer power supplies or substation transformers. Other applications include internal combustion engine cooling, such as fins in a car radiator. It is important to predict the temperature distribution within the fin in order to choose the configuration that offers maximum effectiveness.

In this paper, we will obtain the general exact solution of the of nonlinear heat transfer in rectangular straight fin using λ-symmetry and Lie point symmetry methods.

The following sections will be organized as follows: In section 2, the fin problem description is introduced. In section 3, λ-symmetry method will be used to reduce the order of fin equation, and then we will show the relation between the temperature at the fin tip u_0 and the temperature gradient at the fin base $u'(1)$. In section 4, the reduced ODE will be solved using Lie point symmetry method, then we will get the exact solution of fin equation. Also, we will give some numerical examples and some physical interpretations. In section 5, the fin efficiency will be discussed.

2. Problem Description

In this paper, we consider steady operation with no heat generation, and the fin tip is insulated, for one dimensional case, the energy balance equation is given by Y. Cengel and A. Ghajar (2011)

$$\frac{d}{d\mathcal{X}} A_c \left(K(T) \frac{dT}{d\mathcal{X}} \right) - Ph(T - T_a) = 0, \tag{2.1}$$

where A_c is the cross-sectional area of the fin, $\mathcal{X}$ is the axial distance measured from the fin tip, $k(T)$ is the thermal conductivity of the fin, T is the fin temperature, P is the fin perimeter, h is the heat transfer coefficient and T_a is the ambient temperature .

Momoniat (2011), Khani et al. (2009), Harley (2013) and Arka Bhowmik et al. (2013) imposed that the cross-sectional area of the fin A_c is fixed and take the form of a rectangle with length b and width w which can be neglected (Lau et al., 1973). The thermal conductivity may be considered as a linear relation in the temperature as follows (Kim and Huang, 2007)

$$K(T) = K_a\big(1 + \tau(T - T_a)\big),$$

and the nonlinear heat transfer coefficient is given by

$$h = h_b \left(\frac{T - T_a}{T_b - T_a} \right)^n,$$

where, h_b is the heat transfer coefficient at the base of temperature, K_a is the thermal conductivity of the fin at the ambient temperature T_a , τ is a constant, T_b is the temperature of the heat source which relate fin and the constant n indicates the mode of heat transfer .

Here, the exponent represents transition boiling when $n = -4$, laminar film boiling or condensation when $n = -1/4$, laminar natural convection when $n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when n = 2, radiation when n = 3, and n vanishes for constant heat transfer coefficient. The constant n may vary between −6.6 and 5 (Momoniat, 2011; Arka Bhowmik et al., 2013; Min-Hsing Chang, 2005; Unal, 1998).

After taking the previous assumptions into account, Equation (2.1) becomes

$$A_c K_a \frac{d}{d\mathcal{X}}\left(\left(1+\tau(T-T_a)\right)\frac{dT}{d\mathcal{X}}\right) - P h_b \frac{(T-T_a)^{n+1}}{(T_b-T_a)^n} = 0. \tag{2.2}$$

Equation (2.2) can be made non-dimensional by the set of the transformations (Momoniat, 2011)

$$u = \frac{T-T_a}{T_b-T_a}, \quad k = \frac{K}{K_a}, \quad x = \frac{\mathcal{X}}{b}, \quad \mathcal{E} = \tau(T_b-T_a), \quad N^2 = \frac{P h_b b^2}{K_a A_c}, \tag{2.3}$$

by substituting Equation (2.3) into Equation (2.2), to obtain

$$\left(1+\mathcal{E}\,u(x)\right)u''(x) + \mathcal{E}u'(x)^2 - N^2 u(x)^{n+1} = 0\,. \tag{2.4}$$

The boundary conditions are given by (Momoniat, 2011; Khani et al., 2009; Harley, 2013; Arka Bhowmik et al., 2013).

1. At the fin tip $(\mathcal{X} = 0)$. Because the fin is insulated, the change of temperature is

$$\frac{dT}{d\mathcal{X}} = 0. \tag{2.5}$$

From Equation (2.3), Equation (2.5) becomes

$$\frac{du}{dx} = 0 \qquad \text{or} \qquad u'(0) = 0\,.$$

2. At the fin base $(\mathcal{X} = b)$. The fin temperature is the same temperature as the heat source T_b

$$T(b) = T_b\,. \tag{2.6}$$

From Equation (2.3), Equation (2.6) becomes

$$u(1) = 1.$$

Here, the boundary conditions are

$$u(1) = 1, \qquad u'(0) = 0, \tag{2.7}$$

where, $' = \frac{d}{dx}$.

Approximate solutions of Equation (2.4) are investigated using homotopy analysis method in Khani et al. (2009), by asymptotic analysis method in Harley (2013) and by decomposition and evolutionary methods in Arka Bhowmik et al. (2013). For the special case when $n = 0$, series solutions of Equation (2.4) are investigated using Adomian decomposition method in Huang Chiu et al. (2002). For special case when $\mathcal{E} = 0$, series solutions of Equation (2.4) are investigated using homotopy asymptotic method in Haq and Ishaq (2012) and Adomian decomposition method in Min-Hsing Chang (2005). Exact analytical solution of Equation (2.4) when $\mathcal{E} = 0$ is obtained in Abbasbandy and Shivanian (2010). In this paper, we will obtain the exact solution of Equation (2.4) using symmetry reduction methods for all heat transfer modes n.

3. λ-Symmetry Method

In 2001, Muriel and Romero (2001; 2009) were able to show that many of the known order-reduction processes of ODEs can be explained by the invariance of the equation under some special vector fields that are not Lie point symmetries, but satisfy a new prolongation formula. The components of these vector fields must satisfy a system of determining equations that depends on an arbitrary function λ, which can be chosen to solve the system easily. In fact, if an equation is invariant under a λ-symmetry, one can obtain a complete set of functionally independent invariants and reduce the order of the equation by one as in Lie symmetries.

Soon afterwards, Pucci and Saccomandi have clarified the meaning of λ-prolongation by means of classical theory of characteristics of vector fields (Pucci and Saccomandi, 2002). Several applications of the λ-symmetry approach to relevant equations of the mathematical physics appear in Abdel Kader et al. (2013) and Bhuvaneswari et al. (2011; 2012).

Following Muriel and Romero (2001; 2009), the prolongation formula

$$X^{[\lambda,(2)]} = \xi\partial_x + \eta\partial_u + \eta^{[\lambda,(1)]}\partial_{u'} + \eta^{[\lambda,(2)]}\partial_{u''},$$

$$\eta^{[\lambda,(1)]} = D_x\eta - D_x\xi u' + \lambda(\eta - \xi u'),$$

$$\eta^{[\lambda,(2)]} = D_x\eta^{[\lambda,(1)]} - D_x\xi u'' + \lambda\left(\eta^{[\lambda,(1)]} - \xi u''\right),$$

is applied to Equation (2.4), where D_x denotes the total derivative operator with respect to x. After solving the obtained determining equations we obtained a vector field $X = \partial_u$ which is a λ-symmetry of Equation (2.4) with

$$\lambda = \frac{N^2 u^{1+n} - \mathcal{E}(u')^2}{(1 + \mathcal{E}u)u'}.$$

The first integral of $X^{[\lambda,(1)]}$can be obtained by solving the following equation

$$w_u + \lambda\, w_{u'} = 0.$$

This equation admits a solution in the form

$$w = G\left(x, \frac{-2N^2u^{2+n}(3 + n + 2\mathcal{E}u + \mathcal{E}nu) + (6 + 5n + n^2)(u' + \mathcal{E}uu')^2}{6 + 5n + n^2}\right).$$

Let,

$$Z = \frac{-2N^2u^{2+n}(3 + n + 2\mathcal{E}u + \mathcal{E}nu) + (6 + 5n + n^2)(u' + \mathcal{E}uu')^2}{6 + 5n + n^2}. \quad (3.1)$$

Hence,

$$Z' = 2(1 + \mathcal{E}\, u)u'\left((1 + \mathcal{E}u)u'' + \mathcal{E}u'^2 - N^2u^{1+n}\right). \quad (3.1a)$$

Substituting (2.4) into (3.1a), we obtain

$$Z' = 0. \quad (3.2)$$

Equation (3.2) has a solution $Z = c_1$. From (3.1), we obtain

$$\frac{-2N^2u^{2+n}(3 + n + 2\mathcal{E}u + \mathcal{E}nu) + (6 + 5n + n^2)(u' + \mathcal{E}uu')^2}{6 + 5n + n^2} = c_1, \quad (3.3)$$

where, c_1 is a constant. From (3.3) and (2.5), when $x = 0$, we obtain

$$c_1 = -\frac{2N^2{u_0}^{2+n}(3 + n + \mathcal{E}(2 + n)u_0)}{(6 + 5n + n^2)}, \quad (3.4)$$

where, $u_0 = u(0)$ represents the value of temperature at the fin tip ($x = 0$).

Substituting (3.4) into (3.3), we obtain

$$\begin{aligned}-2N^2u^{2+n}(3 + n + 2\mathcal{E}u + \mathcal{E}nu) + (6 + 5n + n^2)(u' + \mathcal{E}uu')^2 \\ = -2N^2{u_0}^{2+n}(3 + n + \mathcal{E}(2 + n)u_0).\end{aligned} \quad (3.5)$$

By using the boundary condition $u(1) = 1$, Equation (3.5) becomes

$$u'(1) = \frac{1}{(1 + \mathcal{E})}\sqrt{\left(-\frac{2N^2{u_0}^{2+n}(3 + n + (2 + n)\mathcal{E}u_0)}{(2 + n)(3 + n)} + \frac{2N^2(3 + n + (2 + n)\mathcal{E})}{(2 + n)(3 + n)}\right)}. \quad (3.6)$$

Equation (3.6) gives a general relationship among the temperature at the fin tip u_0, the temperature gradient at the base $u'(1)$, the mode of heat transfer n and the fin parameters N and $\mathcal{E}$.

3.1. Numerical Example

The following example shows the relationship between u_0 and $u'(1)$ for different values of heat transfer mode n at a special case when $N = 2$ and $\mathcal{E} = 0.5$ (Momoniat et al. (2009)).

Example 3.1.1: ($N = 2$ and $\mathcal{E} = 0.5$ **)**

Equation (3.6) takes the form

$$u'(1) = \frac{4}{3}\sqrt{\frac{8 + 3n - 2(3+n)u_0^{2+n} - (2+n)u_0^{3+n}}{(2+n)(3+n)}} \, . \tag{3.7}$$

Figure (1) shows that the temperature at the fin tip u_0 decreases to zero when the magnitude of the temperature gradient at the base $u'(1)$ increases and the rate of decay to zero decreases with increasing $u'(1)$ when the value of n decreases.

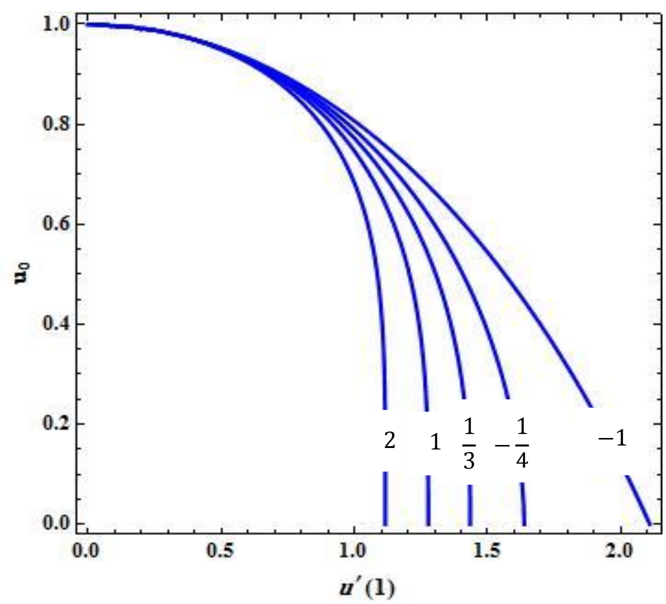

Figure 1. Plot of the relation (3.7) between $u'(1)$ and u_0 with $N = 2$ and $\mathcal{E} = 0.5$ at various values of n.

Equation (3.6) is general for any n. Momoniat et al. (2009) obtained Equation (3.6) at $n = 0$ only. In the next section, Equation (3.5) will be solved using Lie point symmetry method.

4. Lie Point Symmetry Method

It is known that the autonomous ODE (3.5) admits the Lie point symmetry generator (Bluman and Kumei, 1989 ; Hydon , 2000)

$$X = \partial_x , \tag{4.1}$$

The canonical coordinates in this case are given by

$$v(r) = x , \quad r = u(x),$$

which prolong to

$$\dot{v} = \frac{1}{u'(x)} .$$

Hence, Equation (3.5) reduces to

$$\begin{aligned} &-2N^2 r^{2+n}(3+n+\mathcal{E}(2+n)r) + (6+5n+n^2)(1+\mathcal{E}r)^2 \frac{1}{\dot{v}^2} \\ &= -2N^2 {u_0}^{2+n}(3+n+\mathcal{E}(2+n)u_0), \end{aligned} \tag{4.2}$$

where, $\dot{v} = \frac{dv}{dr}$.

By introducing the transformation $\dot{v}^2 = y$, we obtain the following equation

$$y = \frac{(1+\mathcal{E}r)^2}{2N^2 r^{2+n}\left(\frac{1}{2+n} + \frac{\mathcal{E}r}{3+n}\right) - 2N^2 {u_0}^{2+n}\left(\frac{1}{2+n} + \frac{\mathcal{E}u_0}{3+n}\right)} . \tag{4.3}$$

Considering Equation (4.3), we obtain the following exact implicit solution for Equation (3.5)

$$v = x = \int_{u_0}^{u} \frac{\sqrt{(2+n)(3+n)}(1+\mathcal{E}r)}{\sqrt{2N^2(-{u_0}^{2+n}(3+n+(2+n)\mathcal{E}u_0) + r^{2+n}(3+n+(2+n)\mathcal{E}r))}} dr. \tag{4.4}$$

This solution has an unknown parameter, namely u_0. This parameter can be easily obtained with the help of boundary condition $u(1) = 1$ as follows:

$$1 = \int_{u_0}^{1} \frac{\sqrt{(2+n)(3+n)}(1+\mathcal{E}r)}{\sqrt{2N^2(-{u_0}^{2+n}(3+n+(2+n)\mathcal{E}u_0) + r^{2+n}(3+n+(2+n)\mathcal{E}r))}} dr. \tag{4.5}$$

Equation (4.5) shows the relation between the temperature at fin tip u_0 and the thermo-geometric parameter N.

4.1. Numerical Example

Example 4.1.1:

In this example, we will consider the case $n = -4$ (Haq and Ishaq, 2012; Abbasbandy and Shivanian, 2010; Liaw and Yeh, 1994).

Equation (4.4) and Equation (4.5) become

$$x = \frac{3\mathcal{E}\text{Log}\left(u + 2u\mathcal{E}u_0 - \mathcal{E}u_0^2 + \sqrt{u-u_0}\sqrt{1+2\mathcal{E}u_0}\sqrt{u+(1+2u\mathcal{E})u_0}\right)u_0^3(1+\mathcal{E}u_0)^2}{2N(1+2\mathcal{E}u_0)^{5/2}} + \frac{u_0\sqrt{u-u_0}\sqrt{u+(1+2u\mathcal{E})u_0}\,(2+u\mathcal{E}+2\mathcal{E}(2+u\mathcal{E})u_0+3\mathcal{E}^2u_0^2)}{2N(1+2\mathcal{E}u_0)^2} - \frac{3\mathcal{E}\text{Log}\left(u_0(1+\mathcal{E}u_0)\right)u_0^3(1+\mathcal{E}u_0)^2}{2N(1+2\mathcal{E}u_0)^{5/2}}, \tag{4.6}$$

$$N = \frac{3\mathcal{E}\text{Log}\left(1 + 2\mathcal{E}u_0 - \mathcal{E}u_0^2 + \sqrt{1-u_0}\sqrt{1+2\mathcal{E}u_0}\sqrt{1+(1+2\mathcal{E})u_0}\right)u_0^3(1+\mathcal{E}u_0)^2}{2(1+2\mathcal{E}u_0)^{5/2}} + \frac{u_0\sqrt{1-u_0}\sqrt{1+(1+2\mathcal{E})u_0}\,(2+\mathcal{E}+2\mathcal{E}(2+\mathcal{E})u_0+3\mathcal{E}^2u_0^2)}{2(1+2\mathcal{E}u_0)^2} - \frac{3\mathcal{E}\text{Log}\left(u_0(1+\mathcal{E}u_0)\right)u_0^3(1+\mathcal{E}u_0)^2}{2(1+2\mathcal{E}u_0)^{5/2}}. \tag{4.7}$$

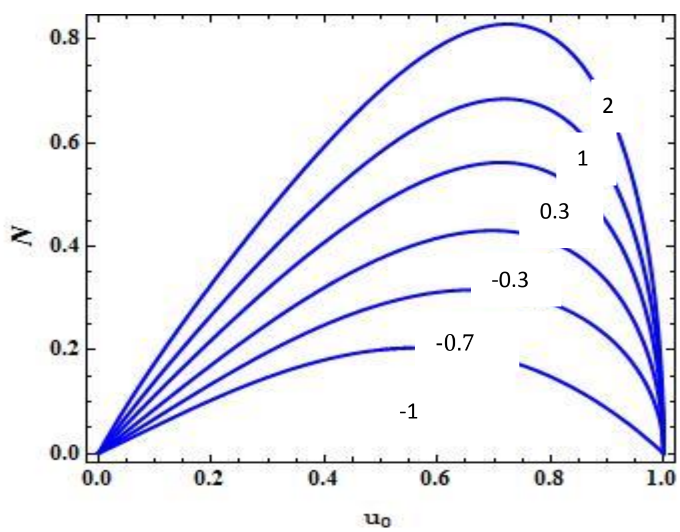

Figure 2. Plot of the relation (4.7) between N and u_0 for various values of $\mathcal{E}$ and $n = 4$.

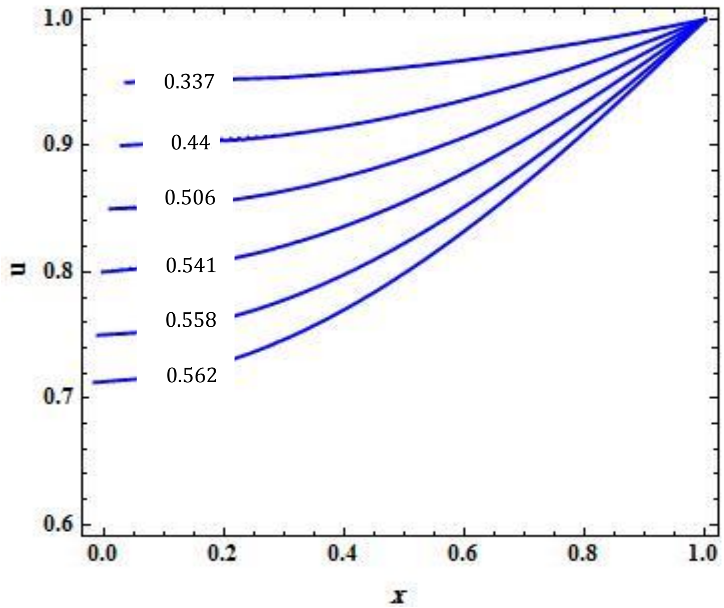

Figure 3. Plot of the relation (4.6) between u and x with $n = -4$ and $\mathcal{E} = 0.3$ at various values of N.

In Figure (2), the relation between the temperature at the fin tip u_0 and the parameter N has been plotted for various values of $\mathcal{E}$ when $n = -4$. Each curve with various values of $\mathcal{E}$ has a peak value. The branch on the right-hand side of the peak corresponds to the physically stable and realizable states (Min-Hsing Chang, 2005). In the branch on the right-hand side of the peak, the temperature of fin tip u_0 increases when the parameter N decreases.

Figure (3) shows that the temperature u increases with increasing x (in other words, the temperature increases when approaching from heat source).

Remark 1:

When $\mathcal{E} = 0$, $n \neq -2$ (Min-Hsing Chang, 2005; Haq and Ishaq, 2012; Abbasbandy and Shivanian, 2010)**,** Equation (4.4) and Equation (4.5) become

$$x = -\frac{\sqrt{2n+4}}{nN} u^{-n/2} \,{}_2F_1\left(\frac{1}{2}, \frac{n}{2n+4}; \frac{3}{2} - \frac{1}{n+2}; \left(\frac{u}{u_0}\right)^{-n-2}\right) - \frac{1}{N}\sqrt{\pi}\sqrt{\frac{n}{2}+1}\; u_0^{-n/2} \frac{\Gamma\left(\frac{n}{2n+4}\right)}{\Gamma\left(-\frac{1}{n+2}\right)}, \tag{4.8}$$

$$N = -\frac{\sqrt{2n+4}}{n} \,{}_2F_1\left(\frac{1}{2}, \frac{n}{4+2n}, \frac{3}{2} - \frac{1}{2+n}, u_0^{2+n}\right) - \sqrt{\pi}\sqrt{\frac{n}{2}+1}\; u_0^{-n/2} \frac{\Gamma\left(\frac{n}{4+2n}\right)}{\Gamma\left(-\frac{1}{2+n}\right)}, \tag{4.9}$$

where, ${}_2F_1$ is the hypergeometric function (Frank et al., 2010), which can be defined as

$$ {}_2F_1(a,b,c,x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-t\,x)^{-a} dt. $$

Equations (4.8)-(4.9) are exactly the same as the solution given by (Abbasbandy and Shivanian, 2010).

Remark 2:

When $\mathcal{E} = 0$, $n = -2$ (Abbasbandy and Shivanian, 2010)**,** Equation (4.4) and Equation (4.5) become

$$ x = \frac{1}{N}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{\mathrm{Log}\left(\frac{u}{u_0}\right)}\right) u_0 \ , \tag{4.10} $$

$$ N = \sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{\mathrm{Log}\left(\frac{1}{u_0}\right)}\right) u_0 \ , \tag{4.11} $$

where, $\mathrm{Erfi}(z) = \int_0^Z e^{-t^2}\, dt$, denotes the Error function.

Equations (4.10)-(4.11) are exactly the same as the solution given by (Abbasbandy and Shivanian, 2010).

5. Fin Efficiency

The fin efficiency can be stated as the ratio of actual heat transfer rate from the fin to ideal heat transfer rate from the fin if the entire fin were at base temperature (Y. Cengel and A. Ghajar, 2011). This leads to (Kim and Huang , 2007)

$$ \eta_{fin} = \frac{K_b A_c \frac{dT}{d\mathcal{X}} |_{\mathcal{X}=b}}{p h_b (T_b - T_a) b}, \tag{5.1} $$

where, K_b is the thermal conductivity of the fin at fin base temperature T_b.

Using Equation (2.3), Equation (5.1) becomes (Momoniat, 2011)

$$ \eta_{fin} = \frac{1+\varepsilon}{N^2}\ u'(1), \qquad N \neq 0\,. \tag{5.2} $$

From Equation (4.5) we obtain

$$N=\int_{u_0}^{1}\frac{1+\mathcal{E}r}{\sqrt{2r^{2+n}\left(\frac{1}{2+n}+\frac{\mathcal{E}r}{3+n}\right)-2u_0{}^{2+n}\left(\frac{1}{2+n}+\frac{\mathcal{E}u_0}{3+n}\right)}}dr\,. \tag{5.3}$$

Substituting Equation (3.6) and Equation (5.3) into Equation (5.2) we obtain

$$\eta_{fin}=\frac{2\sqrt{3+n+(2+n)\mathcal{E}-u_0^{2+n}(3+n+(2+n)\mathcal{E}u_0)}}{(2+n)(3+n)\displaystyle\int_{u_0}^{1}\frac{1+r\mathcal{E}}{\sqrt{r^{2+n}(3+n+(2+n)r\mathcal{E})-u_0^{2+n}(3+n+(2+n)\mathcal{E}u_0)}}dr}. \tag{5.4}$$

5.1. Numerical Example

In this example, we will consider $\mathcal{E}=0$ (Min-Hsing Chang, 2005; Haq and Ishaq, 2012; Abbasbandy and Shivanian, 2010).

Equation (5.4) becomes

$$\eta_{fin}=-\frac{2nu_0^{n/2}\sqrt{1-u_0^{n+2}}\,\Gamma\left(-\frac{1}{n+2}\right)}{(n+2)}\left(2u_0^{n/2}\Gamma\left(-\frac{1}{n+2}\right){}_2F_1\left(\frac{1}{2},\frac{n}{2n+4};\frac{3}{2}-\frac{1}{n+2};u_0^{n+2}\right)+\sqrt{\pi}n\,\Gamma\left(\frac{n}{2n+4}\right)\right)^{-1}. \tag{5.5}$$

Equation (5.5) represents the relation between the efficiency η_{fin} and the temperature of fin tip u_0 for any value of n.

By using Equation (5.2), Equation (3.6) with $\mathcal{E}=0$ and Equation (5.5), we can get relation between the efficiency η_{fin} and both the parameter N and the temperature gradient at the fin base $u'(1)$.

$$\frac{-\sqrt{2}nN\,\Gamma(-\beta)\,P^{\frac{n\beta}{2}}}{\beta^{-\frac{1}{2}}}\left(2\,\Gamma(-\beta)\,P^{\frac{n\beta}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{n\beta}{2};\frac{(3n+4)\beta}{2};P\right)+\sqrt{\pi}n\,\Gamma\left(\frac{n\beta}{2}\right)\right)^{-1}=1, \tag{5.6}$$

$$\sqrt{\eta_{fin}}=\frac{-\sqrt{2}n\sqrt{u'(1)}\,\Gamma(-\beta)\,\psi^{\frac{n\beta}{2}}}{\beta^{-\frac{1}{2}}}\left(2\,\Gamma(-\beta)\,\psi^{\frac{n\beta}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{n\beta}{2};\,\frac{(3n+4)\beta}{2};\,\psi\right)+\sqrt{\pi}n\,\Gamma\left(\frac{n\beta}{2}\right)\right)^{-1}, \tag{5.7}$$

where,

$$\beta = \frac{1}{n+2} \quad , P = 1 - \frac{1}{2\beta}\left(N\,\eta_{fin}\right)^2 \text{ and } \quad \psi = \left(1 - \frac{1}{2\beta}u'(1)\,\eta_{fin}\right).$$

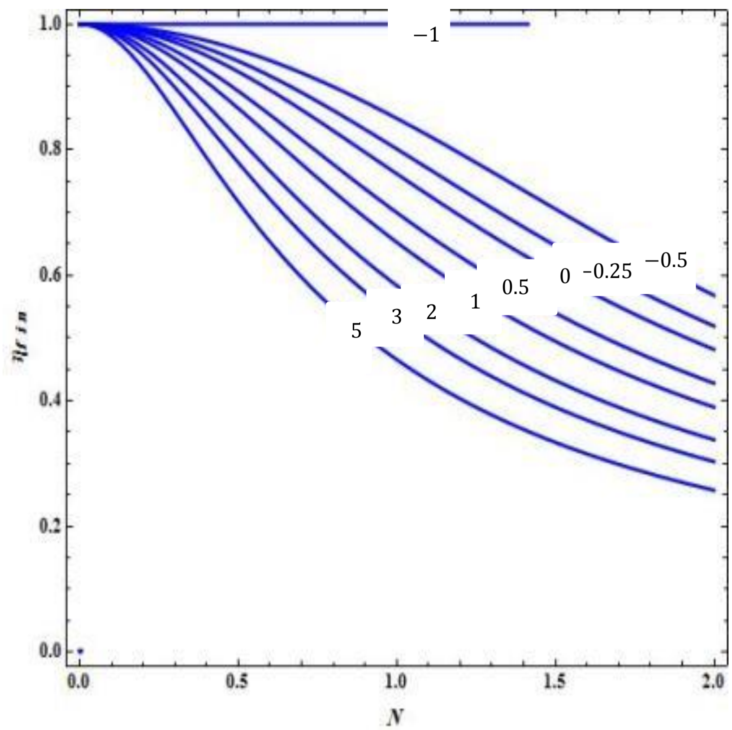

Figure 4. Plot of the relation (5.6) between η_{fin} and N for various values of n and $\mathcal{E} = 0$.

From Figure (4), it is noticed that the fin efficiency η_{fin} decreases when the magnitude of N increases and also, we note that the case $n = -1$ indicates a uniform local heat flux over the whole fin surface and induces the result of $\eta_{fin} = 1$. So Equations (5.5)-(5.7) are valid only for the cases $n \geq -1$ (Min-Hsing Chang , 2005).

6. Conclusions

In this paper, the exact implicit solution of the nonlinear differential equation governing heat transfer in fins (2.4) subject to the boundary conditions (2.7) is obtained using λ-symmetry reduction method and Lie point symmetry method. λ-symmetry method enables us to reduce the original ODE (2.4) into autonomous first order ordinary differential equation (3.5). Then we obtained general relationship among the temperature at the fin tip, the temperature gradient at the fin base , the mode of heat transfer n and the fin parameters N and $\mathcal{E}$. By using Lie point symmetry, the first order ordinary differential equation (3.5) is transformed into algebraic equation (4.3), which is solved analytically in (4.4). Numerical examples show that the temperature of fin increases when approaching from the heat source. We showed that the fin efficiency η_{fin} increases when the parameter N decreases.

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