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K-Total Product Cordial Labelling of Graphs

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Abstract

In this paper we introduce the *k*-Total Product cordial labelling of graphs. Also we investigate the 3-Total Product cordial labelling behaviour of some standard graphs.

Keywords: Path, Cycle, Star, Comb

MSC 2000 No.: 05C78

1. Introduction

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The graph obtained by subdividing each edge of a graph G by a new vertex is denoted by S (G). The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has $p_1$vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy G_2.Terms not defined here are used in the sense of Harary (1969).

Rosa (1967) introduced the concept of β-valued graph and Cahit (1987) was instrumental for the introduction of a weaker version of the above concept, known as cordial labelling. Several authors studied cordial graphs [Gallian (2011)]. Motivated by these definitions, Sundaram et al. (2004) introduced Product cordial labelling of graphs. Some authors are now working on Product cordial graphs [Salehi (2010); Selvaraju (2009); Seoud (2011); Vaidya, (2010), (2011)] and several variations of it [Babujee (2010), Sundaram (2005)]. The authors have introduced a generalized form of Product cordial labelling, known as the k- Product cordial labelling [Ponraj (2012)]. In this paper we introduce a new concept known as the k-Total Product cordial labelling and investigate 3-Total Product cordial labelling behaviour of some standard graphs.

2. *K*-Total Product Cordial Labelling

Definition 2.1.

Let f be a map from $V(G)$ to $\{0,1, ..., k\text{-}1\}$, where k is an integer, $2 \leq k \leq |V(G)|$. For each edge uv, assign the label $f(u) f(v)$ $(mod\ k)$. f is called a K-Total Product cordial labelling of G if

$$|f(i) - f(j)| \leq 1, \quad i, j \in \{0,1, ..., k\text{-}1\},$$

where $f(x)$ denotes the total number of vertices and edges labelled with x ($x = 0,1,2, ..., k$-1).

Theorem 2.2.

Let G be a (p,q) k-Product cordial graph. If $p \equiv 0(mod\ k)$ or $q \equiv 0\ (mod\ k)$ then G is k-Total Product cordial.

Proof:

***Case* (i):** $p \equiv 0(mod\ k)$.

Let $p= kt$. Let f be a k- Product cordial labelling of G. Since f is a k-Product cordial labelling, $v_f(i)=t$ and $|e_f(i) - e_f(j)| \leq 1$, $1 \leq I \leq k\text{-}1, 1 \leq j \leq k\text{-}1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges respectively labeled with x (x =0,1,2,3, …, k-1) .

Now

$$\begin{aligned} |f(i) - f(j)| &= |v_f(i) + e_f(i) - (v_f(j) + e_f(j))| \\ &= |v_f(i) - v_f(j) + e_f(i) - e_f(j)| = |e_f(i) - e_f(j)| \leq 1. \end{aligned}$$

***Case* (ii)**: Similar to (i) since $e_f(i) = e_f(j)$.

Theorem 2.3.

Any path P_n is 3-Total Product cordial.

Proof:

Let P_n be the Path $u_1u_2 \ldots u_n$.

***Case* (i):** $n \equiv 0(mod\ 3)$.

Let $n = 3t$. Define $f(u_i) = 0$, $1 \le i \le t$ and $f(u_{t+i}) = 2$, $1 \le i \le 2t$. Here, $f(0) = 2t$, $f(1) = 2t\text{-}1$, $f(2) = 2t$. Therefore, f is a 3-Total Product cordial labling.

***Case* (ii):** $n \equiv 1(mod\ 3)$.

Let n=3t+1. Define $f(u_i) = 0$, $1 \le i \le t$ and $f(u_{t+i}) = 2$, $1 \le i \le 2t+i$. Since $f(0) = 2t$, $f(1)=2t$, $f(2) = 2t+1$, Here, f is a 3-Total Product cordial labelling.

***Case* (iii):** $n \equiv 2(mod\ 3)$.

Let $n = 3t+2$. Define a map f as follows: $f(u_1) = 0$, $f(u_2) = 1$, $f(u_{2+i}) = 0$, $1 \le i \le t\text{-}1$, $f(u_{t+1+i}) = 2$, $1 \le i \le 2t+1$. Here, $f(0) = f(1) = f(2) = 2t+1$. Therefore, f is a 3-Total Product cordial labelling.

Illustration 2.4.

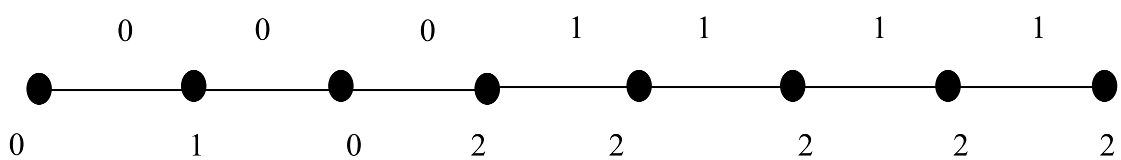

Figure 1. A 3-Total product cordial labelling of P_8.

Theorem 2.5.

The Cycle C_n is 3-Total product cordial labelling iff $n \neq 3, 6$.

Proof:

Let C_n be the cycle $u_1u_2 \ldots u_nu_1$.

***Case* (i):** $n \equiv 0(mod\ 3)$. $n > 6$.

Let $n = 3t$, $t > 2$. Define $f(u_1) = 0$, $f(u_2) = 1$, $f(u_{2+i}) = 0$, $1 \le i \le t\text{-}2$, $f(u_{t+i}) = 2$, $1 \le i \le 2t$. Clearly, $f(0) = f(1) = f(2) = 2t$. Therefore, f is a 3-Total Product cordial labelling.

***Case* (ii):** $n \equiv 1(mod\ 3)$.

Let $n = 3t+1$. Define $f(u_i) = 0$, $1 \le i \le t$ and $f(u_{t+i}) = 2$, $1 \le i \le 2t+1$. Here, $f(1) = 2t+1$, $f(2) = 2t+2$. Therefore, f is a 3-Total Product cordial labelling.

***Case* (iii):** $n \equiv 2(mod\ 3)$.

Let $n = 3t+2$. Define $f(u_i) = 0$, $1 \le i \le t$ and $f(u_{t+i}) = 2$, $1 \le i \le 2t+2$. Here, $f(0) = 2t+1$, $f(1) = 2t+1$, $f(2) = 2t+2$. Therefore, f is a 3-Total Product cordial labelling.

***Case* (iv):** $n = 3$.

Suppose f is a Total Product cordial labelling of C_3. Here, sum of the order and size of C_3 is 6. Clearly, $f(0) \ge 3$, a contradiction.

***Case* (v):** $n = 6$.

Here, sum of the order and size of C_6 is 12. If 0 is labelled with 1 vertex, then $f(0) = 3$. If 0 labelled with any two vertices then $f(0) \ge 5$, which should not happen.

Illustration 2.6.

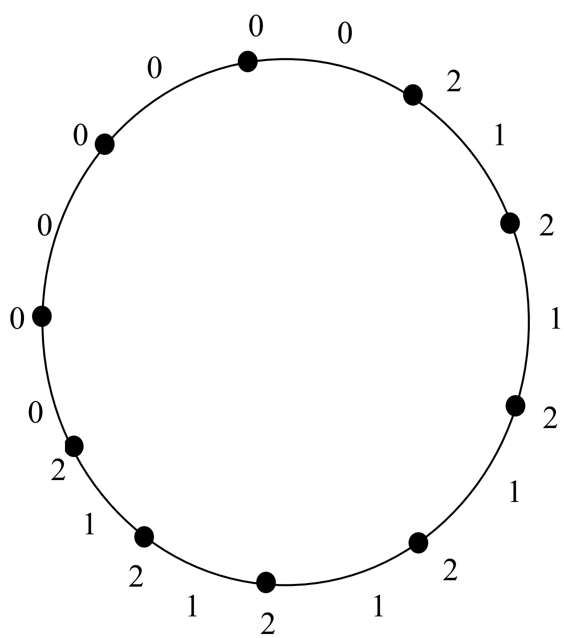

Figure 2. A 3-Total product cordial labelling of C_{10} .

Result 2.7. Ponraj (2012). Any Star is *k*-Product cordial.

Theorem 2.8. The Star $K_{1,n}$ is 3-Total Product cordial iff $n = 0, 2(mod\ 3)$.

Proof:

$V(K_{1,n}) = \{u, v_i, 1 \le i \le n\}$ and $E(K_{1,n}) = \{uv_i, 1 \le i \le n\}$.

***Case* (i):** $n \equiv 0, 2 (mod\ 3)$.

The result follows from theorem 2.2 and 2.7.

***Case* (ii):** $n \equiv 1 (mod\ 3)$.

Let $n = 3t+1$. Here, the sum of order and size of the star is $6t+3$. Clearly, $f(u) \neq 0$.

***Subcase* (i):** $f(u) = 1$.

Suppose x pendant vertices are labelled with 0 and y pendant vertices are labelled with1.Then $n-x-y$ pendant vertices are labelled with 2. Therefore, $f(0) = 2x$, $f(1) = 2y+1$, $f(2) = 2(n-x-y)$. But, $f(0) = f(1) = f(2) = 2t+1$. Therefore, $2x = 2t+1$, an impossibility.

***Subcase* (ii):** $f(u) = 2$.

Similar to Subcase (i), we get a contradiction. Hence $K_{1,n}$ is is 3-Total Product cordial labelling iff $n = 0, 2 (mod\ 3)$.

Illustration 2.9.

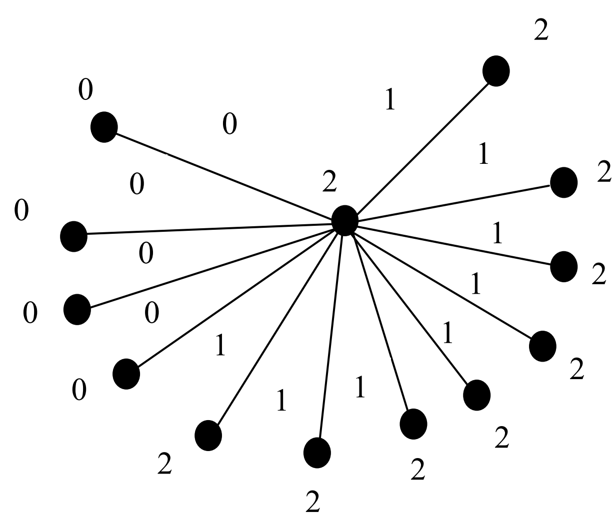

Figure 3. A 3-Total product cordial labelling of $K_{1,10}$.

Remark 2.10.

Any star is k-Product cordial [Ponraj (2012)] and hence a k-Product cordial graph need not be a k- Total Product cordial graph.

Theorem 2.11.

The Comb is 3-Total Product cordial.

Proof:

Let P_n be the path $u_1u_2u_3...u_n$. Also, let v_i be the pendant vertex adjacent to u_i $(1 \le i \le n)$

***Case* (i):** $n \equiv 0(mod\ 3)$.

Let $n=3t$. Define $f(u_i) = f(v_i) = 0$, $1 \le i \le t$, $f(u_{t+i}) = 1$, $1 \le i \le 2t$, $f(v_{t+i}) = 2$, $1 \le i \le 2t$. Here, $f(0) = 4t$, $f(1) = 4t - 1$, $f(2) = 4t$. Therefore, f is a Total Product cordial labelling.

***Case* (ii):** $n \equiv 1(mod\ 3)$.

Let $n = 3t+1$. Define $f(u_i) = 0$, $1 \le i \le t$, $f(v_i) = 0$, $1 \le i \le t-1$, $f(v_t) = 2$, $f(v_{t+1}) = 0$, $f(u_{t+i}) = 1$, $1 \le i \le 2t+1$, $f(v_{t+1+i}) = 2$, $1 \le i \le 2t$. Here, $f(0) = f(1) = f(2) = 4t+1$. Therefore, f is a Total Product cordial labelling.

***Case* (iii):** $n \equiv 2(mod\ 3)$.

Let $n=3t+2$.

Define $f(u_i) = 0$, $1 \le i \le t$, $f(v_i) = 0$, $1 \le i \le t+1$, $f(u_{t+i}) = 1$, $1 \le i \le 2t+2$, $f(v_{t-1+i}) = 2$, $1 \le i \le 2t+1$. Here, $f(0) = 4t+2$, $f(1) = 4t+1$, $f(2) = 4t+2$.

Therefore, f is a Total Product cordial labelling.

Theorem 2.12.

$P_n \Theta 2K_1$ is 3- Total Product cordial.

Proof:

Let P_n be the path $u_1, u_2 ... u_n$. Let v_i and w_i be the pendant vertices which adjacent to u_i, $1 \le i \le n$.

Define $f(u_i) = 1$, $1 \le i \le n$, $f(v_i) = 0$, $1 \le i \le n$, $f(w_i) = 2$, $1 \le i \le n$, $f(0) = 2n$, $f(1) = 2n-1$, $f(2) = 2n$.

Therefore, f is a 3-Total Product cordial.

Illustration 2.13.

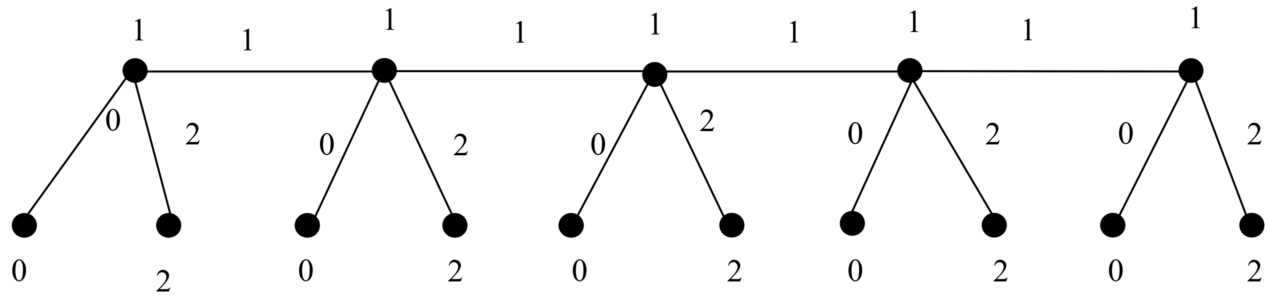

Figure 4. A 3-Total product cordial labelling of $P_5 \Theta 2K_1$.

Theorem 2.14.

K_2+mK_1 is 3-Total Product cordial iff m =2 (mod 3).

Proof:

Let $V(K_2+mK_1) = \{u, v, u_i : 1 \leq i \leq n\}$ and $E(K_2+mK_1) = \{uv, uu_i, vu_i : 1 \leq i \leq n\}$.

***Case* (i):** $m \equiv 0(mod\ 3)$.

Let m=3t. If possible, let there be a 3-Total Product cordial labelling. The sum of the order and size of K_2+mK_1 is 9t+3. Therefore, $f(0) = f(1) = f(2) = 3t+1$.

Clearly, $f(u)$ and $f(v)$ are not equal to zero otherwise $f(0) \geq 3t+2$. Let x, y be the number of vertices in mK_1 labelled with 0 and 1, respectively. Then $3x = 3t+1$, a contradiction.

Case(ii): $m \equiv 1(mod\ 3)$.

Let m=3t+1. Here, the sum of size and order is 9t+6. Here, $f(0) = f(1) = f(2) = 3t+2$. Let x be the number of vertices in mK_1 labelled with 0. Then, $3x=3t+2$, a contradiction.

Case(iii): $m \equiv 2(mod\ 3)$.

Let m=3t+2. Define $f(u) = f(v_i) = 1$, $f(u_i) = 0$, $1 \leq i \leq t+1$, $f(u_{t+1+i}) = 1$, $1 \leq i \leq t$, $f(u_{2t+1+i}) = 2$, $1 \leq i \leq t+1$. Here, $f(0) = f(1) = f(2) = 3t+3$. Therefore f is 3- Total Product cordial.

Theorem 2.15.

$S(K_{1,n})$ is 3-Total Product cordial.

Proof:

Let $V(S(K_{1,n})) = \{u, u_i, v_i : 1 \le i \le n\}$ and $E(S(K_{1,n})) = \{uu_i, u_iv_i : 1 \le i \le n\}$.

Case **(i):** $n \equiv 0(mod\ 3)$.

Let $n=3t$. Define $f(u) = 1$, $f(u_i) = 1$, $1 \le i \le 2t$, $f(u_{2t+i}) = 2$, $1 \le i \le t$, $f(v_i) = 2$, $1 \le i \le t$, $f(v_{t+i}) = 0$, $1 \le i \le 2t$, $f(1) = 4t+1$, $f(0) = f(1) = 4t$. Hence f is 3-Total Product cordial labelling.

***Case* (ii):** $n \equiv 1(mod\ 3)$.

Let $n=3t+1$. Define $f(u) = 1$, $f(u_i) = 1$, $1 \le i \le 2t$, $f(u_{2t+i}) = 2$, $1 \le i \le t+1$, $f(v_i) = 2$, $1 \le i \le t$, $f(v_{t+i}) = 0$, $1 \le i \le 2t+1$, $f(1) = 4t+1$, $f(0) = f(2) = 4t+2$. Hence, f is 3-Total Product cordial labelling.

***Case* (iii):** $n \equiv 2(mod\ 3)$.

Let $n=3t+2$. Define $f(u) = 1$, $f(u_i) = 1$, $1 \le i \le 2t$, $f(u_{2t+1}) = 0$, $f(u_{2t+1+i}) = 2$, $1 \le i \le 2t+1$, $f(v_i) = 2$, $1 \le i \le t$, $f(v_{t+i}) = 0$, $1 \le i \le t$, $f(v_{2t+1}) = 1$, $f(v_{2t+1+i}) = 0$, $1 \le i \le t$, $f(v_{3t+2}) = 2$, $f(0) = f(1) = f(2) = 4t+3$. Hence, f is 3-Total Product cordial labelling.

3. Conclusions

In this paper we have explored the cases when a k-Product cordial graph become K-Total Product cordial and also studied the k-Total Product cordial behaviour of some graphs for the specific value $k = 3$.It shall be interesting to study the K-Total Product behaviour of standard graphs for general k.

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