



6-2013

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Recommended Citation

Borkar, M. S. and Charjan, S. S. (2013). Bianchi Type I Magnetized Cosmological Model with Λ -Term in Bimetric Theory of Gravitation, *Applications and Applied Mathematics: An International Journal (AAM)*, Vol. 8, Iss. 1, Article 7.

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Bianchi Type I Magnetized Cosmological Model with Λ -Term in Bimetric Theory of Gravitation

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Received: June 30, 2012; Accepted: February 14, 2013

Abstract

In this paper, we investigated Bianchi Type I magnetized cosmological model with cosmological constant in Rosen's bimetric theory of gravitation by using the techniques of Letelier and Stachel. The nature of the model is discussed in presence as well as in absence of the magnetic field and cosmological constant. Our model exists and it never goes to vacuum if cosmological constant is not equal to zero and for finite value of cosmic time. In the presence of magnetic field, our model goes to vacuum when the cosmic time tends to minus infinity. Further in the absence of magnetic field the model exists if cosmological constant is negative and n lies between zero and half and our model is filled with dark matter if cosmological constant is positive and n is greater than half. It is realized that, in the absence of cosmological constant our model goes over to the vacuum model.

Keywords: Gravitational theory, electromagnetic fields, cosmology

MSC 2010 No.: 83-02, 83C50, 83F05

1. Introduction

In the early phase of the universe, there is no definite evidence that the present day universe (FRW universe) was of the same type. Therefore it is important to study different Bianchi Type models in the context of the early phase of the universe. Several theories of gravitation have been formulated which are considered to be alternatives to Einstein's theory of gravitation. One of them is Rosen's (1977) bimetric theory of gravitation. The Rosen's bimetric theory is the theory of gravitation based on two metrics. One is the fundamental metric tensor g_{ij} which describes the gravitational potential and the second metric γ_{ij} refers to the flat space-time and describes the inertial forces associated with the acceleration of the frame of reference. The metric tensors g_{ij} determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and describes the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. Therefore, it interacts with g_{ij} but not directly with matter. One can regard γ_{ij} as giving the geometry that would exist if there were no matter. In the absence of matter one would have $g_{ij} = \gamma_{ij}$.

Thus, at every point of space-time, there are two metrics

$$ds^2 = g_{ij} dx^i dx^j, \quad (1)$$

$$d\eta^2 = \gamma_{ij} dx^i dx^j. \quad (2)$$

The field equation of Rosen's (1974) bimetric theory of gravitation is given by

$$N_i^j - \frac{1}{2} N \delta_i^j + \Lambda_i^j = -8\pi k T_i^j, \quad (3)$$

where $N = N_i^i$, $k = \sqrt{\frac{g}{\gamma}}$ together with $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$. Here, the vertical bar ($|$) stands for γ -covariant differentiation and T_i^j is the energy-momentum tensor of matter fields.

Several aspects of bimetric theory of gravitation have been studied by Rosen (1974, 1977), Goldman (1976), Karade (1980), Katore et al. (2006), Isrelit (1981), Khadekar et al. (2007). In particular, Reddy et al. (1998) have obtained some Bianchi Type cosmological models in bimetric theory of gravitation. The purpose of Rosen's bimetric theory is to get rid of the singularities that occur in general relativity that was appearing in the big-bang in cosmological models and therefore, recently, there has been a lot of interest in cosmological models in related to Rosen's bimetric theory of gravitation.

In the context of general relativity cosmic strings do not occur in Bianchi Type models. In it some Bianchi Type cosmological models – two in four and one in higher dimensions– are

studied by Krori et al. (1994). They showed that the cosmic strings do not occur in Bianchi Type V cosmology. Bali and Dave (2003), Bali and Upadhaya (2003), Bali and Singh (2005) have investigated Bianchi Type IX, I and V string cosmological models under different physical conditions in general relativity. Raj Bali and Anjali (2006) have investigated Bianchi Type I magnetized string cosmological model in general relativity by introducing the condition $A=(BC)^n$, where $n > 0$ in Einstein field equations, whereas Raj Bali and Umesh Kumar Pareek (2007) have deduced Bianchi Type I string dust cosmological model with magnetic field in general relativity by imposing the condition $A = N (BC)^{-n}$, where $n > 0$ and N is proportionality constant in Einstein field equations. Further Borkar et al. (2009, 2010), Gaikwad et al. (2011) and Kandalkar et al. (2011) have been investigated many magnetized cosmological models in bimetric theory of gravitation by using the techniques of Letelier and Stachel [(1979), (1983), (1980)].

In this paper, we investigated Bianchi Type I magnetized cosmological model with Λ -term in Rosen's bimetric theory of gravitation by using the techniques of Letelier and Stachel [(1979, 1983), 1980] and using the condition $A=(BC)^n$, where $n > 0$ in Rosen's field equations. The physical and geometrical significance of the model are discussed in the presence as well as in the absence of the magnetic field and cosmological constant Λ .

It is realized that, in the absence of cosmological constant Λ our model goes over to the model of Borkar et al. (2010) and there is no any new contribution than that of it.

2. Solutions of Rosen's Field Equations

We consider Bianchi Type I metric in the form

$$ds^2 = - dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \quad (4)$$

where A , B and C are functions of t alone. Here $B \neq C$ otherwise we get LRS Bianchi Type I model.

The flat metric corresponding to metric (4) is

$$d\eta^2 = - dt^2 + dx^2 + dy^2 + dz^2. \quad (5)$$

The energy momentum tensor T_i^j for the string dust with magnetic field is taken as

$$T_i^j = (\epsilon + p) v_i v^j + p g_i^j - \lambda x_i x^j + E_i^j, \quad (6)$$

with

$$v^i v_i = -x_i, \quad x^i = -1, \tag{7}$$

$$v^i x_i = 0. \tag{8}$$

In this model $\epsilon, \lambda, p, v^i$ and x_i denote the rest energy density, the string tension density, pressure, the flow vector and the direction of strings of the system respectively.

The electromagnetic field E_{ij} is given by Lichnerowicz (1967)

$$E_{ij} = \bar{\mu} \left[|h|^2 \left(v_i v_j + \frac{1}{2} g_{ij} \right) - h_i h_j \right], \tag{9}$$

where $\bar{\mu}$ is the magnetic permeability.

The four velocity vector v_i is given by

$$g_{ij} v^i v^j = -1. \tag{10}$$

The magnetic flux vector h_i is defined by

$$h_i = \frac{\sqrt{-g}}{2\mu} \epsilon_{ijkl} F^{kl} v^j, \tag{11}$$

where F_{kl} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi Civita tensor density.

Assume the comoving coordinates and hence we have

$$v^1 = v^2 = v^3 = 0, \quad v^4 = 1.$$

Further we assume that the incident magnetic field is taken along x -axis, so that

$$h_1 \neq 0, \quad h_2 = h_3 = h_4 = 0.$$

The first set of Maxwell's equation

$$F_{[ij,k]} = 0, \tag{12}$$

yields

$$F_{23} = \text{constant} = H(\text{say}).$$

Due to the assumption of infinite electrical conductivity, we have

$$F_{14} = F_{24} = F_{34} = 0.$$

The only non-vanishing component of F_{ij} is F_{23} . Hence,

$$h_1 = \frac{AH}{\mu BC} \quad (13)$$

and

$$|h|^2 = \frac{H^2}{\mu^2 B^2 C^2}. \quad (14)$$

From equation (9), we obtain

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{H^2}{2\mu B^2 C^2}. \quad (15)$$

Equation (6) of energy momentum tensor yields

$$T_1^1 = \left(p - \lambda - \frac{H^2}{2\mu B^2 C^2} \right), T_2^2 = T_3^3 = \left(p + \frac{H^2}{2\mu B^2 C^2} \right), T_4^4 = \left(-p - \frac{H^2}{2\mu B^2 C^2} \right). \quad (16)$$

The Rosen's field equation (3) for the metric (4) and (5) together with (16) lead to the following system of equations:

$$\frac{-A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC \left(-p + \lambda + \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda, \quad (17)$$

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC \left(-p - \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda, \quad (18)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} = 16\pi ABC \left(-p - \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda, \quad (19)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC \left(p + \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda, \quad (20)$$

where

$$A_4 = \frac{dA}{dt}, \quad B_4 = \frac{dB}{dt}, \quad C_4 = \frac{dC}{dt}, \quad \text{etc.}$$

From equations (18) and (19), we obtain

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{C_4^2}{C^2} - \frac{B_4^2}{B^2}. \quad (21)$$

Equations (17) and (18) lead to

$$2\frac{B_{44}}{B} - 2\frac{A_{44}}{A} + 2\frac{A_4^2}{A^2} - 2\frac{B_4^2}{B^2} = 16\pi ABC \left(\lambda + \frac{2K}{B^2 C^2} \right), \quad (22)$$

where

$$K = \frac{H^2}{2\mu}.$$

Equations (20) and (22), after using string dust condition ($\epsilon = \lambda$) [Zel'dovich (1980)], lead to

$$\frac{B_{44}}{B} - 3\frac{A_{44}}{A} - \frac{C_{44}}{C} + 3\frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} = 16\pi A \left(\frac{K}{BC} \right) + 2\Lambda. \quad (23)$$

The field equations (17)-(20) are the system of four equations in six unknown parameters $A, B, C, \lambda, \epsilon$ and p . Therefore, to deduce a determinate solution, we assume two extra conditions. First is that the component σ_1^1 of shear tensor σ_i^j is proportional to the expansion (θ), which leads to

$$A = (BC)^n, \quad \text{where } n > 0 \quad (24)$$

and second is Zel'dovich condition $\epsilon = \lambda$. Using the first condition (equation (24)), in the equation (23), we obtain

$$(1-3n)\frac{B_{44}}{B} - (3n+1)\frac{C_{44}}{C} + (3n-1)\frac{B_4^2}{B^2} + (3n+1)\frac{C_4^2}{C^2} = 16\pi K(BC)^{n-1} + 2\Lambda. \quad (25)$$

Now, the equation (21), which can be rewritten as

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = \frac{B_4}{B} + \frac{C_4}{C}, \quad (26)$$

which after integrating reduces to

$$C^2 \left(\frac{B}{C} \right)_4 = LBC, \quad (27)$$

where L is a constant of integration.

Assume $BC = \mu$, $\frac{B}{C} = v$. Then, $B^2 = \mu v$, $C^2 = \frac{\mu}{v}$. In view of these relations, equation (27) becomes

$$\frac{v_4}{v} = L. \quad (28)$$

Now equation (25) after using (24) and using assumptions $BC = \mu$ and $\frac{B}{C} = v$ lead to

$$-3n \frac{\mu_{44}}{\mu} + 3n \left(\frac{\mu_4^2}{\mu^2} \right) - \left(\frac{v_4}{v^2} \right)^2 + \frac{v_{44}}{v} = 16\pi\mu^{(n-1)} K + 2\Lambda. \quad (29)$$

The equations (28) and (29) yield

$$2\mu_{44} - 2 \frac{\mu_4^2}{\mu} = -\frac{4}{3n} [8\pi\mu^n K + \Lambda \mu], \quad (30)$$

which reduces to

$$\frac{d}{d\mu} [f^2] - 2 \frac{f^2}{\mu} = -\frac{4}{3n} [8\pi\mu^n K + \Lambda \mu], \quad (31)$$

where $\mu_4 = f(\mu)$. Integrating equation (31), we get

$$f^2 = P\mu^2 - \frac{32\pi K\mu^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n} \mu^2 \log \mu, \quad (32)$$

where P is an integrating constant. From equation (28), we obtain

$$\log v = \int \frac{L d\mu}{\sqrt{P\mu^2 - \frac{32\pi K\mu^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n} \mu^2 \log \mu}} + \log b, \quad (33)$$

where b is a constant of integration.

Now, using $\mu_4 = f(\mu)$ and expression (32), the metric (4) will be

$$ds^2 = - \frac{d\mu^2}{\left[P\mu^2 - \frac{32\pi K\mu^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n} \mu^2 \log \mu \right]} + \mu^{2n} dx^2 + \mu v dy^2 + \frac{\mu}{v} dz^2, \quad (34)$$

where v is determined by equation (33).

After suitable transformation of co-ordinates

$$\mu = T, \quad x = X, \quad y = Y, \quad z = Z,$$

i.e.,

$$d\mu = dT, \quad dx = dX, \quad dy = dY, \quad dz = dZ,$$

the model (34) reduces to

$$ds^2 = - \frac{dT^2}{\left[PT^2 - \frac{32\pi KT^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n} T^2 \log T \right]} + T^{2n} dX^2 + T v dY^2 + \frac{T}{v} dZ^2. \quad (35)$$

Now choosing the cosmic time $u = \pm \log T$, for convenience, we select $u = -\log T$, and then the model (35) is reduced to

$$ds^2 = - \frac{du^2}{\left[P - \frac{32\pi}{3n(n-1)} K(e^{-u})^{n-1} + \frac{4\Lambda}{3n} u \right]} + e^{-u} [(e^{-u})^{2n-1} dX^2 + v dY^2 + \frac{1}{v} dZ^2]. \quad (36)$$

This is the Bianchi Type I magnetized cosmological model with Λ - term in bimetric theory of gravitation.

3. Physical Quantities of the Model

The energy density (ϵ), the string tension density (λ), the pressure (p) for the model (36) (in terms of cosmic time u) are given by

$$\epsilon (= \lambda) = \frac{(2n-1)\Lambda}{24n\pi} (e^u)^{n+1} - \frac{(4n+1)}{3n} Ke^{2u}, \quad 0 < n \neq \frac{1}{2}, \quad (37)$$

$$p = \frac{-\Lambda}{12\pi} (e^u)^{n+1} - \frac{2}{3} K e^{2u}, \quad (38)$$

respectively. For $\Lambda \neq 0$ and finite u , the condition $\epsilon > 0$ and $p > 0$ satisfies if

$$8\pi \frac{(4n+1)}{(2n-1)} K (e^{-u})^{n-1} < \Lambda < -8\pi K (e^{-u})^{n-1}, \quad (39)$$

and $\epsilon = 0$, $p = 0$ gives $n = 0$, which contradicts the fact that $n > 0$. Thus, the model never goes to vacuum model, for finite value of u and for $\Lambda \neq 0$.

The strong energy conditions of Hawking and Ellis (1973): $(\epsilon - p) > 0$ and $(\epsilon + p) > 0$ are satisfied by our model (36) if

$$8\pi \frac{(2n+1)}{(4n-1)} K (e^{-u})^{n-1} < \Lambda < -8\pi (6n+1) K (e^{-u})^{n-1}. \quad (40)$$

The expansion θ is given by $\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$ and for the model (36), it has the value

$$\theta = (n+1)f e^u,$$

which (after using (32)) reduces to

$$\theta = (n+1) \left[P - \frac{32\pi K (e^{-u})^{n-1}}{3n(n-1)} + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}}. \quad (41)$$

The components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{1}{3} \left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} \right],$$

or

$$\sigma_1^1 = \frac{(2n-1)}{3} \left[P - \frac{32\pi K (e^{-u})^{n-1}}{3n(n-1)} + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}}, \quad (42)$$

$$\sigma_2^2 = -\frac{1}{2} \sigma_1^1 + \frac{L}{2}, \quad (43)$$

$$\sigma_3^3 = -\frac{1}{2} \sigma_1^1 - \frac{L}{2}, \quad (44)$$

$$\sigma_4^4 = 0. \quad (45)$$

Special Case

In the absence of magnetic field, *i.e.*, for $K = 0$, the expressions for the energy density (ϵ), the string tension density (λ), the pressure (p), scalar of expansion (θ) and components of shear tensor (σ_i^j) are given by

$$\epsilon (= \lambda) = \frac{(2n-1)\Lambda}{24n\pi} (e^u)^{n+1}, \quad (46)$$

$$p = \frac{-\Lambda}{12\pi} (e^u)^{n+1}, \quad (47)$$

$$\theta = (n+1) \left[P + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}}, \quad (48)$$

$$\sigma_1^1 = \frac{(2n-1)}{3} \left[P + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}}, \quad (49)$$

$$\sigma_2^2 = -\frac{(2n-1)}{6} \left[P + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}} + \frac{L}{2}, \quad (50)$$

$$\sigma_3^3 = -\frac{(2n-1)}{6} \left[P + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}} - \frac{L}{2}, \quad (51)$$

$$\sigma_4^4 = 0. \quad (52)$$

4. Conclusion

There is no big-bang and big crunch singularities in our model (36). It is interesting to note that the cosmological constant Λ is playing an important role in our model. From the equations (37), (38) and (39), it is seen that the rest energy density ϵ , string tension density λ and pressure p all are positive if Λ lies in the open interval in equation (39), for n is not equal to half and for finite u . This shows that such a Bianchi Type I magnetized cosmological model (36) in bimetric theory of gravitation exists when cosmological constant Λ lies in the open interval in the equation (39). It never goes to vacuum model for Λ is not equal to zero and for finite value of u . When $u \rightarrow -\infty$, then $\theta \rightarrow -\infty$, $\sigma \rightarrow -\infty$ and when $u \rightarrow \infty$, then $\theta \rightarrow \infty$, $\sigma \rightarrow \infty$. This

confirms that the model is expanding as well as shearing. The scalar of expansion and the shear in the model increases as cosmic time u increases.

Further when $u \rightarrow \infty$, then $\epsilon = \lambda \rightarrow \infty$, $p \rightarrow \infty$. When $u \rightarrow -\infty$, then $\epsilon = \lambda \rightarrow 0$ and $p \rightarrow 0$, which shows that our model goes over to vacuum model, when the cosmic time u is minus infinity. In the special case, in the absence of magnetic field, i.e., $K = 0$, from the equation (46) and (47), it is seen that our model (36) exists if Λ is negative and n between zero and half and if the cosmological constant $\Lambda > 0$ then our model filled with dark matter for n greater than half, for finite value of cosmic time u . Also the expansion and shear in the model increases, as the cosmic time u increases. Further it is observed that in the absence of cosmological constant Λ i.e. if $\Lambda = 0$ then our model (36) goes over to the model of Borkar et al. (2010) and there is no any new contribution than that of it.

Acknowledgement

The authors express their sincere thanks to UGC, New Delhi, for financial assistance under SAP and under minor research project.

REFERENCES

- Bali, R. and Anjali (2006). Bianchi Type I Magnetized String Cosmological Model in General Relativity, Astrophysics Space Science, Vol. 302.
- Bali, R. and Dave, S. (2003). Bianchi Type -IX String Cosmological Models with Bulk Viscous Fluid in General Relativity, Astrophysics Space Science, Vol. 288.
- Bali, R. and Upadhaya, R. D. (2003). LRS Bianchi Type I String Dust Magnetized Cosmological Models, Astrophysics Space Science, Vol. 283.
- Bali, R. and Singh, D. (2005). Bianchi Type-V Bulk Viscous Fluid String Dust Cosmological Model in General Relativity, Astrophysics Space Science, Vol. 300.
- Bali, R. and Pareek, U. K. (2007). Bianchi Type I String Dust Cosmological Model with Magnetic Field in General Relativity, Astrophysics Space Science, Vol. 312.
- Borkar, M. S. and Charjan, S. S. (2009). Bianchi Type I String Dust Cosmological Model with Magnetic Field in Bimetric Relativity, International Journal of Applied Mathematics, Vol. 22, No. 3.
- Borkar, M. S. and Charjan, S. S. (2010). The Charged Perfect Fluid Distribution in Bimetric Theory of Relativity, Journal of the Indian Academy Mathematics, Vol. 32, No. 1.
- Borkar, M.S. and Charjan, S.S. (2010). Bianchi Type I Bulk Viscous Fluid String Dust Cosmological Model with Magnetic Field in Bimetric Theory of Gravitation, International Journal Applications and Applied Mathematics, Vol. 5, No. 1.
- Borkar, M.S. and Charjan, S.S. (2010). Bianchi Type I Magnetized Dust Cosmological Model in Bimetric Theory of Gravitation, International Journal of Applications and Applied Mathematics, Vol. 5, No. 2.

- Gaikwad, N.P., Borkar, M.S. and Charjan, S.S. (2011). Bianchi Type I Massive String Magnetized Barotropic Perfect Fluid Cosmological Model in Bimetric Theory of Gravitation, Chinese Physics Letter, Vol. 28, No. 8.
- Goldman, I. and Rosen, N. (1976). A Cosmological Model in the Bimetric Gravitation Theory, Astrophysical Journal, Vol. 212.
- Hawking, S.W. and Ellis, G.F.R. (1973). The Large Scale Structure of Space-Time, Cambridge University Press, Cambridge.
- Isrelit, M. (1981). Spherically Symmetric Fields in Rosen's Bimetric Theories of Gravitation, General Relativity and Gravitation, Vol. 13, No. 7.
- Kandalkar, S.P. et al. (2011). String Cosmology in Kantowski Sachs Space-Time with Bulk Viscosity and Magnetic Field, Journal of Vectorial Relativity, Vol. 6.
- Karade, T.M. (1980). Spherically Symmetric Space-Times in Bimetric Relativity Theory I, Indian Journal of Pure-Applied Mathematics, Vol. 11, No. 9.
- Katore, S.D. and Rane, R.S. (2006). Magnetized Cosmological Models in Bimetric Theory of Gravitation, Pramana Journal of Physics, Vol. 67, No. 2.
- Khadekar, G.S. and Tade, S.D. (2007). String Cosmological Models in Five Dimensional Bimetric Theory of Gravitation, Astrophysics Space Science, DOI 10.1007/S10509-007-9410-2.T.
- Krori, K.D., Choudhuri, T. and Mahant, C.R. (1994). String in some Bianchi Type Cosmologies, General Relativity and Gravitation, Vol. 26, No. 3.
- Letelier, P. S. (1979). Clouds of Strings in General Relativity, Physical Review D, Vol. 20, No. 6.
- Letelier, P.S. (1983). String Cosmology, Physical Review D, Vol. 28, No. 10.
- Lichnerowicz, A. (1967). Relativistic Hydrodynamics and Magneto hydrodynamics, Benjamin, New York.
- Reddy, D.R.K. and Venkateshwara Rao, N. (1998). On Some Bianchi Type Cosmological Models in Bimetric Theory of Gravitation, Astrophysics Space Science, Vol. 257.
- Rosen, N. (1974). A Theory of Gravitation, Annls of Physics, Vol. 84.
- Rosen, N. (1977). Topics in Theoretical and Experiential Gravitation Physics, Edited by Sabbata V. De and Weber J., Plenum Press, London.
- Stachel, J. (1980). Thickening the String I, The String Perfect Dust, Physical Review D, Vol. 21, No. 8.
- Zel'dovich, Y.B. (1980). Cosmological Fluctuations Produced Near a Singularity, Monthly Notices of the Royal Astronomical Society, Vol. 192.