



6-2011

## Small Reynolds Number Steady Flow Past a Sphere with a Source at its Centre

Sunil Datta  
*Lucknow University*

Shanu Singhal  
*Lucknow University*

Follow this and additional works at: <https://digitalcommons.pvamu.edu/aam>



Part of the [Fluid Dynamics Commons](#)

### Recommended Citation

Datta, Sunil and Singhal, Shanu (2011). Small Reynolds Number Steady Flow Past a Sphere with a Source at its Centre, *Applications and Applied Mathematics: An International Journal (AAM)*, Vol. 6, Iss. 1, Article 16.

Available at: <https://digitalcommons.pvamu.edu/aam/vol6/iss1/16>

This Article is brought to you for free and open access by Digital Commons @PVAMU. It has been accepted for inclusion in *Applications and Applied Mathematics: An International Journal (AAM)* by an authorized editor of Digital Commons @PVAMU. For more information, please contact [hvkoshy@pvamu.edu](mailto:hvkoshy@pvamu.edu).



Available at  
<http://pvamu.edu/aam>  
Appl. Appl. Math.  
ISSN: 1932-9466

Applications and Applied  
Mathematics:  
An International Journal  
(AAM)

Vol. 6, Issue 1 (June 2011) pp. 201 – 210  
(Previously, Vol. 6, Issue 11, pp. 1942 – 1951)

---

## Small Reynolds Number Steady Flow Past a Sphere with a Source at its Centre

Sunil Datta and Shanu Singhal

Department of Mathematics and Astronomy  
Lucknow University  
Lucknow 226007, India  
[sv.datta@gmail.com](mailto:sv.datta@gmail.com),  
[shanu\\_singhal04@rediffmail.com](mailto:shanu_singhal04@rediffmail.com)

Received: July 13, 2010; Accepted: February 7, 2011

### Abstract

In this paper we study the steady flow, at low Reynolds number  $R$ , past a pervious sphere with a source at its centre. Assuming the source strength  $s$  also to be small, we use the method of matched asymptotic expansions to generate the inner and outer solutions pertaining to the problem. Graphs depicting the streamlines have been drawn to exhibit the effects of  $s$  and  $R$ . Also, the expression of the calculated drag experienced by the sphere shows that the effect of a source ( $s > 0$ ) is to decrease the drag.

**Keywords:** Reynolds number, source, Stream function, Stokes and Oseen expansions, Drag

**MSC 2010:** 76D05, 76D07

### 1. Introduction

Denoting by  $U, a, \rho_a$  and  $\nu$ , respectively, a characteristic velocity, a characteristic length, fluid density and coefficient of kinematic viscosity, we have Reynolds number  $R = Ua/\nu$ . As Reynolds number is small when  $U$  is small, the flow is also known as slow viscous flow. But it

may be remembered that  $R$  will also be small when characteristic length  $a$  is small or when kinematic viscosity  $\nu$  is large. In recent times the study of slow flow phenomena has come into prominence, particularly in the fields of biomechanics, microfluidics and chemical engineering for studying important and useful problems such as locomotion of micro-organisms, movement of mucus in lung, motion of blood cells, flow of dusty gas, smoke dyes and electrophoresis and Brownian movement. Amongst a host of recent papers, we may cite those of Ayoubi and Longuski (2008) Bandyopadhyay (2006), Chadwick (2002), Charalambopoulos and Dassios (2002), Felderhof (2005), Pozrikidis and Farrow (2003), Ranger (2004) and Usha and Vimala (2003). The paper by Veysey and Goldenfeld (2007) very well expounds the theory of low Reynolds number flow past a sphere and a circular cylinder.

The problem of slow steady flow past a sphere placed in a uniform stream of viscous incompressible fluid was considered first by Stokes (1851) under the assumption that the inertia terms are negligible. Later on it was extended by Oseen (1910) by linearizing the inertia terms. A regular perturbation solution of Navier-Stokes equations is non-existent for slow flow problems. The method of singular perturbation/matched asymptotic expansions was introduced by Kaplun (1957) and then used by Kaplun and Lagerstrom (1957). Proudman and Pearson (1957) also employed the same technique to study the flow past a sphere at small Reynolds number  $R$ . The method of matched asymptotic expansions is of continuing interest as exhibited in the article by Veysey and Goldenfeld (2007), wherein its development and extension is discussed.

Some years back the problem of slow flow past a pervious sphere with a source at its centre was considered by Datta (1973) by taking into account the effect of source in the inertia terms of Navier-Stokes equations and suitably linearizing them, under the assumption that the interaction between the source flow and the Stokes flow is small. Placek and Peters (1980) used the results of this paper to estimate the target efficiency of small particles by spherical collectors. Later on Datta and Sthapit (1976) extended the analysis by making use of matching technique to obtain the terms up to  $O(R)$ .

In this paper we consider the problem anew for two reasons; (i) that here attention is given to the full Navier-Stokes equations rather than the reduced equations used by Datta and Sthapit, thereby exhibiting the full effect of source interaction (ii) that the analysis becomes too cumbersome in the paper by Datta and Sthapit when we try to determine higher order approximations but the Matched asymptotic method when suitably applied to the full Navier-Stokes equation effects the simplification. We have used the inner and outer expansions for the stream function as given by Proudman and Pearson (1957) and include the effect of source in the inner and outer expansions. It is found that the analysis becomes easier in finding higher order approximations. Here, we have been able to determine  $O(R^2 \log R)$  terms in the Stokes expansion and that was not possible by the earlier analysis of Datta and Sthapit.

## 2. Basic Equations

We consider the flow of a viscous fluid of uniform velocity  $U$  past a sphere of radius  $a$ , and non-dimensionlize the space coordinates by  $a$ .

Following Proudman and Pearson (1957) the non-dimensional governing equations, obtained from Navier-Stokes equations, in the Stokes region of flow near the sphere in polar coordinates  $(r, \theta)$  for the stream function  $\psi$  is

$$R\left[\frac{1}{r^2} \frac{\partial(\psi, D^2\psi)}{\partial(r, \mu)} + \frac{2}{r^2} D^2\psi L\psi\right] = D^4\psi \tag{1.1}$$

where  $\mu = \cos \theta$  and  $R = \frac{Ua}{\nu}$  is Reynolds number; further

$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{(1-\mu^2)}{r^2} \frac{\partial^2}{\partial \mu^2}, \tag{1.2}$$

$$L = \frac{\mu}{1-\mu^2} \frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \mu}. \tag{1.3}$$

The relations between Stokes and Oseen’s variables are

$$\rho = Rr, \Psi = R^2\psi. \tag{1.4}$$

Far off from the sphere in Oseen region, equation (1.1) becomes

$$\frac{1}{\rho^2} \frac{\partial(\Psi, D^2\Psi)}{\partial(\rho, \mu)} + \frac{2}{\rho^2} D^2\Psi L\Psi = D^4\Psi, \tag{1.5}$$

where  $D^2$  and  $L$  are the same operators as given by (1.2) and (1.3) but with  $r$  replaced by  $\rho$ .

Now taking into account that the stream function for a source is  $-s\mu$ , we take the inner expansion (Stokes expansion) as

$$\psi = -s\mu + \bar{\psi}, \tag{1.6}$$

where with  $f_{n+1}(R)/f_n(R) \rightarrow 0$  as  $R \rightarrow 0$ . We assume the inner expansion (Stokes expansion) as

$$\bar{\psi} = \psi_0 + f_1(R)\psi_1 + f_2(R)\psi_2 + \dots,$$

and with  $F_{n+1}(R)/F_n(R) \rightarrow 0$  as  $R \rightarrow 0$ , the outer expansion (Oseen expansion) as

$$\Psi = -s\mu R^2 + \frac{\rho^2}{2}(1-\mu^2) + F_1(R)\Psi_1 + F_2(R)\Psi_2 + \dots, \tag{1.7}$$

where  $\rho^2(1-\mu^2)/2$  is the stream function for the uniform stream. It may be noted that since the first term (the source term) is of  $O(R^2)$ , its effect will come into play only when such terms in the expansion are taken into consideration. Now the inner boundary conditions (stream surface and no slip condition) are

$$\bar{\psi} = \frac{\partial \bar{\psi}}{\partial r} = 0 \text{ at } r=1 \tag{1.8}$$

and the outer boundary condition corresponding to uniform stream is

$$\Psi \rightarrow \frac{\rho^2}{2}(1-\mu^2) \text{ as } r \rightarrow \infty. \tag{1.9}$$

The undetermined constants in the inner and outer solutions are to be determined from the VanDyke (1964) matching condition (1.10) the  $m$  term inner expansion of the  $n$  term outer expansion = the  $n$  term outer expansion of the  $m$  term inner expansion.

### 3. The Leading Terms of the Expansions

The primary solution in Oseen expansion is the uniform stream  $\frac{\rho^2}{2}(1-\mu^2)$ . The derivation of the leading term of the Stokes expansion,  $\psi_0$ , is straight forward. The equation for  $\psi_0$

$$D^4\psi_0=0.$$

As in Proudman and Pearson (1957), using the boundary conditions (1.8) and matching with the uniform stream, the solution is obtained

$$\psi_0 = \frac{1}{4} \left( 2r^2 - 3r + \frac{1}{r} \right) (1-\mu^2).$$

For the term  $\Psi_1$  in Oseen expansion, using (1.5) and (1.7), the governing equation is

$$\left[ D^2 - \left( \mu \frac{\partial}{\partial \rho} + \frac{1-\mu^2}{\rho} \frac{\partial}{\partial \mu} \right) \right] D^2 \Psi_1 = 0. \tag{2.1}$$

As derived by Proudman and Pearson (1957), the solution, on matching with (2.1), is

$$\Psi_1 = -\frac{3}{2} (1+\mu) \left[ 1 - e^{-\frac{\rho}{2}(1-\mu)} \right]. \tag{2.2}$$

Next, equation for the Stokes part  $\psi_1$  by using (1.1) and (1.6) is

$$D^4 \psi_1 = \frac{s}{r^2} \left( \frac{\partial}{\partial r} - \frac{2}{r} \right) D^2 \psi_0 + \frac{1}{r^2} \left[ \frac{\partial(\psi_0 \cdot D^2 \psi_0)}{\partial(r, \mu)} + 2 \left\{ \frac{\mu}{(1-\mu^2)} \frac{\partial \psi_0}{\partial r} + \frac{1}{r} \frac{\partial \psi_0}{\partial \mu} \right\} D^2 \psi_0 \right],$$

wherein the source term makes its appearance and the resulting solution is evaluated as

$$\psi_1 = \left[ \frac{a}{r} + br + cr^2 + \frac{9}{16}s \right] (1-\mu^2) + \left[ \frac{d}{r^2} + e - \frac{3}{16}r^2 + \frac{9}{32}r + \frac{3}{32r} \right] \mu(1-\mu^2), \tag{2.3}$$

where  $a, b, c, d$  and  $e$  are arbitrary constants. We evaluate values of these constants by using boundary conditions (1.8) and matching with the Oseen’s term (2.2). Thus, we get

$$a = -\frac{9s}{32} + \frac{3}{32}, b = -\frac{9s}{32} - \frac{9}{32}, c = \frac{3}{16}, d = -\frac{3}{32}, e = -\frac{3}{32}.$$

Hence, we obtain

$$\psi_1 = \frac{3}{32} \left[ 2r^2 - 3r + \frac{1}{r} + 6s - \frac{3s}{r} - 3sr \right] (1-\mu^2) - \frac{3}{32} \left[ \frac{1}{r^2} + 1 + 2r^2 - 3r - \frac{1}{r} \right] \mu(1-\mu^2). \tag{2.4}$$

Since higher order terms in the Stokes and Oseen’s expansions are not proportional to the simple power of  $R$ , it seems desirable to give a brief account of the nature of these terms. As in Proudman and Pearson (1957), first consider the third term of the Stokes expansion  $\psi_2$  and take  $f_2(R) = R^2$ . Thus, from equations (1.1) and (1.6), we have

$$D^4 \psi_2 = \frac{s}{r^2} \left[ \frac{\partial}{\partial r} - \frac{2}{r} \right] D^2 \psi_1 + \frac{1}{r^2} \left[ \left( \frac{\partial \psi_0}{\partial r} \frac{\partial}{\partial \mu} D^2 \psi_1 + \frac{\partial}{\partial r} \psi_1 \frac{\partial}{\partial \mu} D^2 \psi_0 \right) - \left( \frac{\partial \psi_0}{\partial \mu} \frac{\partial}{\partial r} D^2 \psi_1 + \frac{\partial \psi_1}{\partial \mu} \frac{\partial}{\partial r} D^2 \psi_0 \right) \right] \\ + \frac{2}{r^2} \left[ \left\{ \left( \frac{\mu}{1-\mu^2} \right) \frac{\partial}{\partial r} \psi_0 + \frac{1}{r} \frac{\partial}{\partial \mu} \psi_0 \right\} D^2 \psi_1 + \left\{ \left( \frac{\mu}{1-\mu^2} \right) \frac{\partial}{\partial r} \psi_1 + \frac{1}{r} \frac{\partial}{\partial \mu} \psi_1 \right\} D^2 \psi_0 \right].$$

Now by using the values of  $\psi_0$  and  $\psi_1$  from equation (2.1) and (2.4) we get

$$\psi_2 = -\frac{3}{40} \left[ C_1 r^2 + C_2 r + \frac{C_3}{r} - 4 \frac{\log r}{r} s^2 + \frac{45}{8} s^2 + 60s + \frac{1}{40r^3} - \frac{3}{16r^2} - \frac{3 \log r}{5r} + \frac{3}{16} + 3r^2 \log r - r^3 \right] Q_1(\mu) \\ + \frac{27}{32} \left[ C_4 r^3 + C_5 + \frac{C_6}{r^2} + \frac{r^2}{3} - \frac{r}{2} - \frac{1}{6r} - \frac{sr^2}{18} + \frac{103}{162} sr - \frac{52}{15} s \log r + \frac{119s}{54r} + \frac{712}{1095} \frac{s}{r^2} \log r \right] Q_2(\mu) \\ + \frac{9}{20} \left[ \frac{C_7}{r} + \frac{C_8}{r^3} + \frac{\log r}{42r^3} + \frac{1}{48r^2} + \frac{4 \log r}{35r} - \frac{1}{3} + \frac{11r}{24} - \frac{43r^2}{120} + \frac{r^3}{9} \right] Q_3(\mu),$$

where  $C_1, C_2, C_3, C_4, C_5, C_6, C_7$  and  $C_8$  are arbitrary constants. The values of these constants are evaluated as in Proudman and Pearson (1957) by using the steps in the Van Dyke matching principle (1.10) with  $m=3$  and  $n=2$  (not counting the source term  $s\mu$ ). It is found that  $C_4 = -4/27$ ,  $C_1 = 3 \log R$  and then  $C_2$  and  $C_3$  must be multiple of  $\log R$  in order to satisfy the no-slip condition

on the sphere. Thus, we should replace  $f_2(R)$  by  $R^2 \log R$ , the other terms vanishing being of  $O(R^2)$  and so we have

$$\psi_2 = \frac{9}{160} (2r^2 - 3r + \frac{1}{r})(1 - \mu^2). \quad (2.6)$$

We see that second order Stokes term remains unaffected by the source. As in Proudman and Pearson, the next higher term of Oseen expansion does not involve  $O(R^2 \log R)$  terms.

## 4. Conclusion

### 4.1. Vorticity and Stream Lines

An important flow quantity is vorticity. In the axial symmetric case, the only non-vanishing component of vorticity is the azimuthal component and that may be expressed, correct up to  $O(R)$  as

$$\zeta = -\frac{1}{r \sin \theta} D^2 \psi = -\frac{3}{2r^2} \left[ 1 + \frac{3R}{8} + \frac{R(r-2)}{16} \left\{ 2 \left( 4 - \frac{1}{r} + \frac{1}{r^2} \right) \mu + \frac{6s}{r} \right\} \right] \sin \theta.$$

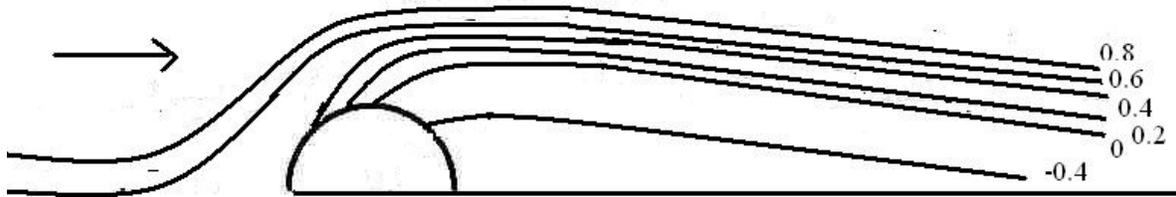
The vorticity will vanish, either at  $\theta = 0, \pi$  (rear and forward ends) or when

$$1 + \frac{3R}{8} + \frac{R(r-2)}{16} \left\{ 2 \left( 4 - \frac{1}{r} + \frac{1}{r^2} \right) \mu + \frac{6s}{r} \right\} = 0.$$

Separation occurs on the sphere  $r = 1$  when  $\zeta = 0$ . Thus, besides  $\theta = 0, \pi$ , since  $|\mu| \leq 1$ , it follows from the above equation that it may occur when  $R \geq 8/(1+3s)$  again at the rear end  $\theta = 0$ . Hence, the minimum value 8 of  $R$  gets reduced because of source. It may be seen that for no source  $s = 0$ , the result reduces to that obtained by Proudman and Pearson through eddy formation provided by stream lines  $\psi = 0$ . From these results it is easy to conclude that for  $R = 8$  the separation point shifts from  $\theta = 0$  to the position  $\theta_s = \cos^{-1}(1 - 3s/4)$ , moving away with an increase in the value of  $s$ . But, as the perturbation expansions are true only for small values of Reynolds number  $R$ , as also observed by Proudman and Pearson, the above statements do not hold good; therefore, no separation takes place and we have only a regular pattern of stream lines ( $\psi = -s\mu + \psi_0 + R\psi_1 = \text{constant}$ ).

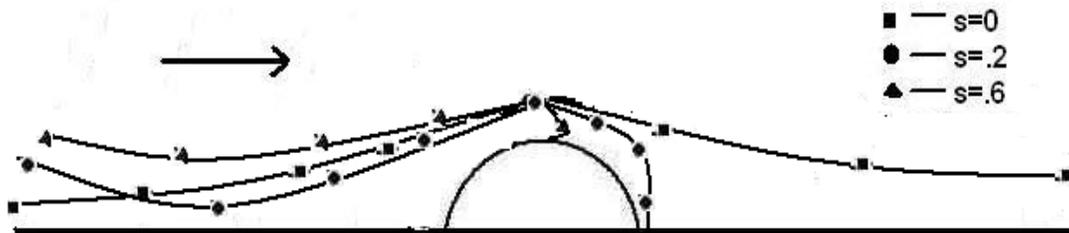
Various streamlines  $\psi (= -0.6, -0.4, 0, 0.2, 0.4, 0.6, 0.8)$  are depicted in Figure 1 for  $R = 0.5$  and  $s = 0.6$ . Keeping in view that for the no source ( $s = 0$ ) case, the stream line  $\psi = 0$  is the line  $\theta = 0$ , we see that the effect of the source is to shift the stream lines away from the sphere and the streamline  $\psi = c = 0.6$  now assumes the position  $\theta = 0$ . Also, we see that while for the values  $c < c_0$  ( $0.4 < c_0 < 0.6$ ) the stream lines emerge from the sphere, the stream lines beyond  $c_0$

are still coming from  $-\infty$  and going to  $\infty$ , thus, the effect of the source on the flow pattern is clearly exhibited.



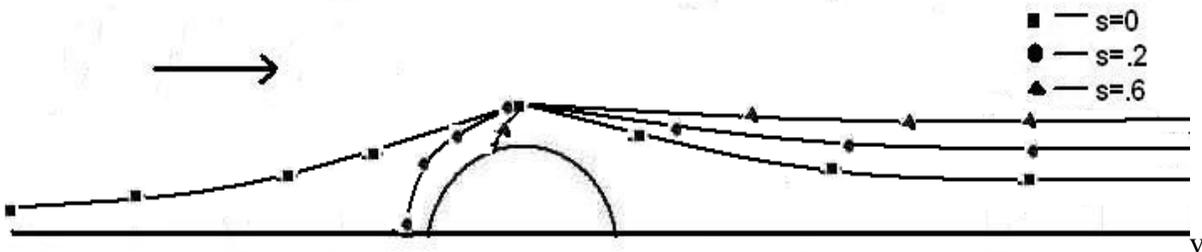
**Figure 1.** Various stream lines for  $R = 0.5$  and  $s = 0.6$  drawn in the upper half

In Figure 2a, we have drawn the stream line  $\psi = 0.2$  when  $R=0.5$  for source strengths  $s = 0, -0.2, -0.6$ , the latter two are seen to be sinks. We find when there is no source, the stream line is seen to come from  $-\infty$  and go to  $\infty$  in a regular manner. The effect of the sink is to displace the streamline towards the sphere on the downward side but on the upward side this is not so near the sphere surface. For the sinks stream lines seem tend towards the boundary surface on the downward side. Thus, there is an inward turning of the streamline separating it from the stream line for  $s = 0$ .



**Figure 2a.** Stream line  $\psi = 0.2$  for  $R = 0.5$ , drawn for  $s = -0.6, -0.2, 0$

In Figure 2b, we have drawn the stream line  $\psi = 0.2$  when  $R=0.5$  for source strengths  $s = 0, 0.2, 0.6$ , the latter two are seen to be sources. The effect of the source is to displace the streamline away from the sphere on the downstream side. But on the upstream side the streamline is seen to emerge from the sphere. This is because of the fluid being issued by the source. The void space in-between indicates the formation of eddies but our analysis is unable to capture that.



**Figure 2b.** Stream line  $\psi = 0.2$  for  $R = 0.5$ , drawn for  $s = 0, 0.2, 0.6$

In Figure 3 Streamline  $\psi = 0.2$  is drawn for  $R = 0, 0.5, 0.8$  when  $s = 0.5$ . Only minute difference is noticed. It implies that small Reynolds number do not affect the stream line pattern up to this order.

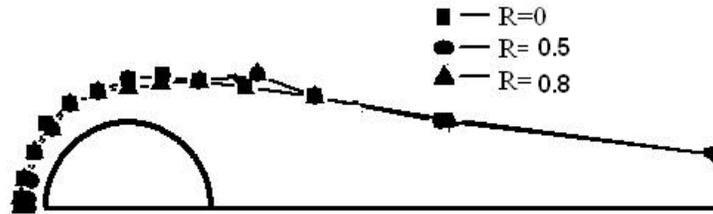


Figure 3. Stream line  $\psi = 0.2$  for  $s = 0.5$ , drawn for  $R = 0, 0.5, 0.8$

#### 4.2. Drag

Drag on the sphere the force exerted on the body by the moving fluid. It is evaluated by summing up the tensor components along the flow direction. Now in the spherical polar coordinates, the relevant non-dimensional stress components are the normal and tangential stress components  $\sigma_{rr}$  and  $\sigma_{r\theta}$  given by

$$\left(-p + 2\frac{\partial q_r}{\partial q_\theta}\right) \text{ and } \left(\frac{1}{r}\frac{\partial q_r}{\partial q_\theta} - \frac{q_\theta}{r} + \frac{\partial q_\theta}{\partial r}\right),$$

where the velocity components  $q_r$  and  $q_\theta$  are expressible in terms of Stokes stream function  $\psi$  as

$$q_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \text{ and } q_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}.$$

The pressure can be determined to within an irrelevant additive constant by integrating the Navier-Stokes equations and then expressed in terms of Stokes stream function

$$\psi = -s\mu + \psi_0 + R\psi_1 + R^2 \log R \psi_2 + \dots.$$

Let  $\sigma_{rr}$  and  $\sigma_{r\theta}$  be the normal and tangential stress components on the surface of the sphere, the dimensional drag on the sphere, omitting the details, is given by

$$\begin{aligned} D &= 2\pi a^2 \int_0^\pi [\sigma_{rr} \cos \theta - \sigma_{r\theta} \sin \theta]_{r=a} \sin \theta d\theta, \tag{2.7} \\ &= 2\pi a^2 \int_0^\pi \left[ \left(-p + 2\rho_d \nu \frac{\partial q_r}{\partial r}\right) \cos \theta - \rho_d \nu \left(\frac{1}{r}\frac{\partial q_r}{\partial q_\theta} - \frac{q_\theta}{r} + \frac{\partial q_\theta}{\partial r}\right) \sin \theta \right]_{r=1} \sin \theta d\theta \\ &= \frac{4}{3} \pi a \rho_d \nu U \left\{ \frac{9}{2} + \frac{27}{16} R + \frac{81}{80} R^2 \log R - \frac{21}{16} sR \right\} \\ &= 6\pi \rho_d \nu a U \left[ 1 + \frac{3}{8} R - \frac{7}{24} sR + \frac{9}{40} R^2 \log R + O(R^2) \right]. \end{aligned}$$

The third term in the above expression shows that the effect of a source ( $s > 0$ ) is to decrease the drag and reverse is the case for a sink ( $s < 0$ ) and that no change occurs in the  $O(R^2)$  term.

### ***Acknowledgements***

*The authors are thankful to the referees for their constructive suggestions.*

### **REFERENCES**

- Ayoubi, Mohammad A. and Longuski, James M. (2008) Analytical solutions for translational motion of spinning up rigid bodies subject to constant body fixed forces and moments, J. Applied Mech., **75**, No. 1, pp. 011004.
- Bandyopadhyay, Promode R. (2006). Stokes mechanism of drag reduction, J. Appl. Mech., **73**, No. 3, pp. 483-489.
- Becker, L. E., Koehler, S. A. and Stone, H. A. (2003). On self-propulsion of micromachines at low Reynolds number: Purcell's three-link swimmer. J. Fluid Mech., **490**, pp.15-35.
- Blake, J. R. and Otto, S. R. (1996). Ciliary propulsion, chaotic filtration and a 'blinking' Stokeslet. J. Engg. Math., **30**, pp. 151-168.
- Chadwick, E. (2002). A slender-body theory in Oseen flow, Proc. Roy. Soc. London, A **458**, pp. 2007-2016.
- Charalambopoulos, Antonis and Dassios, George (2002). Complete decomposition of axisymmetric Stokes flow, Int. J. Eng. Sci., **40**, No. 10, pp. 1099-1111.
- Datta, S. (1973). Stokes flow past a sphere with a source at its centre. Mathematiki Vesnik.**10** (25) pp.227-229.
- Felderhof, B.U. (2005) Sedimentation of spheres at small Reynolds number, J. Chem. Phys., **122**, pp. 214905.
- Ishiyama, K., Sendoh, M., Yamazaki, A. and Aral, K. (2001). Swimming of magnetic micro-machines under a wide range of Reynolds number conditions. IEEE Trans. Magnet. **37**, pp.2868- 2870. **6**, pp. 585-593.
- Kaplun, S. and Lagerstrom, P. A. (1957). Asymptotic expansions of Navier-Stokes solutions for small Reynolds numbers. J. Maths. Mech.,
- Khan, S.K. and Palaniappan, D. (1997). Slow viscous flow about a permeable cylinder, Arch. Mech., **49**, No. 1, pp. 177-199.
- Oseen, C. W. (1910). Uber die Stokessche Formel und uber eine verwandte Aufgabe in der Hydrodynamik. Ask. Mat. Astr. Pys. **6**: No.29.
- Padmavathi, B.S., Rajasekhar, G.P. and Amarnath, T. (1998). A note on complete general solutions of Stokes equations, Quart. J Mech. Appl.Math. **51**, No. 3, pp. 383-388.
- Placek, Timothy D. and Peters, Leonard K. (1980). A Hydridynamic Approach To Partical Target Efficiency In the Presence Of Diffusiophoresis, Aerosol Sci. Vol-**11**, pp.521-533.
- Pozrikidis, C. and Farrow, D.A. (2003). A model of fluid flow in tumors, Ann. Biomed. Eng., **31**, pp. 181-194.

- Proudman, L. and Pearson, J. R. A. (1957). Expansions at small Reynolds number for the flow past a sphere and a circular cylinder *J. Fluid Mech.*, **2**, pp. 237-262.
- Ramkisson, H. and Rahaman, K. (2003b). Wall effects on a spherical particle, *Int. J. Eng. Sci.*, **41**, No. 3-5, pp.283-290.
- Ranger, K.B. (2004). Fluid velocity fields derived from vorticity singularities, *Quart. Appl. Math.*, **62**, pp. 671-685.
- Shankar, P.N. (1998). Three-dimensional Stokes flow in a cylindrical container, *Physics of Fluids*, **10**, No. 3, p. 540.
- Sthapit, Y.R. and Datta, S. (1976). Steady flow at small Reynolds number past a sphere with a source at its centre. *Ganita*, **27**, 101- 108.
- Stokes, G. G. (1851). On the effect of the internal friction of fluids on the motion of pendulums. *Trans. Camb. Philos. Soc.*, **9**, 8-106.
- Usha, R. and Vimala, P. (2003). Axisymmetric rotational disk braking by a curved disk, *ZAMM*, 83, No. **5**, pp. 311-320.
- VanDyke, M. (1964). *Perturbation Methods in Fluid Mechanics*, Academic Press.
- Veysey J. and Goldenfeld, N. "Simple viscous flows: From boundary layers to the renormalization group", *Rev. of Modern Physics*, VOL. 79, July- September, 2007, pp. 883-927.

## Biographical Note

Sunil Datta

Professor & Head, Department of Mathematics & Astronomy, Lucknow University (Retired)

Specialization: Applied Mathematics, Fluid Mechanics. Published more than 70 research papers in National and International journals. Recipient of Merit scholarships, Fellowships & Teaching awards. Member of various National Mathematical Bodies and of AMS & SIAM Authored textbooks on (i) Mechanics (ii) Tensor; Member of Indo-Russian joint project "Cell models of nanofiltration through complex porous membranes".

Shanu Singhal

Research student of Department of Mathematics & Astronomy, Lucknow University, M.Sc. in mathematics from Kanpur University, India. Teaching experience as lecturer (Mathematics Department) in Degree College and after wards in engineering college. Presently pursuing Ph.D. in fluid mechanics from Lucknow University.