Applications and Applied Mathematics: An International Journal (AAM)

Volume 5 | Issue 2 | Article 14

12-2010

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Vivek Kumar
S. D. (P. G.) College

Sudhir Kumar
S. D. (P. G.) College

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Kumar, Vivek and Kumar, Sudhir (2010). On A Couple-Stress Fluid Heated From Below In Hydromagnetics, Applications and Applied Mathematics: An International Journal (AAM), Vol. 5, Iss. 2, Article 14.
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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 05, Issue 2 (December 2010), pp. 432 – 445
(Previously, Vol. 05, Issue 10, pp. 1529 – 1542)

On A Couple-Stress Fluid Heated From Below In Hydromagnetics

Vivek Kumar and Sudhir Kumar
Department of Mathematics
S. D. (P. G.) College
(affiliated to C.C.S. University Meerut)
Muzaffarnagar-251 001
Uttar Pradesh, India
vivek_shrawat@yahoo.co.in;skpundir05@yahoo.co.in

Received: February 22, 2010; Accepted: November 2, 2010

Abstract

The combined effect of dust particles, magnetic field and rotation on a couple-stress fluid heated from below is considered. For the case of stationary convection, dust particles are found to have a destabilizing effect on the system, whereas the rotation is found to have stabilizing effect on the system. Couple-stress and magnetic field are found to have both stabilizing and destabilizing effects under certain conditions. The oscillatory modes are introduced due to the presence of magnetic field and rotation in the system. The results are presented through graphs in each case.

Keywords: Thermal instability; couple-stress fluid; dust particles; magnetic field and rotation

MSC 2000 No.: 76E06, 76E07, 76E15, 76U05

1. Introduction

The theoretical and experimental results on the onset of thermal instability (B'enard convection) in a fluid layer under varying assumptions of hydrodynamics, has been discussed in detail by

Chandrasekhar (1981). With the growing importance of non-Newtonian fluids in technology and industries, the investigations of such fluids are desirable. The theory of couple-stress fluids is proposed by Stokes (1966). Couple-stresses appear in noticeable magnitude in fluids with very large molecules. Applications of couple-stress fluid occur in the attention the study of the mechanism of lubrication of synovial joints, at which currently attract the attention of researchers. A human joint is a dynamically loaded bearing that has an articular cartilage as the bearing and synovial fluid as the lubricant. Normal synovial fluid is clear or yellowish and is a non-Newtonian, viscous fluid. Walicki and Walicka (1999) modeled synovial fluid as couple-stress fluid in human joints because of the long chain of lauronic acid molecules found as additives in synovial fluid. The problem of a couple-stress fluid heated from below in a porous medium is considered by Sharma and Sharma (2001) and Sharma and Thakur (2000).

Recent spacecraft observations have confirmed that the dust particles play a significant role in the dynamics of the atmosphere as well as in the diurnal and surface variations in the temperature of the martain whether. Further, environmental pollution is the main cause of dust to enter the human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromosomes and genetic mutation so observed are likely to breed censer as malformations in the coming progeny. It is essential, therefore to study the presence of dust particles in astrophysical situations and fluid flow. Sunil et al. (2004) have studied the effect of suspended particles on couple-stress fluid heated and soluted from below in a porous medium and found that suspended particles have destabilizing effect on the system.

Sharma and Sharma (2004) have studied the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field and found that rotation has a stabilizing effect while dust particles have a destabilizing effect on the system. Kumar et al. (2004) have studied the thermal instability of Walters' B' visco-elastic fluid permeated with suspended particles in hydromagnetics in a porous medium and found that magnetic fields stabilize the system. The problems on a Rivlin-Ericksen rotating fluid in a porous medium have been discussed by Sharma et al. (1998) and Prakash and Kumar (1999). The problem of thermosolutal convection in Rivlin-Ericksen fluid in a porous medium in the presence of uniform vertical magnetic field and rotation is also considered by Sharma et al. (2001). They have found that rotation has a stabilizing effect on the system.

Sharma and Rana (2002) have studied the thermosolutal instability of Walters' (Model B') visco-elastic rotating fluid permeated with suspended particles and variable gravity field in porous medium. Kumar et al. (2006) have studied the effect of magnetic field on thermal instability of a rotating Rivlin-Ericksen visco-elastic fluid. Kumar et al. (2009) have studied the problem of thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field and found that magnetic field has both stabilizing and destabilizing effects on the system under certain conditions whereas rotation has a stabilizing effect on the system.

Keeping in mind the importance of non-Newtonian fluids, convection in a fluid layer heated from below, dust particles, magnetic field and rotation, we propose to study the Stokes (1966) incompressible couple-stress fluid in the presence of dust particles, magnetic field and rotation in the present paper.

2. Formulation of the Problem

Notations

a	Dimensionless wave number, [-]
F	Couple-stress parameter,
$\mathbf{g}$	Acceleration due to gravity, [m/s^2]
k	Wave number, [1/m]
k_x, k_y	Horizontal wave numbers, [1/m]
n	Growth rate, [1/s]
Q	Chandrasekhar number, [-]
T_A	Taylor number, [-]
R	Rayleigh number, [-]
T	Temperature, [K]
$\mathbf{\Omega}(0,0,\Omega)$	Rotation vector having components $(0,0,\Omega)$
$H(h_x,h_y,h_z)$	Magnetic field having components (h_x,h_y,h_z)
$q(u, v, w)$	Component of velocity after perturbation,
$q_d(l, r, s)$	Component of particles velocity after perturbation,
α	Coefficient of thermal expansion, [1/K]
β	Uniform temperature gradient, [K/m]
Θ	Perturbation in temperature, [K]
k_T	Thermal diffusivity, [m^2/s]
ν	Kinematic viscosity, [m^2/s]
ν'	Kinematic viscoelasticity, [m^2/s]
∇, ∂, D	Del operator, Curly operator and Derivative with respect to $z\ (= d/dz)$

Consider a static state in which an incompressible, Stokes couple-stress fluid layer of thickness d, is arranged between two infinite horizontal planes situated at $z=0$ and $z=d$, which is acted upon by a vertical magnetic field $\mathbf{H}(0, 0, H)$, where H is a constant, uniform rotation $\mathbf{\Omega}(0, 0, \Omega)$ and variable gravity field $\mathbf{g}(0, 0, -g)$. The particles are assumed to be non-conducting .The fluid layer is heated from below leading to an adverse temperature gradient $\beta = \dfrac{T_0 - T_1}{d}$, where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$.

Let $p, \rho, T, \alpha, \nu, \mu', k_T$ and $\mathbf{q}(u, v, w)$ denote respectively pressure, density, temperature, thermal coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity and velocity of the fluid. $\mathbf{q_d}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of particles, respectively. $K = 6\pi\mu\eta$, where η is the particle radius, is a constant and $\bar{x} = (x, y, z)$. Then equation of motion, continuity and heat conduction of couple-stress fluid (Stokes, 1966 and Joseph, 1976) in hydromagnetics are

$$\left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\mathbf{q}\right] = -\frac{1}{\rho_0}\nabla p + \mathbf{g}\left(1+\frac{\delta\rho}{\rho_0}\right) + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\mathbf{q} + \frac{KN}{\rho_0}(\mathbf{q}_d - \mathbf{q}) + 2(\mathbf{q}\times\mathbf{\Omega}) + \frac{\mu_e}{4\pi\rho_0}\left[(\nabla\times\mathbf{H})\times\mathbf{H}\right], \quad (1)$$

$$\nabla . \mathbf{q} = 0, \quad (2)$$

$$\frac{d\mathbf{H}}{dt} = (\mathbf{H}.\nabla).\mathbf{q} + \eta\nabla^2\mathbf{H} \quad (3)$$

and

$$\nabla . \mathbf{H} = 0. \quad (4)$$

The equation of state is

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right], \quad (5)$$

where the suffix zero refers to value at the reference level $z = 0$.

The presence of particles add an extra force term, proportional to the velocity difference between particles and fluid and appears in equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to the exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion for the particles. The buoyancy force on the particles is neglected. Interparticle reactions are not considered for we assume that the distance between particles is quite large as compared with their diameter. The equations of motion and continuity for the particles, under the above approximation, are

$$mN\left[\frac{\partial \mathbf{q}_d}{\partial t} + (\mathbf{q}_d.\nabla)\mathbf{q}_d\right] = KN(\mathbf{q} - \mathbf{q}_d) \quad (6)$$

and

$$\frac{\partial N}{\partial t} + \nabla(N.\mathbf{q}_d) = 0. \quad (7)$$

Here, mN is the mass of the particles per unit volume. Let c_v, c_{pt} denote the heat capacity of the fluid at constant volume and the heat capacity of the particles. Assuming that the particles and fluids are in thermal equilibrium, the equation of heat conduction give

$$\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T+\frac{mNc_{pt}}{\rho_0 c_v}\left(\frac{\partial}{\partial t}+\mathbf{q_d}.\nabla\right)T=k_T\nabla^2 T\,. \tag{8}$$

The kinematic viscosity ν, couple-stress viscosity μ', thermal diffusivity k_T and coefficient of thermal expansion α are all assumed to be constants.

3. Basic State

The basic state is described by

$$\mathbf{q}=(0,0,0),\mathbf{q_d}=(0,0,0),\mathbf{\Omega}=(0,0,\Omega),\mathbf{H}=(0,0,H),T=T_0-\beta z,N=N_0$$

and

$$\rho=\rho(z),p=p(z),T=T(z),\rho=\rho_0\,[1+\alpha\beta z]. \tag{9}$$

4. Perturbation Equations and Normal Mode Analysis

Let $\mathbf{q}(u,v,w)$, $\mathbf{q_d}(l,r,s)$, $\mathbf{h}(h_x,h_y,h_z)$, θ, $\delta\rho$, δp denote respectively the perturbations in fluid velocity, dust particles velocity, magnetic field, temperature, density and pressure. After linearizing the perturbation equations and analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$\left[w,\theta,h_z\zeta,\xi\right]=\left[W(z),\Theta(z),K(z),Z(z),X(z)\right].\exp.\left\{ik_x x+ik_y y+nt\right\}, \tag{10}$$

where k_x and k_y are the wave numbers in x and y directions respectively and $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ are the z-components of the vorticity and current density respectively.

For the considered form of the perturbations in equation (10), equations (1) to (8), after eliminating the physical quantities using the non-dimensional parameters $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $\tau=\frac{m}{K}$, $p_1=\frac{\nu}{k_T}$, $p_2=\frac{\nu}{\eta}$, $q=\frac{\nu}{k_s}$, $F=\frac{\mu'}{\rho_0\,d^2\nu}$, $\tau_1=\frac{\tau\nu}{d^2}$, $M=\frac{mN_0}{\rho_0}$, $B=1+b$ and $D^*=dD$ and dropping (*) for convenience, give

$$\left[\sigma\left(1+\frac{M}{1+\sigma\tau_1}\right)+F(D^2-a^2)^2-(D^2-a^2)\right]\left[D^2-a^2-\sigma Bp_1\right]\left[D^2-a^2-\sigma p_2\right]$$
$$\left[D^2-a^2\right]W-Ra^2\left(\frac{B+\sigma\tau_1}{1+\sigma\tau_1}\right)\left[D^2-a^2-\sigma p_2\right]W+Q\left[D^2-a^2-\sigma Bp_1\right]\left[D^2-a^2\right]D^2W$$
$$+T_A\frac{\left[D^2-a^2-\sigma p_2\right]^2\left[D^2-a^2-\sigma Bp_1\right]D^2W}{\left[\left\{\sigma\left(1+\frac{M}{1+\sigma\tau_1}\right)+F(D^2-a^2)^2-(D^2-a^2)\right\}(D^2-a^2-\sigma p_2)+QD^2\right]}=0. \quad (11)$$

Here, $R=\frac{g\alpha\beta d^4}{\nu k_T}$ is the thermal Rayleigh number, $T_A=\left(\frac{2\Omega d^2}{\nu}\right)^2$ is the Taylor number and $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number.

The perturbation in the temperature is zero at the boundaries because both the boundaries are maintained at constant temperatures. The case of two free boundaries is little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions for the equation (11) are

$$W=0, D^2W=0, D^4W=0, \Theta=0, DZ=0, DK=0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1. \quad (12)$$

From equation (12), it is clear that all the even order derivatives of W vanish on the boundaries. Therefore, the proper solution of equation (11) characterizing the lowest mode is

$$W=W_0\sin\pi z. \quad (13)$$

Here, W_0 is constant. Using equation (13), equation (11) gives

$$R_1=\frac{(1+x)}{x}\left[i\sigma_1\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+F_1(1+x)^2+(1+x)\right]\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)\left[1+x+i\sigma_1 Bp_1\right]$$
$$+Q_1\frac{(1+x)}{x}\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)\frac{\left[1+x+i\sigma_1 p_1\right]}{\left[1+x+i\sigma_1 p_2\right]}$$
$$+\frac{T_{A_1}}{x}\frac{\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)\left[1+x+i\sigma_1 Bp_1\right]\left[1+x+i\sigma_1 p_2\right]}{\left[\left\{i\sigma_1\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+F_1(1+x)^2+(1+x)\right\}\left[1+x+i\sigma_1 p_2\right]+Q_1\right]}, \quad (14)$$

where, $R_1 = \dfrac{R}{\pi^4}$, $i\sigma_i = \dfrac{\sigma}{\pi^2}$, $F_1 = \pi^2 F$, $T_{A_1} = \dfrac{T_A}{\pi^4}$, $Q_1 = \dfrac{Q}{\pi^2}$ and $x = \dfrac{a^2}{\pi^2}$. Equation (14) is the required dispersion relation including the effects of magnetic field, couple-stress, rotation, dust particles and kinematic viscoelasticity in the present problem.

6. Analytical Discussion

6.1. Stationary Convection

At stationary convection, when the instability sets, the marginal state will be characterized by $\sigma = 0$. Thus, putting $\sigma = 0$ in equation (14), we get

$$R_1 = \frac{(1+x)}{xB}\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1 + \frac{T_{A_1}(1+x)}{\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]}\right], \tag{15}$$

which express the Rayleigh number R_1 as a function of the parameters B, F_1, T_{A_1}, Q_1 and dimensionless wave number x. To study the effect of dust particles, couple-stress, rotation and magnetic field, we study the behavior of $\dfrac{dR_1}{dB}, \dfrac{dR_1}{dF_1}, \dfrac{dR_1}{dT_{A_1}}$ and $\dfrac{dR_1}{dQ_1}$ analytically.

From equation (15), we have

$$\frac{dR_1}{dB} = -\frac{(1+x)}{xB^2}\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1 + \frac{T_{A_1}(1+x)}{\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]}\right], \tag{16}$$

which clearly shows that dust particles have a destabilizing effect on the thermal convection in a couple-stress fluid.

From equation (15), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{xB}\left[1 - \frac{T_{A_1}(1+x)}{\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]^2}\right], \tag{17}$$

which shows that couple-stress has a stabilizing or destabilizing effect on the thermal convection under the conditions

$$T_{A_1}(1+x) < \text{ or } > \left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]^2.$$

But, for the permissible values of various parameters, the said effect is stabilizing only if

$$T_{A_1}(1+x) < \left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]^2 .$$

In the absence of rotation $(T_{A_1} = 0)$, equation (17) becomes

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{xB}, \tag{18}$$

which confirm that couple-stress has a stabilizing effect on the thermal convection in the absence of rotation as derived by Sharma and Sharma (2004).
Again from equation (15), we have

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x)^2}{xB\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]}, \tag{19}$$

which clearly shows that rotation has a stabilizing effect on the thermal convection in a couple-stress fluid.

In the absence of magnetic field, equation (19) becomes

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{xB\left\{F_1(1+x)+1\right\}}, \tag{20}$$

which confirm that rotation has a stabilizing effect on the thermal convection in the absence of magnetic field as derived by Sharma and Sharma (2004).
Again from equation (15), we have

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{xB}\left[1 - \frac{T_{A_1}(1+x)}{\left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]^2}\right], \tag{21}$$

which shows that magnetic field has a stabilizing or destabilizing effect on the thermal convection under the conditions

$$T_{A_1}(1+x) < \text{ or } > \left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]^2 .$$

But, for the permissible values of various parameters, the said effect is stabilizing only if

$$T_{A_1}(1+x) < \left[\left\{F_1(1+x)+1\right\}(1+x)^2 + Q_1\right]^2 .$$

In the absence of rotation $(T_{A_1} = 0)$, equation (21) becomes

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{xB}, \tag{22}$$

which confirm that magnetic field has a stabilizing effect on the thermal convection in the absence of rotation as derived by Sharma and Sharma (2004).

6.2. Stability of the System and Oscillatory Modes

Using equations (1) to (8) with the boundary condition (12), we get

$$\sigma\left(1+\frac{M}{1+\sigma\tau_1}\right)I_1 + I_2 + FI_3 - \frac{g\alpha k_T a^2}{\nu\beta}\left(\frac{1+\sigma^*\tau_1}{B+\sigma^*\tau_1}\right)\left[I_4 + \sigma^* Bp_1 I_5\right]$$
$$+\frac{\mu_e\eta}{4\pi\rho_0\nu}[I_6 + \sigma^* p_2 I_7] + d^2\left[\sigma^*\left(1+\frac{M}{1+\sigma^*\tau_1}\right)I_8 + FI_9 + I_{10}\right] + \frac{\mu_e\eta d^2}{4\pi\rho_0\nu}[I_{11} + \sigma p_2 I_{12}] = 0, \tag{23}$$

where

$$I_1 = \int\left(|DW|^2 + a^2|W|^2\right)dz,\ \ I_2 = \int\left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz,$$

$$I_3 = \int\left(|D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2\right)dz, I_4 = \int\left(|DQ|^2 + a^2|Q|^2\right)dz,$$

$$I_5 = \int |Q|^2\,dz,\ I_6 = \int\left(|D^2K|^2 + a^4|K|^2 + 2a^2|DK|^2\right)dz,\ I_7 = \int\left(|DK|^2 + a^2|K|^2\right)dz,$$

$$I_8 = \int |Z|^2\,dz,\ I_9 = \int\left(|D^2Z|^2 + 2a^2|DZ|^2 + a^4|Z|^2\right)dz,$$

$$I_{10} = \int\left(|DZ|^2 + a^2|Z|^2\right)dz,\ I_{11} = \int\left(|DX|^2 + a^2|X|^2\right)dz \text{ and } I_{12} = \int |X|^2\,dz,$$

where σ^* is the complex conjugate of σ. All the integrals I_1 to I_{12} are positive definite, Putting $\sigma = i\sigma_i$ in equation (23) and equating the imaginary parts, we obtain

$$\sigma_i\Bigg[\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)I_1 + \frac{g\alpha k_T a^2}{\nu\beta}\left\{\frac{\tau_1(B-1)}{B^2+\sigma_i^2\tau_1^2}I_4 + \frac{B+\sigma_i^2\tau_1^2}{B^2+\sigma_i^2\tau_1^2}Bp_1 I_5\right\} - \frac{\mu_e\eta}{4\pi\rho_0\nu}p_2 I_7$$
$$-d^2\left\{1+\frac{M}{1+\sigma_i^2\tau_1^2}\right\}I_8 + \frac{\mu_e d^2\eta}{4\pi\rho_0\nu}p_2 I_{12}\Bigg] = 0. \tag{24}$$

In the absence of magnetic field and rotation, equation (24) becomes

$$\sigma_i\left[\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)I_1+\frac{g\alpha k_T a^2}{\nu\beta}\left\{\frac{\tau_1(B-1)}{B^2+\sigma_i^2\tau_1^2}I_4+\frac{B+\sigma_i^2\tau_1^2}{B^2+\sigma_i^2\tau_1^2}Bp_1I_5\right\}\right]=0\,. \tag{25}$$

From equation (25), it is obvious that all the terms in the bracket are positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed in the system and Principle of Exchange of Stabilities (PES) is satisfied in the absence of magnetic field and rotation. It is evident from equation (24) that presence of magnetic field and rotation brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence.

7. Numerical Computations

Graphs have been plotted between critical Rayleigh number R_1 and parameters B, Q_1 and T_{A_1} by giving some numerical values to them.

In Figure 1, critical Rayleigh number R_1 is plotted against dust particles parameter B for fixed value of $F_1 = 10, T_{A_1} = 100$ and $Q_1 = 100, 300, 500$. The critical Rayleigh number R_1 decreases with increase in dust particles parameter B which shows that dust particles have destabilizing effect on the system.

In Figure 2, critical Rayleigh number R_1 is plotted against rotation parameter T_{A_1} for fixed value of $F_1 = 10, B = 20$ and $Q_1 = 100, 400, 700$. The critical Rayleigh number R_1 increases with increase in rotation parameter T_{A_1} which shows that rotation has a stabilizing effect on the system.

In Figure 3, critical Rayleigh number R_1 is plotted against rotation parameter T_{A_1} for fixed value of $F_1 = 10, Q_1 = 500$ and $B = 5, 10, 15$. The critical Rayleigh number R_1 increases with increase in rotation parameter T_{A_1} which shows that rotation has a stabilizing effect on the system.

In Figure 4, critical Rayleigh number R_1 is plotted against magnetic field Q_1 for fixed value of $F_1 = 10, B = 20$ and $T_{A_1} = 100, 500, 1000$. The critical Rayleigh number R_1 increases with increase in magnetic field Q_1 which shows that magnetic field has a stabilizing effect on the system.

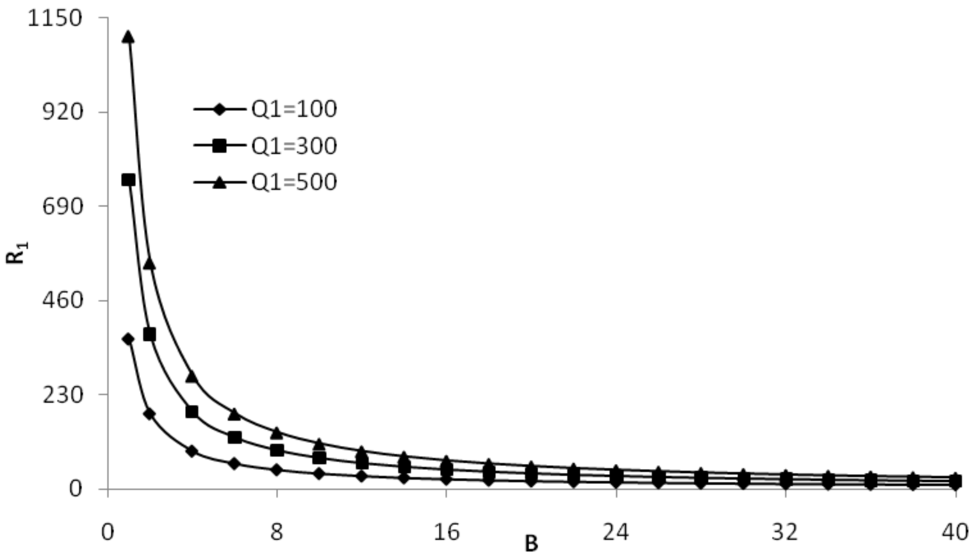

Figure 1. Variation of critical Rayleigh number R_1 with dust B for fixed value of $F_1 = 10, T_{A_1} = 100$ and $Q_1 = 100, 300, 500$.

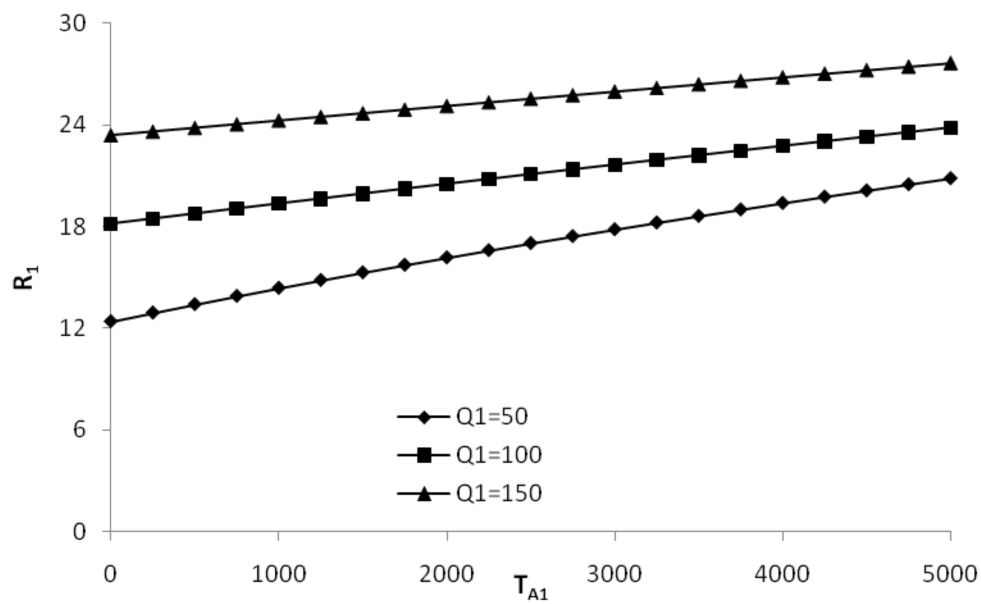

Figure 2. Variation of critical Rayleigh number R_1 with rotation T_{A_1} for fixed value of $F_1 = 10,\ B = 20$ and $Q_1 = 50, 100, 150$.

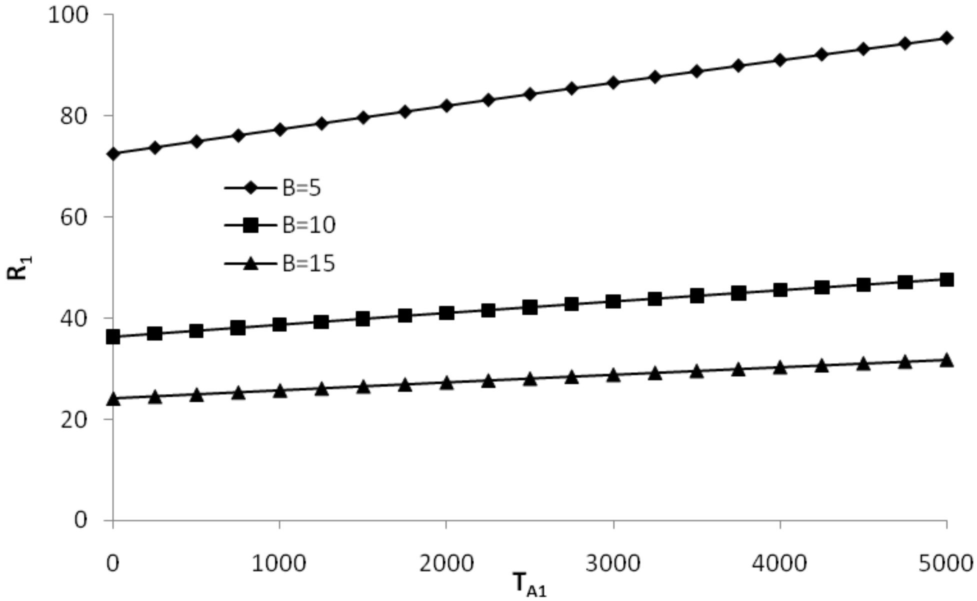

Figure 3. Variation of critical Rayleigh number R_1 with rotation T_{A_1} for fixed value of $F_1 = 10,\ Q_1 = 500$ and $B = 5,10,15.$

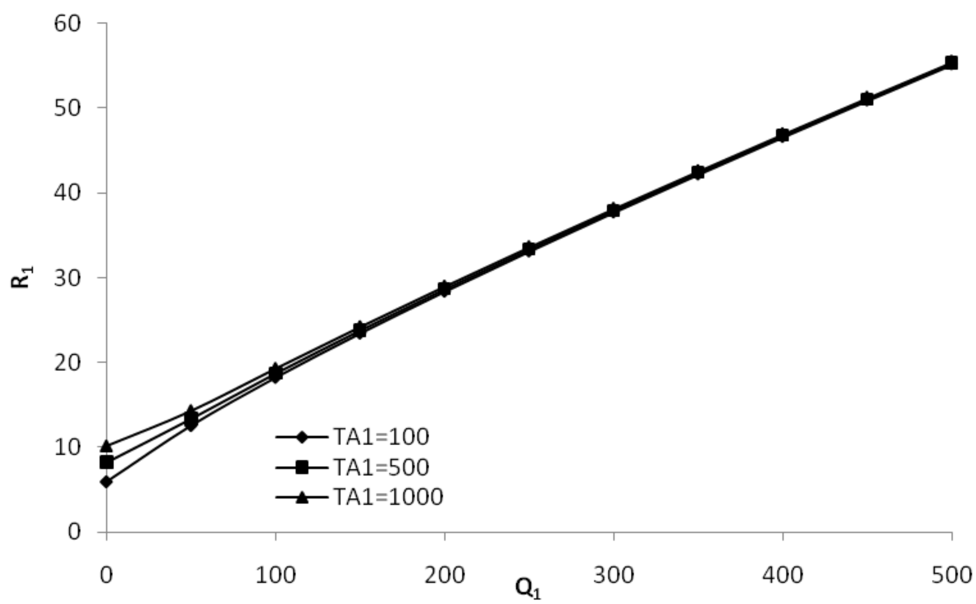

Figure 4. Variation of critical Rayleigh number R_1 with magnetic field Q_1 for fixed value of $F_1 = 10,\ B = 20$ and $T_{A_1} = 100, 500, 1000.$

8. Conclusions

In this present paper, the combined effect of dust particles, magnetic field and rotation on couple-stress fluid heated from below is considered. Dispersion relation governing the effects of dust particles, couple-stress, rotation and magnetic field is derived. The main results from the analysis are summarized as follows:

(i) For the case of stationary convection, dust particles have a destabilizing effect on the system as can be seen from equation (16), and graphically from Figure 1.
(ii) Couple-stress has stabilizing/destabilizing effects on the system for the permissible values of various parameters as can be seen from equation (17). In the absence of rotation, couple-stress clearly has a stabilizing effect on the system as can be seen from equation (18) as derived by Sharma and Sharma (2004).
(iii) For the case of stationary convection, the rotation has a stabilizing effect on the system as can be seen from equation (19), and graphically, from Figure 2 and Figure 3.
(iv) Magnetic field has stabilizing/destabilizing effect on the system for the permissible values of various parameters as can be seen from equation (21), and graphically from fig. 4. In the absence of rotation, magnetic field has a stabilizing effect on the system as can be seen from equation (22) as derived by Sharma and Sharma (2004). The reason for the stabilizing effect of rotation and magnetic field is given by Chandrasekher (1981).
(v) The Principle of Exchange of Stabilities (PES) is found to hold true in the absence of magnetic field and rotation. It is evident from equation (24) that presence of magnetic field and rotation brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence.

Acknowledgement

The authors are grateful to the referees for their technical comments and valuable suggestions, resulting in a significant improvement of the paper.

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