



12-2013

## Graphic Illustration of the Transmission Resonances for the DKP Particles

B. Boutabia-Chéraitia  
*Université Badji-Mokhtar*

Abdenacer Makhlouf  
*Université de Haute Alsace*

Follow this and additional works at: <https://digitalcommons.pvamu.edu/aam>



Part of the [Dynamic Systems Commons](#), and the [Partial Differential Equations Commons](#)

### Recommended Citation

Boutabia-Chéraitia, B. and Makhlouf, Abdenacer (2013). Graphic Illustration of the Transmission Resonances for the DKP Particles, *Applications and Applied Mathematics: An International Journal (AAM)*, Vol. 8, Iss. 2, Article 25.

Available at: <https://digitalcommons.pvamu.edu/aam/vol8/iss2/25>

This Article is brought to you for free and open access by Digital Commons @PVAMU. It has been accepted for inclusion in *Applications and Applied Mathematics: An International Journal (AAM)* by an authorized editor of Digital Commons @PVAMU. For more information, please contact [hvkoshy@pvamu.edu](mailto:hvkoshy@pvamu.edu).



## Graphic Illustration of the Transmission Resonances for the *DKP* Particles

### B. Boutabia-Chéraitia

Laboratoire de Probabilités et Statistiques (LaPS)  
Université Badji-Mokhtar  
BP 12, Annaba 23000, Algérie  
[bboutheina@hotmail.com](mailto:bboutheina@hotmail.com);

### Abdenacer Makhlouf

Laboratoire de Mathématiques, Informatique, et Applications (LMIA)  
Université de Haute Alsace  
4, Rue des frères Lumière, Mulhouse 68093, France  
[Abdenacer.makhlouf@uha.fr](mailto:Abdenacer.makhlouf@uha.fr)

Received: July 22, 2013; Accepted: July 22, 2013

### Abstract

We consider the Duffin-Kemmer-Petiau (*DKP*) equation in the presence of a spatially one-dimensional Woods-Saxon (*WS*) potential and we show by graphics how the zero-reflection condition on the Klein interval depends on the shape of the potential.

**Keywords:** *DKP* algebra; *KG* operator; Dirac particle; *WS* potential; Transmission coefficient; reflection coefficient

**AMS-MSC 2010 No.:** 35P25, 37K20

### 1. Introduction

The *DKP* equation Petiau (1936), Duffin (1938), Kemmer (1939) and Géhéniau (1938) is similar in structure to the Dirac equation. Over the succeeding years, a great number of papers dealing with this equation were published. However, after the early 1950's until approximately 1970, physical interest in it waned because it was ultimately believed that the Klein-Gordon (*KG*) and

the *DKP* equations are equivalent, since for many classes of processes such as the quantum electrodynamics of spin-0 mesons, calculations based on the *DKP* and *KG* equations yield identical results including one-loop corrections. Afterwards, with the discovery of the parity violation and with the creation of the unified theory of electroweak interaction (the Weinberg-Salam theory of standard model), it was concluded that the *DKP* formalism in some cases yields different results from a second-order formalism, and this renewed the interest in the *DKP* equation and its corresponding algebra.

The *KG* equation in the *WS* potential well was solved in Rojas and Villalba (2005), Villalba and Rojas (2006) and it was shown that there is a critical value for the potential where the bound antiparticle mode appears. Also, transmission resonances for the *KG* particle in the *WS* potential barrier have been computed in Rojas and Villalba (2005), and it has been shown that the transmission coefficient as a function of the energy and the potential amplitude shows a behavior that resembles the one obtained for the Dirac particle.

Let us briefly recall that the *DKP* equation is a natural manner to extend the covariant Dirac formalism to the case of scalar (spin0) and vectorial (spin1) particles when interacting with an electromagnetic field, and will be written as ( $\hbar = c = 1$ ):

$$i\beta^\mu[(\partial_\mu + ieA_\mu) - m]\Psi(\mathbf{r}, t) = 0, \quad (1)$$

the matrices  $\beta^\mu$  verifying the *DKP* algebra:

$$\beta^\mu\beta^\nu\beta^\lambda + \beta^\lambda\beta^\nu\beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\nu\lambda}\beta^\mu. \quad (2)$$

The convention for the metric tensor is  $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ . The algebra (2) has three irreducible representations whose degrees are 1, 5 and 10. The first one is trivial, having no physical content, the second and the third ones correspond respectively to the scalar and vectorial representations. For the spin 0, the  $\beta^\mu$  are given by

$$\beta^0 = \begin{pmatrix} \boldsymbol{\theta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \beta^i = \begin{pmatrix} \mathbf{0} & \rho^i \\ -\rho_T^i & \mathbf{0} \end{pmatrix}, i = 1, 2, 3, \quad (3)$$

where

$$\begin{aligned} \rho^1 &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \rho^3 &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \boldsymbol{\theta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \end{aligned} \quad (4)$$

The  $\rho_T$  denoting the transposed matrix of  $\rho$ , and  $\mathbf{0}$  denoting the zero matrix. For the spin 1, the  $\beta^\mu$  are given by

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \bar{0}^T & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

$$\beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix}, i = 1,2,3, \tag{5}$$

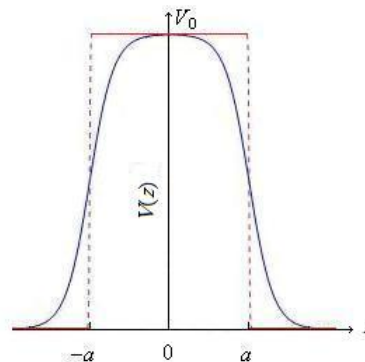
with

$$e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1), \bar{0} = (0,0,0). \tag{6}$$

The  $s_i$  being the standard nonrelativistic ( $3 \times 3$ ) spin 1 matrices, and  $\mathbf{0}$  and  $\mathbf{1}$  denoting respectively the zero matrix and the unity matrix.

We consider the case when the particle is interacting with the scalar and independent of time, potential of  $WS$  given by

$$V(z) = \frac{V_0}{1 + \exp\left(\frac{|z| - a}{r}\right)}, \tag{7}$$



**Figure 1.** The  $WS$  scalar potential.  $a = 2, r = (1/3)$  for solid lined and  $a = 2, r = (1/100)$  for dotted lined.

$V_0$  is real and positive. The parameters of shape  $r$ , and of width  $a$ , are real, positive and adjustable. For this potential, the equation (1) will be written as

$$\left[ i\beta^0 \left( \frac{\partial}{\partial t} + ieV \right) + i\beta^3 \frac{d}{dz} - m \right] \Psi(z, t) = 0. \tag{8}$$

The stationary states  $\Psi(z, t)$  having the form

$$\Psi(z, t) = e^{-iEt} \widehat{\phi}(z), \tag{9}$$

so

$$\left[ \beta^0(E - eV) + i\beta^3 \frac{d}{dz} - m \right] \overline{\phi(z)} = 0. \quad (10)$$

By taking the limit  $r \rightarrow 0^+$ , the potential (7) becomes of a square barrier form

$$V(z) = V_0 \theta(a - |z|), \quad (11)$$

that leads to the following eigenvalue equation

$$\left[ \beta^0(E - eV) + i\beta^3 \frac{d}{dz} - m \right] \kappa(z) = 0, \quad (12)$$

where  $\kappa(z)^T = (\varphi, A, B, C)$ ,  $A, B$  and  $C$  being respectively vectors of components  $A_i, B_i$  and  $C_i; i = 1, 2, 3$ . The square barrier potential is one among the simple models of potentials realizing the concrete physical situation of the pair-creation phenomena Sugg et al. (1993), Calogeracos and Dombey (1999), Dombey and Calogeracos (1999), which is intimately related to the tunnel phenomena. This last is one of important and strange effects predicted by the quantum mechanics. Although it can be explained using the wave mechanics, it doesn't yet delivered its secret.

According to the equations they satisfy, one gathers the components of  $\kappa(z)$  this way Boutabia-Chéraitia and Boudjedaa (2005)

$$\begin{aligned} \Psi^T &= (A_1, A_2, B_3), \Phi^T = (B_1, B_2, A_3), \\ \Theta^T &= (C_2, -C_1, \varphi) \text{ and } C_3 = 0, \end{aligned} \quad (13)$$

with

$$\Psi = 0, \begin{pmatrix} \Phi \\ \Theta \end{pmatrix} = \begin{pmatrix} E - eV \\ m \\ i \frac{d}{dz} \\ m \frac{d}{dz} \end{pmatrix} \otimes \Psi. \quad (14)$$

$O_{KG} = \frac{d^2}{dz^2} + [(E - eV)^2 - m^2]$  being the  $KG$  operator. One will then designate by  $\phi(z)^T = (\Psi, \Phi, \Theta)$  the solution of (12).

## 2. The *DKP* Scattering States

When a wave representing a particle is incident on a potential, it is partially transmitted and partially reflected. The asymptotic forms of diverging waves can be determined at time very long after the interaction. The vectorial *DKP* particle we consider is subjected to the barrier potential (11). As  $|z| \rightarrow \infty, V(z) \rightarrow 0$  sufficiently fast so that  $\phi(z)$  solution of equation (8) becomes that

of a free particle. By what follows, we will try to examine the transmission-reflection problem in which the particle is incident, say, from the left ( $-\infty$ ), or from the right ( $+\infty$ ). The asymptotic form for the wave function  $\phi(z)$  is given in Boutabia-Chéraitia and Boudjedaa (2005) by

$$\phi(z) \xrightarrow{z \rightarrow -\infty} A e^{-ik(z+a)} \begin{pmatrix} 1 \\ E \\ m \\ -i\mu \\ rm \end{pmatrix} \otimes \mathbf{V} + B e^{ik(z+a)} \begin{pmatrix} 1 \\ E \\ m \\ i\mu \\ rm \end{pmatrix} \otimes \mathbf{V}, \quad (15)$$

and

$$\phi(z) \xrightarrow{z \rightarrow +\infty} C e^{ik(z-a)} \begin{pmatrix} 1 \\ E \\ m \\ i\mu \\ rm \end{pmatrix} \otimes \mathbf{V} + D e^{-ik(z-a)} \begin{pmatrix} 1 \\ E \\ m \\ -i\mu \\ rm \end{pmatrix} \otimes \mathbf{V}, \quad (16)$$

where  $\mu^2 = r^2(m^2 - E^2)$ ,  $\mu = irk$  with  $k$  real ( $|E| > m$ ) and  $\mathbf{V}$  is a constant vector of dimension  $(3 \times 1)$  which components are related to the three directions of spin 1.  $B$  and  $D$  are, respectively, the coefficients of the incoming waves from  $-\infty \rightarrow 0$  and from  $+\infty \rightarrow 0$ .  $A$  and  $C$  are, respectively, the coefficients of the reflected and transmitted wave. The reflection and transmission coefficients (respectively,  $\mathbf{R}$  and  $\mathbf{T}$ ) along each direction  $i$  of the spin are given in Boutabia-Chéraitia and Boudjedaa (2005) by

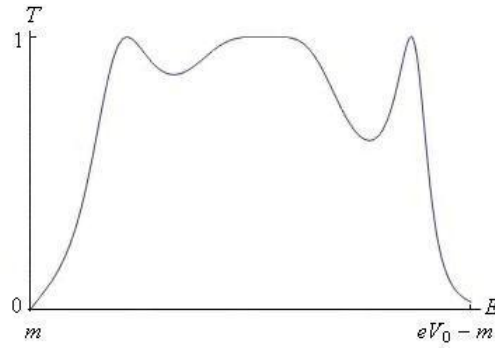
$$\mathbf{R} = \frac{\left(\frac{k^2 - p^2}{2pk}\right)^2 \sin^2 2pa}{1 + \left(\frac{k^2 - p^2}{2pk}\right)^2 \sin^2 2pa}, \quad (17)$$

$$\mathbf{T} = \frac{1}{1 + \left(\frac{k^2 - p^2}{2pk}\right)^2 \sin^2 2pa}, \quad (18)$$

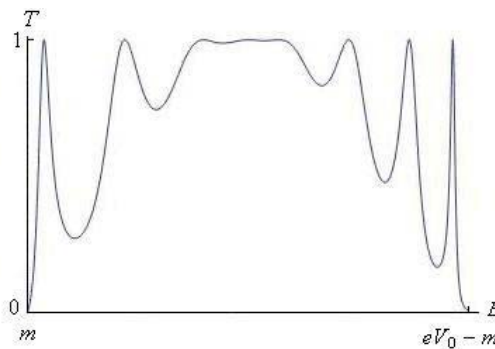
which verify the equality

$$\mathbf{R} + \mathbf{T} = 1. \quad (19)$$

We propose to resolve numerically the equation (18) then we get the following graphics. When varying the energy  $E$  we obtain

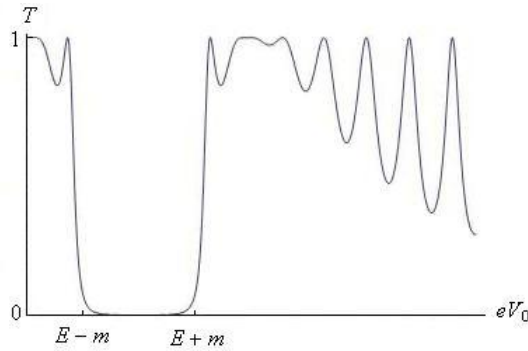


**Figure 2.** The plot illustrates  $T$  for varying energy  $E$  with  $a = 2, eV_0 = 4, m = 1$

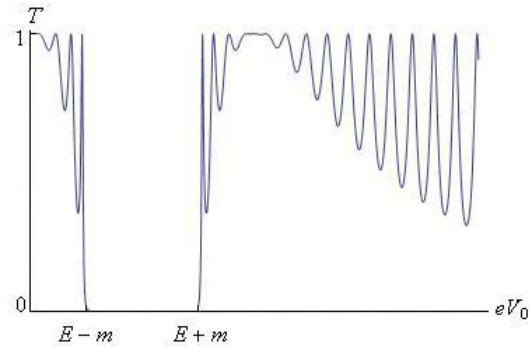


**Figure 3.** The plot illustrates  $T$  for varying energy  $E$  with  $a = 4, eV_0 = 4, m = 1$

and when varying the barrier height  $eV_0$  we obtain



**Figure 4.** The plot illustrates  $T$  for varying barrier height  $eV_0$  with  $a = 2, E = 2m, m = 1$



**Figure 5.** The plot illustrates  $T$  for varying barrier height  $eV_0$  with  $a = 4, E = 2m, m = 1$

From figures 2 and 3, one can see that analogous to the Dirac particle Sugg et al. (1993), Calogeracos and Dombey (1999), Dombey and Calogeracos (1999) and to the  $KG$  particle Rojas and Villalba (2005), the  $DKP$  particle exhibits transmission resonances in the Klein interval ( $m < E < eV_0 - m$ ).

Also, figures 4 and 5 show that analogous to the Dirac and  $KG$  cases, transmission resonances appear for  $eV_0 > E + m$  (thus  $E < eV_0 - m$ ). The zero-transmission happening for values of potential strength  $E - m < eV_0 < E + m$  (thus  $eV_0 - m < E < eV_0 + m$ ).

One can also mention for the four figures, that the occurrence of the transmission resonances increases with the width  $a$  of the square barrier.

### 3. Conclusions

We have showed a similarity in behavior between  $DKP$ ,  $KG$  and Dirac particles when interacting with a one-dimensional scalar potential. For the  $DKP$  and  $KG$  particles this can be interpreted as a demonstration of the equivalence between  $DKP$  and  $KG$  theories. For  $DKP$  and Dirac particles, it gives us the hope of establishing in the future, some similarities in their behavior in the case of the potential well. This work is in pending.

### Acknowledgments

*The authors would like to express their thanks to the referees for their valuable comments and to the editorial board for their support and efficiency.*



## REFERENCES

- Boutabia-Chéraitia, B. and Boudjedaa, T. (2005). Solution of *DKP* equation in Woods-Saxon potential, *Phys. Lett. A*, **338**, pp. 97-107.
- Calogeracos, A. and Dombey, N. (1999). Klein tunneling and the Klein paradox, *Int. J. Mod. Phys. A*, **14**, pp. 631-644.
- Dombey, N. and Calogeracos, A. (1999). Seventy years of Klein paradox, *Phys. Rev.*, **315**, pp. 41-58.
- Duffin, R. J. (1938). On the characteristic matrices of covariant systems, *Phys. Rev.*, **54**, pp. 1114.
- Géhéniau. J. (1938). Mécanique ondulatoire de l'électron et du photon, *Acad. R. Belg. Cl. Sci. Mém. Collect.*, **818, No.1**, 142 p.
- Kemmer, N. (1939). The particle aspect of meson theory, *Proc. Roy. Soc. A*, **173**, pp. 91-116.
- Petiau, G. (1936). Contribution à la théorie des équations d'ondes corpusculaires, *Acad. Roy. De Belg. Mem. Collect.*, **16**, pp. 1-115.
- Rojas, Clara and Villalba, Victor M. (2005). The Klein-Gordon equation with the Woods-Saxon potential well, *Revista Mexicana de Fisica S*, **52 (3)**, pp. 127-129.
- Rojas, Clara and Villalba, Victor M. (2005). Scattering of a Klein-Gordon particle by a Woods-Saxon potential, *Physical Review A*, **71** 052101, pp. 1-4.
- Sugg, R. K., Siu, G. G. and Chou, X. (1993). Barrier penetration and the Klein paradox, *J. Phys. A*, **26**, pp. 1001-1005.
- Villalba, Victor M. and Rojas, Clara (2006). Bound states of the Klein-Gordon equation in the presence of short-range potentials, *Int. J. Mod. Phys. A*, **21**, pp. 313-326.