Applications and Applied Mathematics: An International Journal (AAM)

Volume 6 | Issue 1 Article 6

6-2011

Empirical Comparison of Some Test Statistics for Testing the Mean of a Poisson Distribution

B. M. Golam Kibria
Florida International University

Florence George
Florida International University

Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 6, Issue 1 (June 2011) pp. 74 – 89
(Previously, Vol. 6, Issue 11, pp. 1814 – 1829)

Applications and Applied Mathematics:
An International Journal
(AAM)

Empirical Comparison of Some Test Statistics for Testing the Mean of a Poisson Distribution

B. M. Golam Kibria and Florence George

Department of Mathematics and Statistics
Florida International University FIU
Modesto A. Maidique Campus
Miami, FL 33193, USA
kibriag@fiu.edu; fgeorge@fiu.edu

Received: September 1, 2010; Accepted: January 26, 2011

Abstract

This paper considers the problem of hypotheses testing of the mean of a Poisson distribution. Accordingly we consider the following test statistics: Wald, WCC, Score (S), FT, VS, RVS, Exact and Bayes test statistics. A simulation study based on both one and two sided alternatives has been conducted to compare the performances of the test statistics. The study suggests that for a large sample size, all proposed test statistics except VCC and FT perform well in the sense of correct type I error rate of the test and power. However, for a small sample size, Score and VS have better type I error rate and power properties than the other test statistics.

Keywords: Hypothesis Testing; Poisson Distribution; Power; Simulation

AMS 2010 No.: Primary 62F03, Secondary 62F40

1. Introduction

Many times problems are encountered in which the definitions of success and of failure are easily made, and even though it may be possible to count the number of successes, it may not be possible to count the number of failures. Examples are the number of defects in a finished automobile, the number of diseases in an individual, number of deaths per month by road accidents in a city etc. Poisson distribution is theoretically the one that should be used to model in these conditions of real life rare counting data (traffic accident, number of phone calls, number of patients visit the emergency room). Poisson distribution is used in a number of fields, ie, accident analysis, industry and epidemiology among others [Byrne and Kabaila (2005), Breiman (1962), Edwards et al. (1978), and Flowerdew and Aitkin (2006)]. Even, we might know that the data are from a Poisson distribution, it might be necessary to know the mean of the distribution. To make inference about the unknown mean of the Poisson distribution is an important problem. This can be done by either confidence interval or hypothesis testing.

A considerable literature exists to form the confidence intervals for a Poisson parameter [Agresti and Coull (1998), Anscombe (1948), Garwood (1936), Barker (2002), Byrne and Kabaila (2001, 2005), Casella and Robert (1989) and Freeman and Turkey (1950)]. However, the literature on the test statistics for testing the mean of a Poisson distribution is limited. This paper made an attempt to consider various test statistics for testing the mean of a Poisson distribution and compare them under the same simulation condition. Therefore, our objective is to find some good test statistics for testing the mean of a Poisson distribution based on type I error rate and power of the test. Since a theoretical comparison is not possible, a simulation study has been made to compare performances of the considered test statistics. The organization of the paper is as follows: In section 2, we reviewed various test statistics. A simulation study has been conducted in section 3. Finally, some concluding remarks are made in section 4.

2. Statistical Methodology

In this section we will discuss various test statistics for testing the mean of a Poisson distribution. Suppose $X_1, X_2,.....X_n$ be a *iid* random sample from a Poisson population with mean λ. Consider the following hypothesis:

$$\begin{aligned} &\textit{Null hypothesis}: H_0 : \lambda = \lambda_0 \\ &\textit{Alternative hypothesis}: H_a : \lambda = \lambda_0 \pm k \end{aligned} \tag{2.1}$$

and k is a positive constant. Here we are interested in a two tailed test. However, one can easily follow the same procedure for left or right tailed tests. When $k = 0$, we get type I error ratee of the test (α). When $k \neq 0$, we get powers $(1-\beta)$ of the test statistic. Our objective is to test against a proposed value of the parameter λ with a specific significance level. Since the references for all proposed test statistics are available, we briefly discussed them in this section.

1. Wald Method:

Historically, this is one of the first methods for testing a parameter [Laplace (1812), Barker (2002)] of a distribution. Wald method uses the asymptotic normality of the test statistic,

$$Z = \frac{(\overline{X} - \lambda_0)}{\sqrt{\overline{X}/n}},$$

where $\bar{X}$ denotes the sample mean. At α level of significance, the null hypothesis in (2.1) will be rejected, when

$$|z_{10}| = \left| \frac{(\bar{x} - \lambda_0)}{\sqrt{\bar{x}/n}} \right| > z_{\alpha/2},$$

where $z_{\alpha/2}$ is the upper critical value of a standard normal variate.

2. Wald with Continuity Correction(WCC):

Since Wald interval uses a continuous distribution (normal) to approximate a discrete distribution (Poisson), a continuity correction might make this approximation more accurate, Barker (2002). The Wald test with continuity correction is given by

$$Z = \frac{(\bar{X} - \lambda_0)}{\sqrt{(\bar{X} + 0.5)/n}}.$$

At α level of significance, the null hypothesis in (2.1) will be rejected, when

$$|z_{20}| = \left| \frac{(\bar{x} - \lambda_0)}{\sqrt{(\bar{x} + 0.5)/n}} \right| > z_{\alpha/2}.$$

3. Scores (S):

The Score method [Wilson (1927)] is on the basis of asymptotic normality of the test statistic $Z = \frac{(\bar{X} - \lambda)}{\sqrt{\lambda/n}}$. The score test will be rejected if $\left| \frac{(\bar{x} - \lambda_0)}{\sqrt{\lambda_0/n}} \right| > z_{\alpha/2}$.

4. Variance Stabilizing (VS):

The variance stabilizing transformation [Bartlett (1947)] for a Poisson distribution is the square root transformation. The stabilized variance of the transformed variable is approximately 0.25. Hence, the statistic $Z = \frac{\sqrt{\bar{X}} - \sqrt{\lambda}}{\sqrt{1/(4n)}}$ is asymptotically standard normal. The VS test will be rejected if $\left| \frac{\sqrt{\bar{x}} - \sqrt{\lambda_0}}{\sqrt{1/(4n)}} \right| > z_{\alpha/2}$.

5. Recentered Variance Stabilizing (RVS):

For any positive constant c, $\frac{\sqrt{\overline{X}+c}-\sqrt{\lambda+c}}{\sqrt{1/(4n)}}$ is asymptotically standard normal. Anscombe (1948) showed that, as λ tends to ∞, $var\sqrt{(\overline{X}+c)} = \frac{1+\frac{3-8c}{8\lambda}}{4n}+o(1/\lambda)$. If c is 3/8, the variance of $\sqrt{\overline{X}+0.5}$ is $\frac{1}{4n}+O(1/\theta)$. Hence, the statistic $Z=\frac{\sqrt{\overline{X}+c}-\sqrt{\lambda+c}}{\sqrt{(1/4n)}}$ is asymptotically normal. Thus, the VS test will be rejected if

$$\left|\frac{\sqrt{\overline{x}+3/8}-\sqrt{\lambda_0+3/8}}{\sqrt{(1/4n)}}\right| > z_{\alpha/2}.$$

6. Freeman and Turkey (FT):

Freeman and Turkey (1950) showed that $Z=\sqrt{n}([\sqrt{\overline{X}}+\sqrt{\overline{X}+1}]-[\sqrt{\lambda}+\sqrt{\lambda+1}])$ has an asymptotically standard normal distribution. Therefore, the FT test will be rejected is

$$\left|\sqrt{n}([\sqrt{\overline{x}}+\sqrt{\overline{x}+1}]-[\sqrt{\lambda_0}+\sqrt{\lambda_0+1}])\right| > z_{\alpha/2}.$$

7. Exact Method:

This method is based on an exact relationship between Poisson and Chi-square distribution [Garwood (1936), Agresti and Coull (1998)]. The exact method is designed to guarantee at least $100(1-\alpha)\%$ coverage. The lower and upper limits of exact intervals are

$$\frac{\chi^2_{\alpha/2,df_1}}{2n} \quad \text{and} \quad \frac{\chi^2_{1-alpha/2,df_2}}{2n},$$

respectively, where $df_1 = 2*\sum X$ and $df_2 = 2*\sum X+1$. Based on the confidence intervals, the empirical type I error rate and power of the test will be computed.

8. Bayes Approach:

The non-informative Jefferys prior plays a special role in the Bayesian analysis. The Jefferys prior for Poisson parameter λ is proportional to $\lambda^{-1/2}$ which is improper and the posterior is $Gamma(shape=\sum X+\frac{1}{2}, scale=\frac{1}{n}, rate=n)$ [Cai (2005)]. Hence, the lower limit and upper limit of confidence interval for λ are

$$G_{\alpha/2, shape=\sum X+0.5, rate=n} \quad \text{and} \quad G_{1-\alpha/2, shape=\sum X+0.5, rate=n},$$

respectively, where G represents the cdf of a gamma distribution. Based on the confidence intervals, the empirical type I error rate and power of the test will be computed.

3. A Simulation Study

The main objective of this paper is to identify some appropriate statistics for testing the mean of a Poisson distribution. Since, however a theoretical comparison is not possible, a simulation study is done here to compare the performances of the test statistics in the sense of empirical type I error rate of the test and power.

3.1. Simulation Technique

The plan of the simulation study is as follows:

(i) Sample sizes n = 5, 10, 15, 20, 30, 50 are used.

(ii) Random samples are generated from a Poisson distribution with the following pdf:

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}.$$

In each case, 2500 random samples are generated. The most common 5% level of significance ($\alpha = 0.05$) is used to compute the empirical power. We compare the performance of the test statistics based on empirical type I error rates and powers, which are calculated as the fraction of the rejections of the null hypothesis out of 2500 simulation replications. Empirical type I error rate and power of the statistics for testing the following two sided alternative are calculated by

$$H_0 : \lambda_a = \lambda_0 \quad vs \quad H_a : \lambda_a = \lambda_0 \pm k$$

with $k = 0.5, 1.0, 1.5$ and 2.0. We get type I error rate of the test (α) when $k = 0$, otherwise powers of the considered test statistics. For $\alpha = 0.05$, the empirical power and type I error rate of the test for n = 5, 10, 15, 20, 30 and 50 for $\lambda_0 = 2$ and $\lambda_0 = 5$ are presented in Tables 3.1 and 3.2, respectively. Since, Poisson is a right skewed distribution, it would be interesting to see the performance of the test statistics under one sided alternative. Therefore, the empirical type I error rate and power of the statistics for testing the following right tailed test are calculated by

$$H_0 : \lambda_a = \lambda_0 \quad vs \quad H_a : \lambda_a = \lambda_0 + k$$

with $k = 0.5, 1.0, 1.5, 2.0$ and 2.5. We get type I error rates of the test (α) when $k = 0$, otherwise powers of the considered test statistics. For $\alpha = 0.05$, the empirical power and type I error rate of the test are presented for n = 5, 10, 15, 20, 30 and 50 and $\lambda_0 = 2$ and $\lambda_0 = 5$ in Table 3.3 and 3.4 respectively. We note that the power for exact and Bayes methods are computed based on the confidence intervals, which are calculated as the fraction of the rejections of the null hypothesis out of 2500 simulated confidence intervals.

The simulation has been done by R, the online free version of software. More on simulation procedures we refer Shi and Kibria (2007), Shipra and Kibria (2010a,b) among others. Since a graph provides better facility to make a comparison, we also produce graph for λ vs power for two tailed test for testing $\lambda = 5$ and presented them in Figures 3.1 to 3.6 for n=5, 10, 15, 20, 30 and 50, respectively. Similarly, for testing $\lambda_0 = 2$, we presented them in Figures 3.7 to 3.12 for *n*=5, 10, 15, 20, 30, 50 respectively. For one sided test and testing for $\lambda_0 = 2$, the λ vs power for *n*=5,10,15,20,30 and 50 are presented in Figures 3.13 to 3.18, respectively.

3.2. Results Discussion

From Table 3.1 to 3.4, and Figures 3.1 to 3.18, we observed that as n increases the power of the test increases and approaches to 1 and type I error rate of the test gets closer to nominal type I error rate 0.05. We note that for large n, all considered test statistics except FT and VCC performing well in the sense of accurate nominal type I error rate and power of the test. However, the empirical nominal type I error rates differ from actual type I error rate 0.05 for small sample. The type I error rates of RVS and FT are found under the nominal level for small sample sizes. For all sample sizes, the test statistics, Wald, Score (S) and VS performing well. Where as, RVS, exact and Bayes tests are doing well for large sample sizes. Overall score test is the best. For different sample sizes, the λ vs power of the tests are plotted in Figure 3.1 to 3.6. These graphs also supported the above discussion.

4. Some Concluding Remarks

In this paper we have considered some test statistics for testing the mean of a Poisson distribution. Since a theoretical comparison is not possible, a simulation study has been made to compare the performance of the test statistics. Our simulation indicates that for a small sample size, Wald, Score (S) and VS have better type I error rate and power properties than the other test statistics. We also observed that RVS, Exact and Bayes perform well in the sense of correct type I error rate of the test and high power for large sample size. Overall the Score test is the best preferred due to the correct type I error rate of the test and high power and its ease to computation.

Acknowledgments

Authors are thankful to Prof. A. M. Haghighi, the Editor-in-Chief and the referees for their valuable and constructive comments/suggesstions which certainly improved the quality and presentation of the paper. This paper was written while the first author was on sabbatical leave (2010-2011). He is grateful to Florida International University for awarding him the sabbatical leave that provided the opportunity and excellent research facilities.

Table 3.1. Empirical type I error rate and power of tests for $\alpha = 0.05$, $n = 5,10,15$ and $\lambda_0 = 2$ (Two tailed test)

	λ_a						
Tests	0.5	1.0	1.5	2.0	2.5	3.0	3.5
			n=5				
Wald	0.952	0.622	0.243	0.081	0.069	0.179	0.391
WCC	0.886	0.454	0.136	0.036	0.028	0.124	0.300
S	0.748	0.278	0.070	0.036	0.131	0.338	0.574
FT	0.886	0.454	0.136	0.046	0.086	0.251	0.474
VS	0.886	0.454	0.136	0.046	0.086	0.251	0.474
RVS	0.886	0.454	0.136	0.046	0.086	0.251	0.474
Exact	0.748	0.278	0.070	0.024	0.084	0.250	0.474
Bayes	0.886	0.454	0.137	0.058	0.133	0.339	0.574
			n=10				
Wald	0.999	0.788	0.277	0.047	0.146	0.449	0.766
WCC	0.994	0.684	0.193	0.027	0.107	0.387	0.711
S	0.994	0.684	0.193	0.048	0.241	0.592	0.865
FT	0.994	0.684	0.193	0.032	0.145	0.449	0.766
VS	0.999	0.788	0.277	0.054	0.194	0.528	0.816
RVS	0.994	0.684	0.193	0.032	0.145	0.449	0.766
Exact	0.994	0.684	0.193	0.039	0.192	0.528	0.816
Bayes	0.994	0.684	0.193	0.039	0.192	0.528	0.816
			n=15				
Wald	1.000	0.950	0.452	0.068	0.200	0.643	0.920
WCC	1.000	0.881	0.295	0.028	0.130	0.519	0.873
S	1.000	0.881	0.295	0.057	0.295	0.749	0.952
FT	1.000	0.881	0.295	0.040	0.198	0.643	0.920
VS	1.000	0.922	0.373	0.060	0.244	0.699	0.938
RVS	1.000	0.881	0.295	0.040	0.198	0.643	0.920
Exact	1.000	0.881	0.295	0.047	0.244	0.699	0.938
Bayes	1.000	0.881	0.295	0.047	0.244	0.699	0.938

Note: Column with $\lambda_a = 2.0$ gives the empirical type I error rate of the test.

Table 3.1. continued for $n = 20,30,50$ and $\lambda_0 = 2$ (Two tailed test)

	λ_a						
Tests	0.5	1.0	1.5	2.0	2.5	3.0	3.5
				n=20			
Wald	1.000	0.980	0.478	0.061	0.264	0.774	0.974
WCC	1.000	0.948	0.333	0.032	0.224	0.727	0.966
S	1.000	0.948	0.333	0.053	0.354	0.846	0.988
FT	1.000	0.948	0.333	0.038	0.263	0.774	0.974
VS	1.000	0.965	0.398	0.056	0.307	0.812	0.980
RVS	1.000	0.948	0.333	0.038	0.263	0.774	0.974
Exact	1.000	0.948	0.333	0.045	0.307	0.812	0.980
Bayes	1.000	0.965	0.398	0.064	0.355	0.846	0.988
				n=30			
Wald	1.000	0.998	0.605	0.060	0.361	0.905	0.999
WCC	1.000	0.993	0.491	0.029	0.287	0.869	0.996
S	1.000	0.993	0.491	0.046	0.441	0.931	1.000
FT	1.000	0.993	0.491	0.037	0.361	0.905	0.999
VS	1.000	0.996	0.551	0.053	0.402	0.920	1.000
RVS	1.000	0.993	0.491	0.037	0.361	0.905	0.999
Exact	1.000	0.993	0.491	0.041	0.402	0.920	1.000
Bayes	1.000	0.996	0.551	0.058	0.441	0.931	1.000
				n=50			
Wald	1.000	1.000	0.825	0.050	0.623	0.992	1.000
WCC	1.000	1.000	0.720	0.025	0.551	0.984	1.000
S	1.000	1.000	0.759	0.046	0.692	0.996	1.000
FT	1.000	1.000	0.720	0.031	0.589	0.989	1.000
VS	1.000	1.000	0.791	0.047	0.660	0.993	1.000
RVS	1.000	1.000	0.720	0.031	0.589	0.989	1.000
Exact	1.000	1.000	0.759	0.042	0.660	0.993	1.000
Bayes	1.000	1.000	0.759	0.042	0.660	0.993	1.000

Note: Column with $\lambda_a = 2.0$ gives the empirical type I error rate of the test.

Table 3.2. Empirical type I error rate and power of tests for $\alpha = 0.05$, $n = 5,10,15$ and $\lambda_0 = 5$ (Two tailed test)

	λ_a								
Tests	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
					n=5				
Wald	0.653	0.431	0.207	0.082	0.050	0.066	0.125	0.239	0.395
WCC	0.653	0.431	0.207	0.081	0.047	0.053	0.097	0.187	0.333
S	0.551	0.324	0.148	0.063	0.058	0.110	0.210	0.369	0.526
FT	0.551	0.324	0.146	0.056	0.033	0.062	0.125	0.237	0.395
VS	0.653	0.431	0.207	0.085	0.060	0.090	0.163	0.302	0.458
RVS	0.551	0.324	0.146	0.056	0.033	0.062	0.125	0.237	0.395
Exact	0.551	0.324	0.146	0.058	0.042	0.087	0.162	0.300	0.458
Bayes	0.551	0.324	0.146	0.058	0.042	0.087	0.162	0.300	0.458
					n=10				
Wald	0.912	0.661	0.365	0.133	0.048	0.087	0.227	0.471	0.692
WCC	0.912	0.661	0.365	0.132	0.046	0.070	0.200	0.422	0.651
S	0.875	0.603	0.306	0.104	0.047	0.125	0.308	0.570	0.779
FT	0.875	0.603	0.306	0.102	0.036	0.084	0.227	0.471	0.692
VS	0.912	0.661	0.365	0.134	0.052	0.109	0.267	0.520	0.741
RVS	0.875	0.603	0.306	0.102	0.036	0.084	0.227	0.471	0.692
Exact	0.875	0.603	0.306	0.103	0.040	0.106	0.267	0.520	0.741
Bayes	0.875	0.603	0.306	0.103	0.040	0.106	0.267	0.520	0.741
					n=15				
Wald	0.985	0.839	0.483	0.157	0.053	0.096	0.296	0.613	0.845
WCC	0.985	0.839	0.483	0.157	0.053	0.096	0.296	0.613	0.845
S	0.973	0.803	0.436	0.123	0.060	0.170	0.419	0.728	0.903
FT	0.973	0.803	0.436	0.123	0.048	0.114	0.337	0.655	0.873
VS	0.973	0.803	0.436	0.123	0.055	0.141	0.375	0.690	0.887
RVS	0.973	0.803	0.436	0.123	0.048	0.114	0.337	0.655	0.873
Exact	0.973	0.803	0.436	0.123	0.055	0.141	0.375	0.690	0.887
Bayes	0.973	0.803	0.436	0.123	0.055	0.141	0.375	0.690	0.887

Note: Column with $\lambda_a = 5.0$ gives the empirical type I error rate of the test.

Table 3.2. continued for n =25, 30, 50 and $\lambda_0 = 5$
(Two tailed test)

	λ_a								
Tests	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
					n=25				
Wald	0.999	0.933	0.615	0.201	0.052	0.128	0.438	0.753	0.946
WCC	0.997	0.924	0.571	0.173	0.041	0.109	0.398	0.718	0.938
S	0.995	0.904	0.532	0.146	0.050	0.177	0.511	0.806	0.961
FT	0.995	0.904	0.532	0.146	0.039	0.127	0.438	0.753	0.946
VS	0.997	0.924	0.571	0.173	0.048	0.151	0.473	0.779	0.955
RVS	0.995	0.904	0.532	0.146	0.039	0.127	0.438	0.753	0.946
Exact	0.995	0.904	0.532	0.146	0.041	0.151	0.472	0.779	0.955
Bayes	0.995	0.904	0.532	0.146	0.041	0.151	0.472	0.779	0.955
					n=30				
Wald	1.000	0.982	0.752	0.249	0.045	0.187	0.608	0.909	0.993
WCC	1.000	0.980	0.725	0.223	0.037	0.166	0.582	0.894	0.992
S	1.000	0.975	0.699	0.202	0.046	0.232	0.663	0.934	0.995
FT	1.000	0.975	0.699	0.202	0.037	0.187	0.608	0.909	0.993
VS	1.000	0.980	0.725	0.223	0.049	0.232	0.663	0.934	0.995
RVS	1.000	0.980	0.725	0.223	0.044	0.211	0.635	0.923	0.994
Exact	1.000	0.975	0.699	0.202	0.046	0.232	0.663	0.934	0.995
Bayes	1.000	0.980	0.725	0.223	0.049	0.232	0.663	0.934	0.995
					n=50				
Wald	1.000	1.000	0.923	0.411	0.052	0.315	0.848	0.991	1.000
WCC	1.000	1.000	0.913	0.385	0.042	0.263	0.820	0.987	1.000
S	1.000	1.000	0.913	0.385	0.055	0.356	0.873	0.993	1.000
FT	1.000	1.000	0.898	0.364	0.041	0.288	0.835	0.989	1.000
VS	1.000	1.000	0.913	0.385	0.052	0.335	0.861	0.993	1.000
RVS	1.000	1.000	0.898	0.364	0.041	0.288	0.835	0.989	1.000
Exact	1.000	1.000	0.898	0.364	0.049	0.335	0.861	0.993	1.000
Bayes	1.000	1.000	0.913	0.385	0.052	0.335	0.861	0.993	1.000

Note: Column with $\lambda_a = 5.0$ gives the empirical type I error rate of the test.

Table 3.3. Empirical type I error rate and power of tests for $\alpha = 0.05$, $n = 5,10,15,50$ and $\lambda_0 = 2$ (Right tailed test)

	λ_a					
Tests	2.0	2.5	3.0	3.5	4.0	4.5
			n=5			
Wald	0.0285	0.1310	0.3495	0.5625	0.7615	0.9040
WCC	0.0155	0.0865	0.2725	0.4620	0.6985	0.8545
S	0.0570	0.1935	0.4395	0.6565	0.8390	0.9390
FT	0.0285	0.1310	0.3495	0.5625	0.7615	0.9040
VS	0.0570	0.1935	0.4395	0.6565	0.8390	0.9390
RVS	0.0285	0.1310	0.3495	0.5625	0.7615	0.9040
Exact	0.0155	0.0865	0.2725	0.4620	0.6985	0.8545
Bayes	0.0285	0.1310	0.3495	0.5625	0.7615	0.9040
			n=10			
Wald	0.0400	0.2305	0.6025	0.8600	0.9695	0.9960
WCC	0.0265	0.1800	0.5260	0.8150	0.9580	0.9950
S	0.0585	0.2970	0.6740	0.8975	0.9835	0.9975
FT	0.0400	0.2305	0.6025	0.8600	0.9695	0.9960
VS	0.0400	0.2305	0.6025	0.8600	0.9695	0.9960
RVS	0.0400	0.2305	0.6025	0.8600	0.9695	0.9960
Exact	0.0265	0.1800	0.5260	0.8150	0.9580	0.9950
Bayes	0.0265	0.1800	0.5260	0.8150	0.9580	0.9950
			n=15			
Wald	0.0345	0.2960	0.7495	0.9535	0.9975	1.0000
WCC	0.0255	0.2425	0.6980	0.9415	0.9960	1.0000
S	0.0455	0.3575	0.7960	0.9675	0.9985	1.0000
FT	0.0345	0.2960	0.7495	0.9535	0.9975	1.0000
VS	0.0455	0.3575	0.7960	0.9675	0.9985	1.0000
RVS	0.0345	0.2960	0.7495	0.9535	0.9975	1.0000
Exact	0.0255	0.2425	0.6980	0.9415	0.9960	1.0000
Bayes	0.0255	0.2425	0.6980	0.9415	0.9960	1.0000
			n=50			
Wald	0.0500	0.7530	0.9970	1.0000	1.0000	1.0000
WCC	0.0290	0.6920	0.9950	1.0000	1.0000	1.0000
S	0.0610	0.7830	0.9975	1.0000	1.0000	1.0000
FT	0.0375	0.7235	0.9965	1.0000	1.0000	1.0000
VS	0.0500	0.7530	0.9970	1.0000	1.0000	1.0000
RVS	0.0375	0.7235	0.9965	1.0000	1.0000	1.0000
Exact	0.0245	0.6635	0.9930	1.0000	1.0000	1.0000
Bayes	0.0245	0.6635	0.9930	1.0000	1.0000	1.0000

Note: Column with $\lambda_a = 2.0$ gives the empirical type I error rate of the test.

Table 3.4. Empirical type I error rate and power of tests for $\alpha = 0.05$, $n = 5,10,20,50$ and $\lambda_0 = 5$ (Right tailed test)

	λ_a						
	5.0	5.5	6.0	6.5	7.0	7.5	8
				n=5			
Wald	0.0260	0.0960	0.2045	0.3340	0.5370	0.6970	0.8065
WCC	0.0145	0.0710	0.1525	0.2770	0.4700	0.6340	0.7580
S	0.0445	0.1290	0.2550	0.3985	0.6015	0.7490	0.8520
FT	0.0260	0.0960	0.2045	0.3340	0.5370	0.6970	0.8065
VS	0.0445	0.1290	0.2550	0.3985	0.6015	0.7490	0.8520
RVS	0.0260	0.0960	0.2045	0.3340	0.5370	0.6970	0.8065
Exact	0.0145	0.0710	0.1525	0.2770	0.4700	0.6340	0.7580
Bayes	0.0145	0.0710	0.1525	0.2770	0.4700	0.6340	0.7580
				n=10			
Wald	0.0315	0.1345	0.3230	0.5755	0.7900	0.9215	0.9690
WCC	0.0315	0.1345	0.3230	0.5755	0.7900	0.9215	0.9690
S	0.0520	0.1960	0.4190	0.6805	0.8535	0.9515	0.9810
FT	0.0400	0.1615	0.3670	0.6350	0.8210	0.9350	0.9745
VS	0.0400	0.1615	0.3670	0.6350	0.8210	0.9350	0.9745
RVS	0.0400	0.1615	0.3670	0.6350	0.8210	0.9350	0.9745
Exact	0.0225	0.1130	0.2830	0.5285	0.7550	0.8975	0.9625
Bayes	0.0225	0.1130	0.2830	0.5285	0.7550	0.8975	0.9625
				n=20			
Wald	0.0490	0.2285	0.5820	0.8720	0.9800	0.9945	0.9995
WCC	0.0375	0.2010	0.5500	0.8495	0.9735	0.9930	0.9995
S	0.0600	0.2600	0.6150	0.8875	0.9850	0.9980	0.9995
FT	0.0490	0.2285	0.5820	0.8720	0.9800	0.9945	0.9995
VS	0.0490	0.2285	0.5820	0.8720	0.9800	0.9945	0.9995
RVS	0.0490	0.2285	0.5820	0.8720	0.9800	0.9945	0.9995
Exact	0.0265	0.1490	0.4800	0.8025	0.9620	0.9900	0.9990
Bayes	0.0265	0.1490	0.4800	0.8025	0.9620	0.9900	0.9990
				n=50			
Wald	0.0415	0.4095	0.9125	0.9970	1.0000	1.0000	1.0000
WCC	0.0375	0.3920	0.9040	0.9970	1.0000	1.0000	1.0000
S	0.0465	0.4350	0.9180	0.9970	1.0000	1.0000	1.0000
FT	0.0415	0.4095	0.9125	0.9970	1.0000	1.0000	1.0000
VS	0.0465	0.4350	0.9180	0.9970	1.0000	1.0000	1.0000
RVS	0.0415	0.4095	0.9125	0.9970	1.0000	1.0000	1.0000
Exact	0.0230	0.3210	0.8675	0.9955	1.0000	1.0000	1.0000
Bayes	0.0230	0.3210	0.8675	0.9955	1.0000	1.0000	1.0000

Note: Column with $\lambda_a = 5.0$ gives the empirical type I error rate of the test.

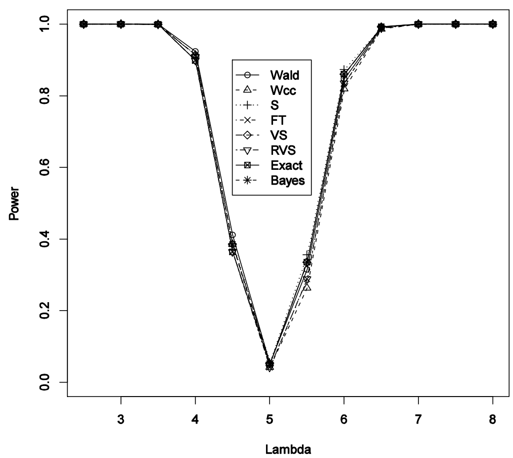

Figure 3.1. Power vs λ, n=5,

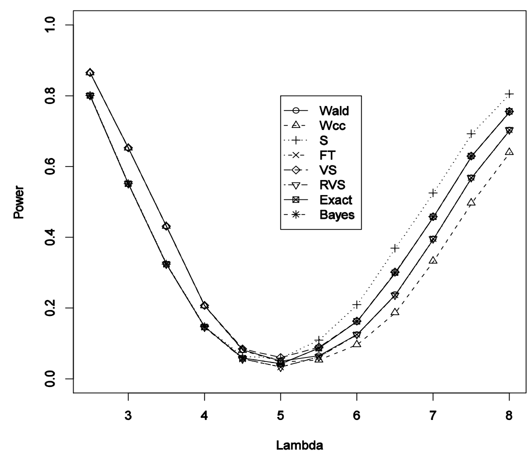

Figure 3.2. Power vs λ, n=10

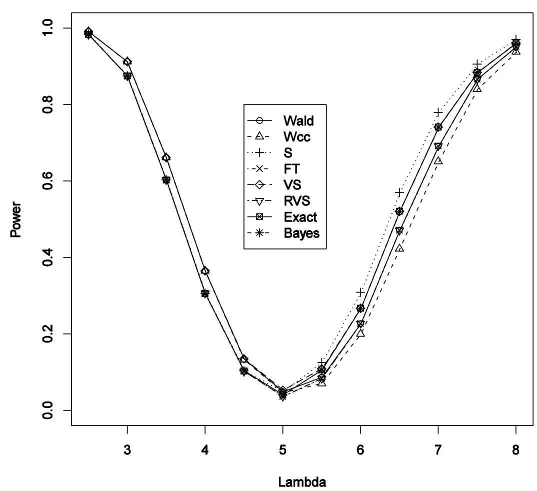

Figure 3.3. Power vs λ, n=15,

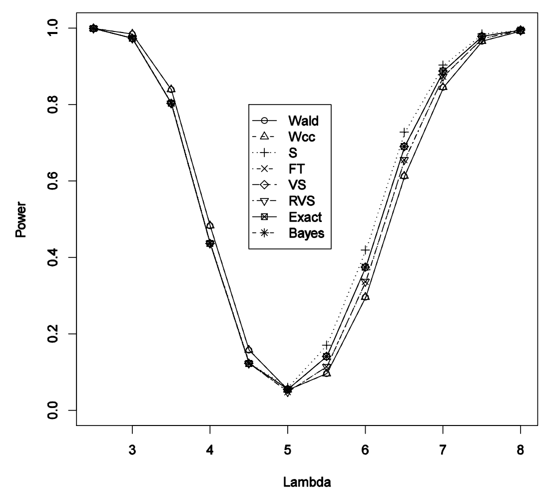

Figure 3.4. Power vs λ, n=20

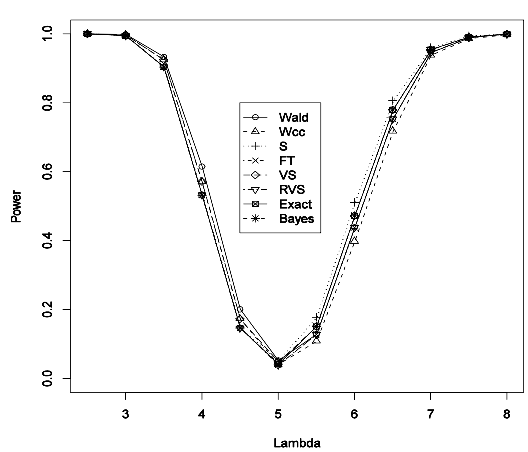

Figure 3.5. Power vs λ, n=30

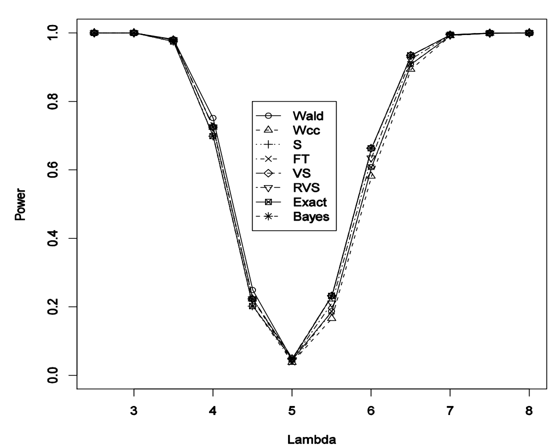

Figure 3.6. Power vs λ, n=50

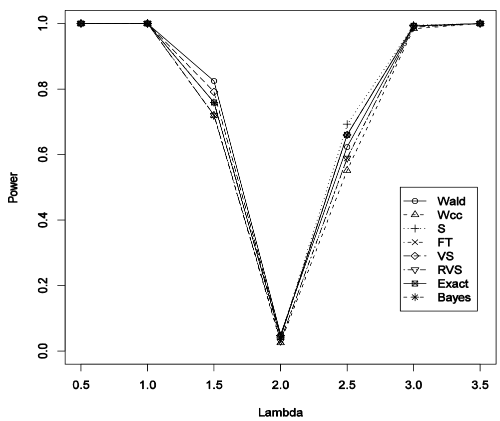

Figure 3.7. Power vs λ, n=5

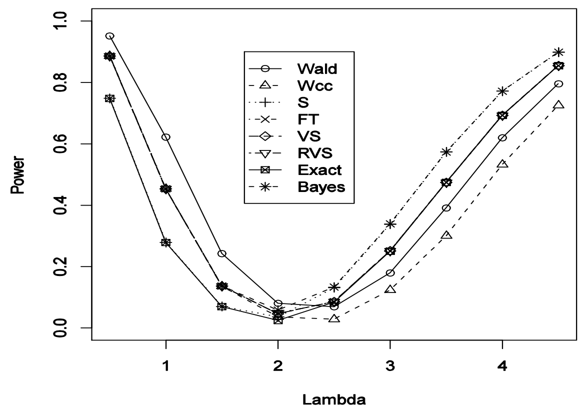

Figure 3.8. Power vs λ, n=10

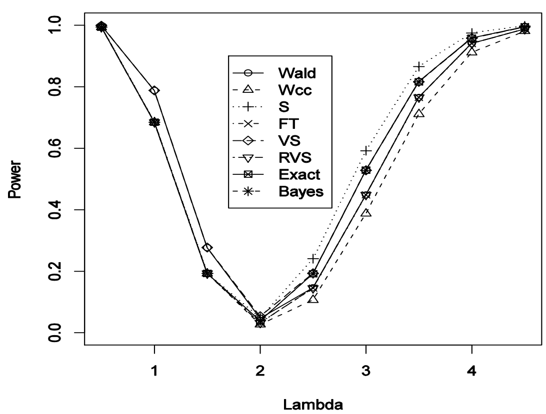

Figure 3.9. Power vs λ, n=15

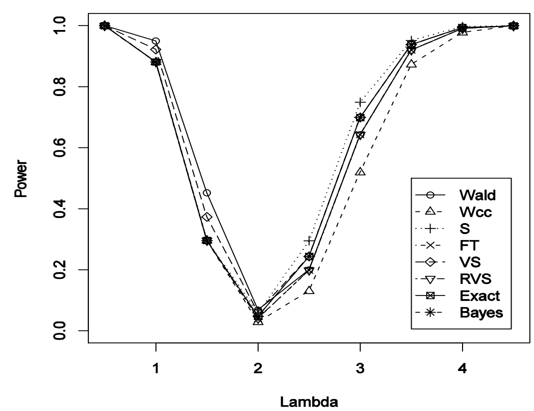

Figure 3.10. Power vs λ, n=20

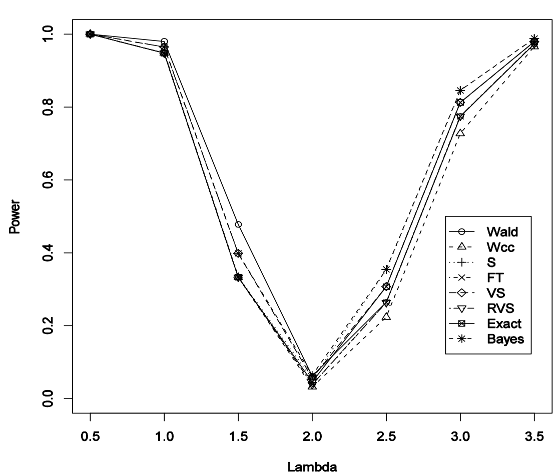

Figure 3.11. Power vs λ, n=30

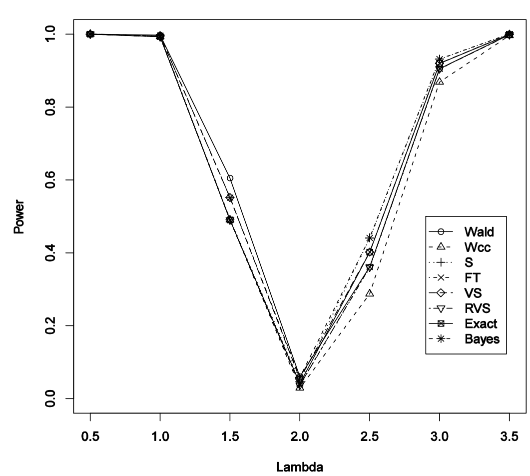

Figure 3.12. Power vs λ, n=50

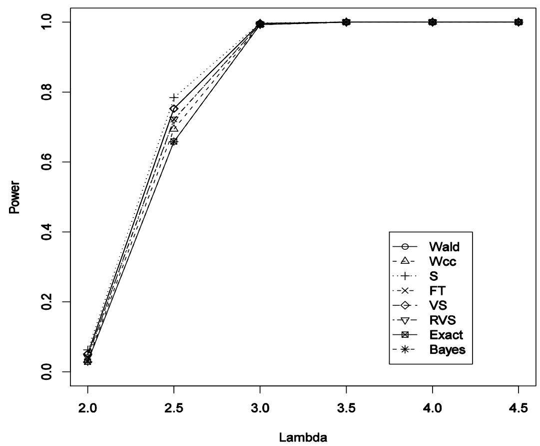

Figure 3.13. Power vs λ, n=5

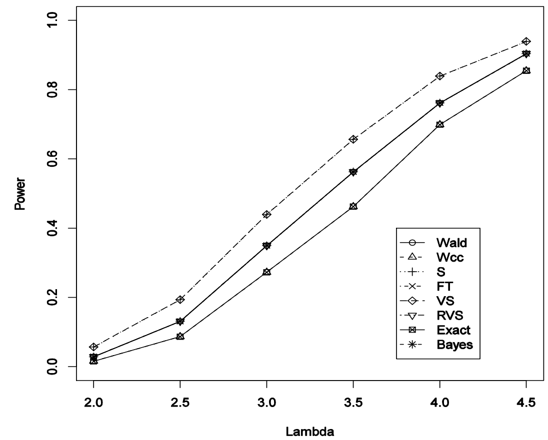

Figure 3.14. Power vs λ, n=10

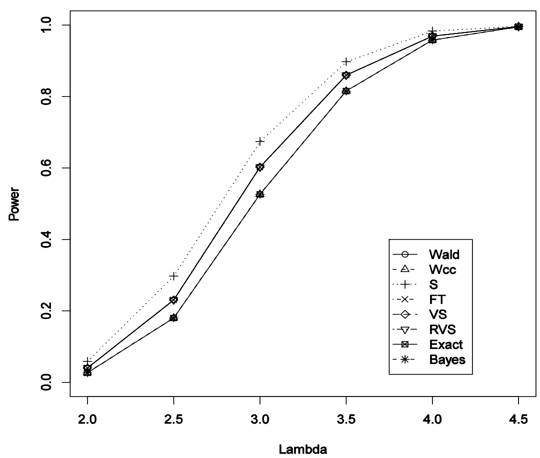

Figure 3.15. Power vs λ, n=15

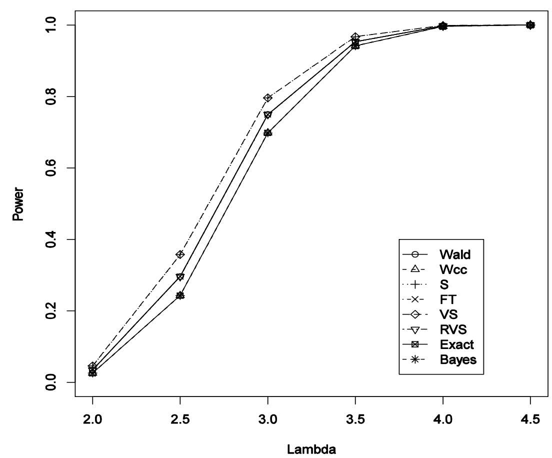

Figure 3.16. Power vs λ, n=20

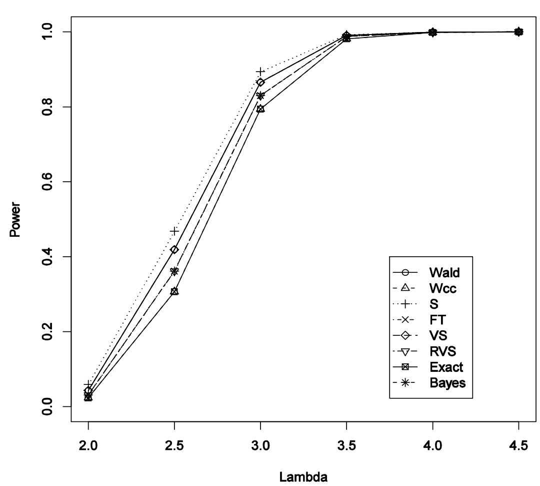

Figure 3.17. Power vs λ, n=30

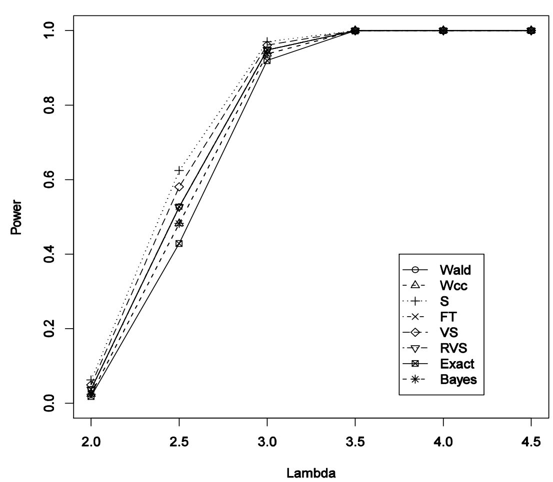

Figure 3.18. Power vs λ, n=50

REFERENCES

Agresti, A. and Coull, B. A. (1998). Approximate is better than Exact for Interval estimation of Binomial proportions, The American Statistician, Vol. 52, No. 2, pp. 119-126.

Anscombe, F. J. (1948). The transformation of Poisson, Binomial and negative Binomial Data, Biometrika Vol. 35, pp. 246-254.

Baklizi, A. and Kibria, B. M. G. (2009). One and Two Sample Confidence Intervals for estimating the mean of Skewed Populations: An Empirical Comparative Study, Journal of Applied Statistics, Vol. 36, pp. 601-609.

Barker, L. (2002). A comparison of Nine Confidence Intervals for a Poisson parameter when the Expected number of events is ≤ 5, The American Statistician, Vol. 56, No. 2, pp85-89.

Bartlett, M.S. (1947). The Use of Transformations, Biometrics, Vol. 13, pp. 39–52.

Breiman, L. (1962). The Poisson tendency in traffic distribution, The Annals of Mathematical Statistics, Vol. 34, pp. 308-311.

Byrne, J. and Kabaila, P. (2001). Short exact confidence confidence intervals for the Pisson mean, Communications in Statistics- Theory and Methods, Vol 30, pp.257-261.

Byrne, J. and Kabaila, P. (2005). Comparison of Poisson confidence intervals, Communications in Statistics- Theory and Methods, Vol 34, pp. 545-556.

Cai, T. T. (2005). One-sided confidence intervals in discrete distributions, Journal of Statistical planning and inference, Vol. 131, No. 1, pp. 63-68.

Casella, G and Robert, C. (1989). Refining Poisson onfidence intervals, The Canadian Journal of Statistics, Vol. 17, pp. 45-57.

Edwards, A.A., Lloyd, D.C. and Purrott, R.J. (1979). Radiation induced chromosome abberrations and the Poisson distribution, Radiation and Environmental Biophysics, Vol. 16, pp. 89-100.

Flowerdew, R. and Aitkin, M. (2006). A method of fitting the gravity model based on the Poisson distribution. Journal of Regional Science, Vol 22, pp. 191-202.

Freeman, M. F., and Tukey, J. W. (1950). Transformations Related to the Angular and the Square Root, Annals of Mathematical Statistics Vol. 21, pp. 607–611.

Garwood, F. (1936). Fiducial Limits for the Poisson Distribution, Biometrika Vol. 28, pp. 437-442.

Hoque, M. M. (2003). *Injuries from road traffic accidents: A serious health threat to the children*. Proceedings published on the World Health Day.

Laplace, P.S. (1812). *Theoretic analytique des probabilities*, Paris, France, Courier.

Shi, W. and Kibria, B. M. G. (2007). On some confidence intervals for estimating the mean of a skewed population, International Journal of Mathematical Education in Science and Technology, Vol. 38, No. 3, pp. 412--421.

Shipra, B. and Kibria, B. M. G. (2010a). Comparison of Some Test Statistics for Testing the Mean of a Right Skewed Distribution, Journal of Statistical Theory and Applications, Vol. 9, No. 1, pp. 77-90.

Shipra, B. and Kibria, B. M. G. (2010b). Comparison of Some Parametric and Nonparametric Type One Sample Confidence Intervals for Estimating the Mean of a Positively Skewed Distribution, Communications in Statistics-Simulation and Computation Vol. 39, pp. 361–389.

Wilson, E.B. (1927). Probable inference, the law of succession, and statistical inference, Journal of the Americal Statistical Association, Vol. 22, pp. 209-212.