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N. Taghizadeh
University of Guilan

S. R. Moosavi Noori
University of Guilan

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Two Reliable Methods for Solving the Modified Improved Kadomtsev-Petviashvili Equation

N. Taghizadeh and S.R. Moosavi Noori
Department of Pure Mathematics
University of Guilan
P.O. Box 1914, Rasht, Iran
taghizadeh@guilan.ac.ir; moosavinoori@msc.guilan.ac.ir

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Abstract

In this paper, the tanh-coth method and the extended (G'/G)-expansion method are used to construct exact solutions of the nonlinear Modified Improved Kadomtsev-Petviashvili (MIKP) equation. These methods transform nonlinear partial differential equation to ordinary differential equation and can be applied to nonintegrable equation as well as integrable ones. It has been shown that the two methods are direct, effective and can be used for many other nonlinear evolution equations in mathematical physics.

Keywords: Tanh-Coth method, Extended (G'/G)-expansion method, Modified Improved Kadomtsev-Petviashvili (MIKP) equation

MSC 2010 No.: 47F05, 35Q53, 35Q80, 35G25

1. Introduction

The investigation of the exact solutions to nonlinear partial differential equations (NLPDEs) plays an important role in the study of many physical phenomena. With the help of exact solutions, when they exist, the mechanism of complicated physical phenomena and dynamical processes modeled by these NLPDEs can be better understood. They can also help to analyze the stability of these solutions and to check numerical analysis for these NLPDEs.

In recent years, reducing PDEs into ordinary differential equations (ODEs) has proved a successful idea to generate exact solutions of nonlinear wave equations. Many approaches to exact solutions in the literature follow such an idea, which contains the tanh and extended tanh methods [Malfliet (1992) , Malfliet and Hereman (1996), Wazwaz (2007), Taghizadeh and Mirzazadeh (2010), Taghizadeh et al. (2011)], (G'/G)-expansion method [Wang et al. (2008), Zhang et al. (2008), Zhu (2010), Taghizadeh and Mirzazadeh (2011), Biazar and Ayati (2011)], the homogeneous balance method [Wang (1995), Khalfallah (2009)], the Jacobi elliptic function method [Inc and Erg¨ut (2005)], the exp-function method [He and Wu (2006)], the F-expansion method [Zhang (2006)], the sine-cosine method [Wazwaz (2004)]and so on.

As we know, over the past two decades or so, several expansion methods for finding travelling-wave solutions to nonlinear evolution equations have been proposed, developed and extended. Two of the basic methods are the tanh-function expansion method [Malfliet (1992)] and the basic (G'/G)-expansion method [Wang et al. (2008)]. One extension of the former is the tanh-coth method [Wazwaz (2007)], and one extension of the latter is the extended (G'/G)-expansion method [Zhu (2010)]. In the present paper these two extended methods are applied to the modified improved Kadomtsev-Petviashvili (MIKP) equation in the form [Taghizadeh and Mirzazadeh (2010)]

$$(u_t + u^2 u_x + a u_{xxx})_x + b u_{yy} = 0.$$

The outcomes of the two methods are compared.

2. The Two Methods

A PDE

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{xxx}, \ldots) = 0, \tag{1}$$

can be converted to an ODE

$$G(U, U', U'', U''', \ldots) = 0, \tag{2}$$

upon using a wave variable $u(x,t) = U(\xi)$, $\xi = x - ct$. If possible, integrating Equation (2) term by term one or more times yields constant(s) of integration. For simplicity the integration constant(s) can be set to zero .

2.1. The Tanh-Coth Method

This method introduces a new independent variable

$$Y = \tanh(\mu\xi), \tag{3}$$

that leads to the change of derivatives:

$$\frac{d}{d\xi} = \mu(1-Y^{2})\frac{d}{dY},$$

$$\frac{d^{2}}{d\xi^{2}} = -2\mu^{2}Y(1-Y^{2})\frac{d}{dY} + \mu^{2}(1-Y^{2})^{2}\frac{d^{2}}{dY^{2}}. \tag{4}$$

The tanh-coth method admits the use of the finite expansion

$$U(\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^{k} + \sum_{k=1}^{M} b_k Y^{-k}, \tag{5}$$

where M is a positive integer, in most cases, that will be determined. Expansion (5) reduces to the standard tanh method for $b_k = 0,\ (k = 1,...,M)$. Substituting (5) into the ODE (2) results in an algebraic equation in powers of Y.

To determine the parameter M, we usually balance the linear terms of the highest order in the resulting equation with the highest order nonlinear terms. We then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $a_k\,(k = 0,...,M)$, $b_k\,(k = 1,...,M)$, μ and c. Having determined these parameters we obtain an analytic solution $u(x,t)$ in a closed form.

2.2. The Extended (G'/G)-Expansion Method

Introduces the solution $U(\xi)$ of Equation (2) in the finite series form

$$U(\xi) = \sum_{i=0}^{M} \alpha_i \left(\frac{G'(\xi)}{G(\xi)}\right)^{i} + \sum_{i=1}^{M} \beta_i \left(\frac{G'(\xi)}{G(\xi)}\right)^{-i}, \tag{6}$$

where α_i , β_i are real constants to be determined, M is a positive integer to be determined, and the function $G(\xi)$ is the general solution of the auxiliary linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \gamma G(\xi) = 0, \tag{7}$$

where λ , γ are real constants to be determined.

Expansion (6) reduces to the (G'/G)-expansion method for $\beta_i = 0,\ (i = 1,...,M)$.

To determine the parameter M, we usually balance the linear terms of the highest order in the resulting equation with the highest order nonlinear terms.

Substituting (6) together with (7) into Equation (2) yields an algebraic equation involving powers of (G'/G). Equating the coefficients of each power of (G'/G) to zero gives a system of algebraic equations for α_i , β_i , λ , γ and c . Then, we solve the system with the aid of a computer algebra system, such as Maple, to determine these constants. On the other hand, depending on the sign of the discriminant $\Delta = \lambda^2 - 4\gamma$, the solutions of Equation (7) are well known to us. So, we can obtain exact solutions of equation (1).

3. Exact Solutions of MIKP Equation

3.1. Using the Tanh-Coth Method

We consider the MIKP equation

$$u_{tx} + 2uu_x^2 + u^2 u_{xx} + au_{xxxx} + bu_{yy} = 0, \tag{8}$$

using the variable $u(x,t) = U(\xi)$, $\xi = x + ly - \beta t$, transforms Equation (8) into the ODE

$$-\beta U'' + 2U(U')^2 + U^2 U'' + aU'''' + bl^2 U'' = 0. \tag{9}$$

Twice integrating Equation (9) and setting the constant of integrating to zero, we will have

$$(bl^2 - \beta)U + \frac{U^3}{3} + aU'' = 0, \ . \tag{10}$$

Balancing U'' with U^3 in Equation (10) gives

$$M + 2 = 3M ,$$

then $M = 1$. In this case, the tanh-coth method in the form (5) admits the use of the finite expansion

$$U(\xi) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{11}$$

Substituting the form (11) into Equation (10) and using (4), collecting the coefficients of Y we obtain:

Coefficients of Y^3 : $\frac{1}{3}a_1^3 + 2aa_1\mu^2$.

Coefficients of Y^2 : $a_0 a_1^2$.

Coefficients of Y^1: $a_0^2 a_1 + a_1^2 b_1 + (bl^2 - \beta)a_1 - 2aa_1\mu^2$.

Coefficients of Y^{0} : $\frac{1}{3}a_0^3 + 2a_0a_1b_1 + (bl^2 - \beta)a_0$.

Coefficients of Y^{-1} : $a_0^2b_1 + a_1b_1^2 + (bl^2 - \beta)b_1 - 2ab_1\mu^2$.

Coefficients of Y^{-2} : $a_0b_1^2$.

Coefficients of Y^{-3} : $\frac{1}{3}b_1^3 + 2ab_1\mu^2$.

Setting these coefficients equal to zero, and solving the resulting system, using Maple, we find the following sets of solutions:

The first set: (i)

$$a_0 = 0, \quad a_1 = \pm\sqrt{3(\beta - bl^2)}, \quad b_1 = 0, \quad \mu = \pm\sqrt{\frac{bl^2 - \beta}{2a}}. \tag{12}$$

The second set: (ii)

$$a_0 = 0, \quad a_1 = 0, \quad b_1 = \pm\sqrt{3(\beta - bl^2)}, \quad \mu = \pm\sqrt{\frac{bl^2 - \beta}{2a}}. \tag{13}$$

The third set: (iii)

$$a_0 = 0, \quad a_1 = \pm\frac{1}{2}\sqrt{3(\beta - bl^2)}, \quad b_1 = \pm\frac{1}{2}\sqrt{3(\beta - bl^2)}, \quad \mu = \pm\frac{1}{2}\sqrt{\frac{bl^2 - \beta}{2a}}. \tag{14}$$

For $bl^2 - \beta < 0$, $a < 0$ the sets (12)-(14) give the solitons solutions

$$u_1(x,t) = \pm\sqrt{3(\beta - bl^2)}\tanh[\sqrt{\frac{bl^2 - \beta}{2a}}(x + ly - \beta t)], \tag{15}$$

$$u_2(x,t) = \pm\sqrt{3(\beta - bl^2)}\coth[\sqrt{\frac{bl^2 - \beta}{2a}}(x + ly - \beta t)], \tag{16}$$

$$u_3(x,t) = \pm\frac{1}{2}\sqrt{3(\beta - bl^2)}\left(\tanh[\frac{1}{2}\sqrt{\frac{bl^2 - \beta}{2a}}(x + ly - \beta t)] + \coth[\frac{1}{2}\sqrt{\frac{bl^2 - \beta}{2a}}(x + ly - \beta t)]\right). \tag{17}$$

Note that, by use of the identity [Parkes (2010)]

$$\tanh(\frac{\theta}{2}) + \coth(\frac{\theta}{2}) = 2\coth(\theta),$$

(17) is identical to (16).

However, for $bl^2 - \beta > 0$, we obtain the travelling wave solutions

$$u_4(x,t) = \pm\sqrt{3(bl^2-\beta)}\tan[\sqrt{\frac{\beta - bl^2}{2a}}(x + ly - \beta t)], \tag{18}$$

$$u_5(x,t) = \pm\sqrt{3(bl^2-\beta)}\cot[\sqrt{\frac{\beta - bl^2}{2a}}(x + ly - \beta t)], \tag{19}$$

$$u_6(x,t) = \pm\frac{1}{2}\sqrt{3(bl^2-\beta)}\left(\tan[\frac{1}{2}\sqrt{\frac{\beta - bl^2}{2a}}(x + ly - \beta t)] - \cot[\frac{1}{2}\sqrt{\frac{\beta - bl^2}{2a}}(x + ly - \beta t)]\right). \tag{20}$$

Note that, by use of the identity [Parkes (2010)]

$$\cot(\frac{\theta}{2}) - \tan(\frac{\theta}{2}) = 2\cot(\theta),$$

(20) is identical to (19).

3.2. Using the Extended (G'/G)-Expansion Method

Recall that $M = 1$ as derived before. Using (6), the extended (G'/G)-expansion method admits the use of the finite expansion

$$U(\xi) = \alpha_0 + \alpha_1\left(\frac{G'}{G}\right) + \beta_1\left(\frac{G'}{G}\right)^{-1}. \tag{21}$$

Substituting (21) into (10), setting coefficients of $\left(G'/G\right)^i$ to zero, we obtain the following underdetermined system of algebraic equations for α_i, β_i, λ, γ and β:

$$\left(\frac{G'}{G}\right)^3 : \frac{1}{3}\alpha_1^3 + 2a\alpha_1 = 0,$$

$$\left(\frac{G'}{G}\right)^2 : \alpha_0\alpha_1^2 + 3a\lambda\alpha_1 = 0,$$

$$\left(\frac{G'}{G}\right)^1 : \alpha_0^2\alpha_1 + (b-\beta+a\lambda^2+2a\gamma)\alpha_1 + \alpha_1^2\beta_1 = 0,$$

$$\left(\frac{G'}{G}\right)^0 : \frac{1}{3}\alpha_0^3 + (b-\beta)\alpha_0 + 2\alpha_0\alpha_1\beta_1 + a\lambda\gamma\alpha_1 + a\lambda\beta_1 = 0,$$

$$\left(\frac{G'}{G}\right)^{-1} : \alpha_0^2\beta_1 + (b-\beta+a\lambda^2+2a\gamma)\beta_1 + \alpha_1\beta_1^2 = 0,$$

$$\left(\frac{G'}{G}\right)^{-2} : \alpha_0\beta_1^2 + 3a\lambda\gamma\beta_1 = 0,$$

$$\left(\frac{G'}{G}\right)^{-3} : \frac{1}{3}\beta_1^3 + 2a\gamma^2\beta_1 = 0.$$

Solving this system using Maple gives

Case 1:

$$\alpha_0 = \pm\frac{\lambda}{2}\sqrt{-6a}, \quad \alpha_1 = \pm\sqrt{-6a}, \quad \beta_1 = 0, \quad \beta = \frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2. \tag{22}$$

Case 2:

$$\alpha_0 = \pm\frac{\lambda}{2}\sqrt{-6a}, \quad \alpha_1 = 0, \quad \beta_1 = \pm\gamma\sqrt{-6a}, \quad \beta = \frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2. \tag{23}$$

Case 3:

$$\alpha_0 = \pm\frac{\lambda}{2}\sqrt{-6a}, \quad \alpha_1 = \pm\sqrt{-6a}, \quad \beta_1 = \pm\gamma\sqrt{-6a}, \quad \beta = \frac{-1}{2}a\lambda^2 + 8a\gamma + bl^2. \tag{24}$$

where λ and γ are arbitrary constants. Substituting Equation (22) into Equation (21) yields

$$U(\xi) = \pm\frac{\lambda}{2}\sqrt{-6a} \pm \sqrt{-6a}\left(\frac{G'}{G}\right), \tag{25}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2)t$.

Substituting general solutions of Equation (7) into Equation (25), we have three types of traveling wave solutions of the MIKP equation as follow.

When $\lambda^2 - 4\gamma > 0$

$$u_{11}(\xi) = \frac{1}{2}\sqrt{-6a(\lambda^2 - 4\gamma)}\left(\frac{A\,\sinh[\frac{1}{2}\sqrt{\lambda^2 - 4\gamma}\xi] + B\,\cosh[\frac{1}{2}\sqrt{\lambda^2 - 4\gamma}\xi]}{A\,\cosh[\frac{1}{2}\sqrt{\lambda^2 - 4\gamma}\xi] + B\,\sinh[\frac{1}{2}\sqrt{\lambda^2 - 4\gamma}\xi]}\right), \tag{26}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2)t$.

When $\lambda^2 - 4\gamma < 0$

$$u_{12}(\xi) = \frac{1}{2}\sqrt{-6a(4\gamma - \lambda^2)}\left(\frac{-A\,\sin[\frac{1}{2}\sqrt{4\gamma - \lambda^2}\xi] + B\,\cos[\frac{1}{2}\sqrt{4\gamma - \lambda^2}\xi]}{A\,\cos[\frac{1}{2}\sqrt{4\gamma - \lambda^2}\xi] + B\,\sin[\frac{1}{2}\sqrt{4\gamma - \lambda^2}\xi]}\right), \tag{27}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2)t$.

When $\lambda^2 - 4\gamma = 0$

$$u_{13}(\xi) = \pm\frac{\sqrt{-6a}B}{A + B\xi}, \tag{28}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2)t$.

In solutions (26) − (28), A and B are left as free parameters.

In particular, if $A \neq 0,\ B = 0$ and $\lambda = 0$, then u_{11} becomes

$$u_{11}(\xi) = \pm\sqrt{3(\beta - bl^2)}\tanh[\sqrt{\frac{bl^2-\beta}{2a}}(x+ly-\beta t)], \tag{29}$$

where $bl^2 - \beta < 0$, $a < 0$ and u_{12} becomes

$$u_{12}(\xi) = \pm\sqrt{3(bl^2 - \beta)}\tan[\sqrt{\frac{\beta - bl^2}{2a}}(x+ly-\beta t)], \tag{30}$$

where $bl^2 - \beta > 0$, $a < 0$.

The solutions (29) and (30) are same the solutions (15) and (18) respectively.

These special results show that the extended (G'/G)-expansion method obtains general solutions and it can be seen that the solutions obtained by using tanh-coth method, are special cases of these solutions. Our observations are in agreement with those in Biazar and Ayati (2011).

Substituting Equation (23) into Equation (21) yields

$$U(\xi) = \pm\frac{\lambda}{2}\sqrt{-6a} \pm \gamma\sqrt{-6a}\left(\frac{G'}{G}\right)^{-1}, \tag{31}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2)t$.

Substituting general solutions of Equation (7) into Equation (31), we have three types of traveling wave solutions of the MIKP equation as follow.

When $\lambda^2 - 4\gamma > 0$

$$u_{21}(\xi) = \pm\frac{\lambda}{2}\sqrt{-6a}$$
$$\pm\gamma\sqrt{-6a}\left(\frac{\sqrt{\lambda^2-4\gamma}}{2}\left(\frac{A\sinh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi] + B\cosh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi]}{A\cosh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi] + B\sinh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi]}\right) - \frac{\lambda}{2}\right)^{-1}, \tag{32}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 2a\gamma + bl^2)t$.

When $\lambda^2 - 4\gamma < 0$

$$u_{22}(\xi)=\pm\frac{\lambda}{2}\sqrt{-6a}$$
$$\pm\gamma\sqrt{-6a}\left(\frac{\sqrt{4\gamma-\lambda^2}}{2}\left(\frac{-A\sin[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]+B\cos[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]}{A\cos[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]+B\sin[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]}\right)-\frac{\lambda}{2}\right)^{-1}, \quad (33)$$

where $\xi=x+ly-(\frac{-1}{2}a\lambda^2+2a\gamma+bl^2)t$.

When $\lambda^2-4\gamma=0$

$$u_{23}(\xi)=\pm\frac{\lambda}{2}\sqrt{-6a}\pm\gamma\sqrt{-6a}\left(\frac{B}{A+B\xi}-\frac{\lambda}{2}\right)^{-1}, \quad (34)$$

where $\xi=x+ly-(\frac{-1}{2}a\lambda^2+2a\gamma+bl^2)t$.

In solutions (32) − (34), A and B are left as free parameters.

In particular, if $A\neq 0$, $B=0$ and $\lambda=0$, then u_{21} becomes

$$u_{21}(\xi)=\pm\sqrt{3(\beta-bl^2)}\coth[\sqrt{\frac{bl^2-\beta}{2a}}(x+ly-\beta t)], \quad (35)$$

where $bl^2-\beta<0$, $a<0$ and u_{22} becomes

$$u_{22}(\xi)=\pm\sqrt{3(bl^2-\beta)}\cot[\sqrt{\frac{\beta-bl^2}{2a}}(x+ly-\beta t)], \quad (36)$$

where $bl^2-\beta>0$, $a<0$.

The solutions (35) and (36) are same the solutions (16) and (19) respectively.

These special results show that the extended (G'/G)-expansion method obtains general solutions and it can be seen that the solutions obtained by using tanh-coth method, are special cases of these solutions. Our observations are in agreement with those in [Biazar and Ayati (2011)].

Substituting Equation (24) into Equation (21) yields

$$U(\xi) = \pm\frac{\lambda}{2}\sqrt{-6a} \pm \sqrt{-6a}\left(\frac{G'}{G}\right) \pm \gamma\sqrt{-6a}\left(\frac{G'}{G}\right)^{-1}, \tag{37}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 8a\gamma + bl^2)t$.

Substituting general solutions of Equation (7) into Equation (37), we have three types of traveling wave solutions of the MIKP equation as follow.

When $\lambda^2 - 4\gamma > 0$

$$\begin{aligned} u_{31}(\xi) &= \pm\frac{\lambda}{2}\sqrt{-6a} \\ &\pm\sqrt{-6a}\left(\frac{\sqrt{\lambda^2-4\gamma}}{2}\left(\frac{A\sinh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi] + B\cosh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi]}{A\cosh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi] + B\sinh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi]}\right) - \frac{\lambda}{2}\right) \\ &\pm\gamma\sqrt{-6a}\left(\frac{\sqrt{\lambda^2-4\gamma}}{2}\left(\frac{A\sinh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi] + B\cosh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi]}{A\cosh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi] + B\sinh[\frac{1}{2}\sqrt{\lambda^2-4\gamma}\xi]}\right) - \frac{\lambda}{2}\right)^{-1}, \end{aligned} \tag{38}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 8a\gamma + bl^2)t$.

When $\lambda^2 - 4\gamma < 0$

$$\begin{aligned} u_{32}(\xi) &= \pm\frac{\lambda}{2}\sqrt{-6a} \\ &\pm\sqrt{-6a}\left(\frac{\sqrt{4\gamma-\lambda^2}}{2}\left(\frac{-A\sin[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi] + B\cos[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]}{A\cos[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi] + B\sin[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]}\right) - \frac{\lambda}{2}\right) \\ &\pm\gamma\sqrt{-6a}\left(\frac{\sqrt{4\gamma-\lambda^2}}{2}\left(\frac{-A\sin[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi] + B\cos[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]}{A\cos[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi] + B\sin[\frac{1}{2}\sqrt{4\gamma-\lambda^2}\xi]}\right) - \frac{\lambda}{2}\right)^{-1}, \end{aligned} \tag{39}$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 8a\gamma + bl^2)t$.

When $\lambda^2 - 4\gamma = 0$

$$u_{33}(\xi) = \pm\frac{\lambda}{2}\sqrt{-6a} \pm \sqrt{-6a}\left(\frac{B}{A+B\xi} - \frac{\lambda}{2}\right) \pm \gamma\sqrt{-6a}\left(\frac{B}{A+B\xi} - \frac{\lambda}{2}\right)^{-1}, \quad (40)$$

where $\xi = x + ly - (\frac{-1}{2}a\lambda^2 + 8a\gamma + bl^2)t$.

In solutions (38) − (40), A and B are left as free parameters.

In particular, if $A \neq 0$, $B = 0$ and $\lambda = 0$, then u_{31} becomes

$$u_{31}(\xi) = \pm\frac{1}{2}\sqrt{3(\beta - bl^2)}\left(\tanh[\frac{1}{2}\sqrt{\frac{bl^2-\beta}{2a}}(x + ly - \beta t)] + \coth[\frac{1}{2}\sqrt{\frac{bl^2-\beta}{2a}}(x + ly - \beta t)]\right), \quad (41)$$

where $bl^2 - \beta < 0,\ a < 0$ and u_{32} becomes

$$u_{32}(\xi) = \pm\frac{1}{2}\sqrt{3(bl^2 - \beta)}\left(\tan[\frac{1}{2}\sqrt{\frac{\beta - bl^2}{2a}}(x + ly - \beta t)] - \cot[\frac{1}{2}\sqrt{\frac{\beta - bl^2}{2a}}(x + ly - \beta t)]\right), \quad (42)$$

where $bl^2 - \beta > 0,\ a < 0$.

The solutions (41) and (42) are same as the solutions (17) and (20) respectively. These special results show that the extended (G'/G)-expansion method obtains general solutions and it can be seen that the solutions obtained by using tanh-coth method, are special cases of these solutions. Our observations are in agreement with those in [Biazar and Ayati (2011)].

4. Conclusion

In this paper, the tanh-coth and the extended (G'/G)-expansion methods have been successfully applied to find the exact solutions for the modified improved Kadomtsev-Petviashvili equation.

The two methods are powerful and are applicable to many nonlinear evolution equation. Comparing the tanh-coth method with the extended (G'/G)-expansion method, shows that the two methods under special conditions are equivalent. In fact, the exact traveling wave solutions obtained by using the extended (G'/G)-expansion method are more general and the tanh-coth method is a special case of this method.

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