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Numerical Comparison of Methods for Hirota-Satsuma Model

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Abstract

This paper outlines the implementation of the modified decomposition method (MDM) to solve a very important physical model namely Hirota-Satsuma model which occurs quite often in applied sciences. Numerical results and comparisons with homotopy perturbation (HPM) and Adomian's decomposition (ADM) methods explicitly reveal the complete reliability of the proposed MDM. It is observed that the suggested algorithm (MDM) is more user-friendly and is easier to implement compared to HPM and ADM.

Keywords: Modified decomposition method, Hirota-Satsuma coupled KdV systems, nonlinear problems, homotopy perturbation method, and Adomian's polynomials.

MSC (2000) No: 65 N 10

1. Introduction

The Hirota-Satsuma model occurs very frequently in a number of physical problems and has been studied by many researchers, [see Abbasbandy (2007, 2008), Abdou et al. (2005), Abassy et al. (2007), He (2006), Kaya (1999), Ma et al. (2004), Mohyud-Din et al. (2009)]. Recently, Geijji et al. (2006) introduced a very reliable and efficient technique the modified decomposition method (MDM), which has been applied to a whole class of diversified linear and nonlinear problems of physical nature, [see Noor et al. (2006), Mohyud-Din et al. (2007, 2010)].

This paper seeks to implement and extend this technique (MDM) to solve Hirota Satsuma model which arises quite often in applied sciences. It is observed that the proposed MDM is extremely useful, very simple and highly accurate. Numerical results clearly reveal the complete reliability of the proposed modified decomposition method (MDM). Moreover, we have also compared our results with homotopy perturbation method (HPM) which was developed by He (1999, 2004, 2005, 2006, 2008) by merging the standard homotopy and perturbation, [see Mohyud-Din (2009, 2010)].

It is to be highlighted that the comparison of MDM has also been made with the traditional Adomian's decomposition method (ADM) which has been modified by Wazwaz (1999, 2000). It is observed that the proposed algorithm (MDM) is easier to implement and is more user friendly as compare to HPM and ADM. The suggested algorithm (MDM) is independent of the complexities arising in the calculation of so-called Adomian's polynomials. Moreover, the proposed scheme (MDM) does not require perturbation and hence reduces the computational work to a tangible level.

It is to be highlighted that the results obtained by MDM are fully compatible with HPM and ADM. It only avoids lengthy calculations and unnecessary complicated procedures.

2. Modified Decomposition Method (MDM)

Consider the following general functional equations:

$$f(x) = 0\,. \tag{1}$$

To convey the idea of the modified decomposition method, we rewrite the above equation as, Geijji et al. (2006):

$$y = N(y) + c, \tag{2}$$

where N is a nonlinear operator from a Banach space $B \to B$ and *f* is a known function. We are looking for a solution of Equation (1) having the series form:

$$y = \sum_{i=0}^{\infty} y_i\,. \tag{3}$$

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The nonlinear operator N can be decomposed as

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{4}$$

From Equations (3) and (4), Equation (2) is equivalent to

$$\sum_{i=0}^{\infty} y_i = c + N(y_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{5}$$

We define the following recurrence relation:

$$\begin{cases} y_0 = c, \\ y_1 = N(y_0), \\ y_{m+1} = N(y_0 + \ldots + y_m) - N(y_0 + \ldots + y_{m-1}), \quad m = 1,2,3,\ldots, \end{cases} \tag{6}$$

Then,

$$(y_1 + \ldots + y_{m+1}) = N(y_0 + \ldots + y_m), \quad m = 1,2,3,\ldots,$$

and

$$y = f + \sum_{i=1}^{\infty} y_i,$$

if N is a contraction, i.e., $\| N(x) - N(y) \| \le \| x - y \|,\ 0 < K < 1$, then

$$\| y_{m+1} \| = \| N(y_0 + \cdots + y_m) - N(y_0 + \cdots + y_{m-1}) \| \le K \| y_m \| \le K^m \| y_0 \|, \ m = 0,1,2,3,\cdots,$$

and the series $\sum_{i=1}^{\infty} = y_i$ absolutely and uniformly converges to a solution of Equation (1) [see Geijji et al. (2006), Noor et al. (2006), Mohyud-Din et al. (2007, 2010)], which is unique, in view of the Banach fixed-point theorem.

3. Solution Procedure

Consider the following Hirota-Satsuma coupled KdV system

$$u_t - \frac{1}{2} u_{xxx} + 3uu_x - 3(vw)x = 0,$$

$$v_t - v_{xxx} - 3uv_x = 0,$$
$$w_t + w_{xxx} - 3uw_x = 0,$$

with initial conditions

$$u(x,0) = \frac{1}{3}\left(\beta - 2k^2\right) + 2k^2 \tan h^2(kx),$$
$$v(x,0) = \frac{-4k^2 c_0\left(\beta + k^2\right)}{3c_1^2} + \frac{4k^2\left(\beta + k^2\right)}{3c_1}\tan h(kx),$$
$$w(x,0) = c_0 + c_1 \tan h(kx),$$

where c_0, c_1 and β are constants. The exact solution of the problem is given by

$$u(x,t) = \frac{1}{3}\left(\beta - 2k^2\right) + 2k^2 \tan h^2\left(k(x + \beta t)\right),$$
$$v(x,t) = \frac{-4k^2 c_0\left(\beta + k^2\right)}{3c_1^2} + \frac{4k^2\left(\beta + k^2\right)}{3c_1}\tan h\left(k(x + \beta t)\right),$$
$$w(x,0) = c_0 + c_1 \tan h\left(k(x + \beta t)\right).$$

Applying the modified decomposition method (MDM), following approximants are obtained

$$\begin{cases} u_0(x,t) = c, \\ u_0(x,\ t) = \frac{1}{3}\left(\beta - 2k^2\right) + 2k^2 \tan h^2(kx), \\ v_0(x,t) = c,, \\ v_0(x,\ t) = \frac{-4k^2 c_0\left(\beta + k^2\right)}{3c_1^2} + \frac{4k^2\left(\beta + k^2\right)}{3c_1}\tan h(kx), \\ w_0(x,t) = c, \\ w_0(x,y,t) = c_0 + c_1 \tan h(kx), \end{cases}$$

$$\begin{cases} u_1(x,t) = N\, u_0(x,t), \\ u_1(x,\ t) = \frac{1}{3}\left(\beta - 2k^2\right) + 2k^2 \tan h^2(kx) - \frac{2\cos hx = 2t \sinh x}{\cos^3 x}, \\ v_1(x,t) = N\, u_0(x,t), \\ v_1(x,\ t) = \frac{-4k^2 c_0\left(\beta + k^2\right)}{3c_1^2} + \frac{4k^2\left(\beta + k^2\right)}{3c_1}\tan h(kx) + \frac{\cosh^2 x \cosh x + t \sinh x}{\cos h^2 x}, \\ w_1(x,t) = N\, u_0(x,t), \\ w_1(x,y,t) = c_0 + c_1 \tan h(kx) + 2\left(\frac{-\cosh^2 + \cosh x + t \sinh x}{\cos h^2 x}\right), \end{cases}$$

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$\vdots$

The closed form solution is given as

$$(u, v, w) = (e^{x+y-t}, e^{x-y+t}, e^{-x+y+t}). \tag{7}$$

Now, we apply homotopy perturbation method (HPM) on Hirota-Satsuma coupled KdV system, we get

$$\begin{cases} \dfrac{\partial u_0}{\partial t} + p\dfrac{\partial u_1}{\partial t} + p^2\dfrac{\partial u_2}{\partial t} + \cdots = \dfrac{1}{3}\left(\beta - 2k^2\right) + 2k^2 \tan h^2(kx) + \dfrac{1}{2} p\displaystyle\int_0^t \left(\dfrac{\partial^3 u_0}{\partial x^3} + p\dfrac{\partial^3 u_1}{\partial x^3} + p^2\dfrac{\partial^3 u_2}{\partial x^3} + \cdots\right)dt \\ \qquad -3p\displaystyle\int_0^t \left(\begin{array}{l} (u_0 + pu_1 + \cdots)\left(\dfrac{\partial u_0}{\partial x} + p\dfrac{\partial u_1}{\partial x} + \cdots\right) \\ +3\left(\dfrac{\partial v_0}{\partial x} + p\dfrac{\partial v_1}{\partial x} + \cdots\right)\left(\dfrac{\partial w_0}{\partial x} + p\dfrac{\partial w_1}{\partial x} + \cdots\right)dt \end{array}\right), \\ \dfrac{\partial v_0}{\partial t} + p\dfrac{\partial v_1}{\partial t} + p^2\dfrac{\partial v_2}{\partial t} + \cdots = \dfrac{-4k^2 c_0\left(\beta + k^2\right)}{3c_1^2} + \dfrac{4k^2\left(\beta + k^2\right)}{3c_1}\tan h(kx) + p\displaystyle\int_0^t \left(\dfrac{\partial^3 v_0}{\partial x^3} + p\dfrac{\partial^3 v_1}{\partial x^3} + \cdots\right)dt \\ \qquad +3p\displaystyle\int_0^t \left((u_0 + pu_1 + \cdots)\left(\dfrac{\partial v_0}{\partial x} + p\dfrac{\partial v_1}{\partial x} + \cdots\right)dt\right), \\ \dfrac{\partial w_0}{\partial t} + p\dfrac{\partial w_1}{\partial t} + p^2\dfrac{\partial w_2}{\partial t} + \cdots = c_0 + c_1 \tan h(kx) + p\displaystyle\int_0^t \left(\dfrac{\partial^3 w_0}{\partial x^3} + p\dfrac{\partial^3 w_1}{\partial x^3} + p^2\dfrac{\partial^3 w_2}{\partial x^3} + \cdots\right)dt \\ \qquad +3p\displaystyle\int_0^t \left((u_0 + pu_1 + \cdots)\left(\dfrac{\partial w_0}{\partial x} + p\dfrac{\partial w_1}{\partial x} + p^2\dfrac{\partial w_1}{\partial x} + \cdots\right)dt\right). \end{cases}$$

Comparing the co-efficient of like powers of *p*, we get

$$p^0 \begin{cases} u_0(x,t) = c, \\ u_0(x,\, t) = \dfrac{1}{3}\left(\beta - 2k^2\right) + 2k^2 \tan h^2(kx), \\ v_0(x,t) = c,, \\ v_0(x,\, t) = \dfrac{-4k^2 c_0\left(\beta + k^2\right)}{3c_1^2} + \dfrac{4k^2\left(\beta + k^2\right)}{3c_1}\tan h(kx), \\ w_0(x,t) = c, \\ w_0(x, y, t) = c_0 + c_1 \tan h(kx), \end{cases}$$

$$p^{1}\begin{cases} u_1(x,t) = N\,u_0(x,t), \\ u_1(x,\ t) = \dfrac{1}{3}\left(\beta-2k^2\right)+2k^2\tan h^2(kx)-\dfrac{2\cos hx = 2t\sinh x}{\cos^3 x}, \\ v_1(x,t) = N\,u_0(x,t), \\ v_1(x,\ t)= \dfrac{-4k^2c_0\left(\beta+k^2\right)}{3c_1^2}+\dfrac{4k^2\left(\beta+k^2\right)}{3c_1}\tan h(kx)+\dfrac{\cosh^2 x\cosh x+t\sinh x}{\cos h^2 x}, \\ w_1(x,t) = N\,u_0(x,t), \\ w_1(x,y,t) = c_0+c_1\tan h(kx)+2\left(\dfrac{-\cosh^2+\cosh x+t\sinh x}{\cos h^2 x}\right), \end{cases}$$

$\vdots$.

The closed form solution is given as

$$(u,v,w)=(e^{x+y-t},e^{x-y+t},e^{-x+y+t}),$$

which is the same as (7). Now, Applying Adomian's decomposition method (ADM), we get

$$\begin{cases} u_{n+1}(x,t)=\dfrac{1}{3}\left(\beta-2k^2\right)+2k^2\tan h^2\left(kx\right)+\displaystyle\int_0^t\left(\dfrac{1}{2}\sum_{n=0}^{\infty}\left(\dfrac{\partial^3 u_n}{\partial x^3}\right)-3\sum_{n=0}^{\infty}A_n+3\sum_{n=0}^{\infty}B_n\right)dt, \\ v_{n+1}(x,t)=\dfrac{-4k^2c_0\left(\beta+k^2\right)}{3c_1^2}+\dfrac{4k^2\left(\beta+k^2\right)}{3c_1}\tan h\left(kx\right)+\displaystyle\int_0^t\left(\sum_{n=0}^{\infty}\left(\dfrac{\partial^3 v_n}{\partial x^3}\right)+3\sum_{n=0}^{\infty}C_n\right)dt, \\ w_{n+1}(x,t)=c_0+c_1\tan h\left(kx\right)-\displaystyle\int_0^t\left(\sum_{n=0}^{\infty}\left(\dfrac{\partial^3 w_n}{\partial x^3}\right)+3\sum_{n=0}^{\infty}D_n\right)dt, \end{cases}$$

where A_n, B_n, C_n, D_n are the so-called Adomian's polynomials which can be evaluated by the algorithms developed in Wazwaz (1999, 2000). Consequently, we get the same approximants which ultimately yield the same closed form solution as that of (7).

$$b=0, k=t=1$$

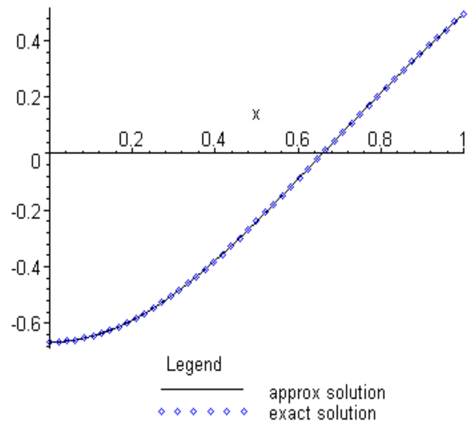


Figure 1 (u).

$$b=0, k=t=c_0=c_1=1$$

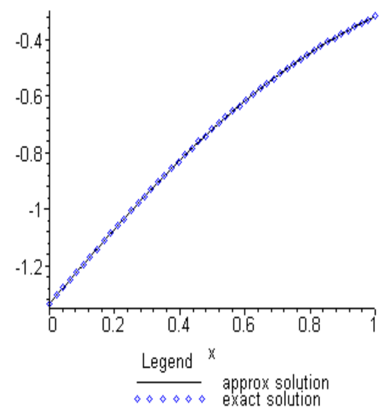


Figure 2 (v).

$$b=0.01, k=t=c_0=c_1=1$$

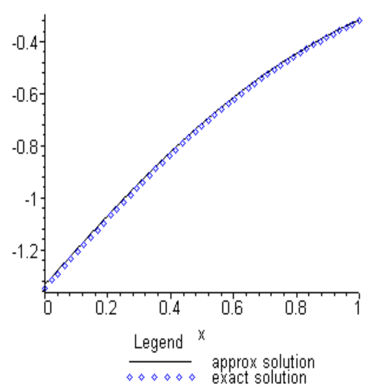
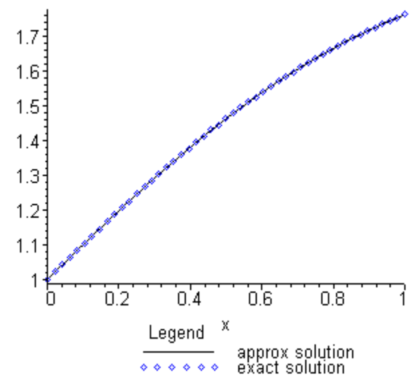
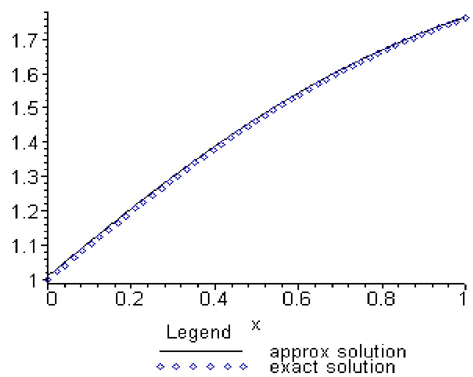


Figure 3 (w).

$$b = 0, k = t = c_0 = c_1 = 1$$



$$b = 0.01, k = t = c_0 = c_1 = 1$$



4. Conclusion

In this paper, we applied the modified decomposition method (MDM) for solving Hirota-Satsuma coupled KdV systems. The method is applied in a direct way without using linearization, transformation, discretization or restrictive assumptions. It may be concluded that the MDM is very powerful and efficient in finding the analytical solutions for a wide class of boundary value problems. The method gives more realistic series solutions that converge very rapidly in physical problems. It is worth mentioning that the method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result. It is concluded that the proposed MDM is more user friendly and is easier to implement compared to homotopy perturbation (HPM) and Adomian's decomposition (ADM) methods.

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