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Comparison Differential Transformation Technique with Adomian Decomposition Method for Dispersive Long-wave Equations in (2+1)-Dimensions

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Abstract

In this paper, we will introduce two methods to obtain the numerical solutions for the system of dispersive long-wave equations (DLWE) in (2+1)-dimensions. The first method is the differential transformation method (DTM) and the second method is Adomian decomposition method (ADM). Moreover, we will make comparison between the solutions obtained by the two methods. Consequently, the results of our system tell us the two methods can be alternative ways for solution of the linear and nonlinear higher-order initial value problems.

Keywords: Systems of partial differential equations, Differential transform method, Adomian decomposition method, Dispersive Long-wave Equations

MSC (2000): 34K28; 35G25; 34K17

1. Introduction

A variety of methods, exact, approximate, and purely numerical are available for the solution of systems of differential equations. Most of these methods are computationally intensive because they are trial-and-error in nature, or need complicated symbolic computations. Integral transforms such as Laplace and Fourier transforms are commonly used to solve differential equations and usefulness of these integral transforms lies in their ability to transform differential equations into algebraic equations which allows simple and systematic solution procedures. However, using integral transform in nonlinear problems may increase its complexity. In the present work, some partial differential equations with nonhomogeneous initial conditions aimed to solve by the differential transformation method and comparison with Adomian decomposition method which was introduced by G. Adomian in 1984. The differential transformation is a numerical method for solving differential equations. The concept of differential transform was introduced by Zhou (1986), who solved linear and nonlinear initial value problems in electric circuit analysis.

Ayaz [(2003), (2004a), Ayaz (2004b)], and Kangalgil and Ayaz (2008) developed this method for PDEs and obtained closed form series solutions for linear and nonlinear initial value problems. The differential transforms method an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally taken long time for large orders. The present method reduces the size of computational domain and applicable to many problems easily. Adomian decomposition method which is given by Jin and Liu (2005), for approximate solution of linear and nonlinear differential equations and to the solutions of various scientific models such that in El-Wakil and Abdou (2007), Jin and Liu (2005), Khalifa et al. (2007). A distinctive practical feature of the differential transformation method DTM is ability to solve linear or nonlinear differential equations. In fact, DTM and ADM are very efficient methods to find the numerical and analytic solutions of differential-difference equations, delay differential equations as well as integral equations we can see that in Karakoc and Bereketoglu (2009), Arikoglu and Ozkol (2006) and Rahman and Fatt (2009). Higher-order dimensional differential transformations are applied to a few some initial value problems to show that the solutions obtained by the proposed method DTM coincide with the approximate solution ADM and the analytic solutions.

System of Dispersive Long-wave Equations

In this paper we will study the system of dispersive long-wave equations (DLWE) in (2+1)-dimensions by using differential transformation method (DTM) and Adomian decomposition method (ADM) and Compare them with exact solution

$$u_{yt} + v_{xx} + \frac{1}{2}\left(u^2\right)_{xy} = 0, \tag{1.1}$$

$$v_t + \left(uv + u + u_{xy}\right)_x = 0. \tag{1.2}$$

Equations (1.1) and (1.2) can be reduced to the (1+1)-dimensions model:

$$u_t + v_z + \frac{1}{2}\left(u^2\right)_z = 0, \tag{1.3}$$

$$v_t + \left(uv + u + u_{zz}\right)_z = 0, \tag{1.4}$$

for $u = u(z,t) = u(x+y,t)$ and $v = v(z,t) = v(x+y,t)$ with initial conditions

$$u(z,0) = 2 + 2\ Tanh\ [z], \quad \text{and} \quad v(z,0) = -1 + 2\ \ Sech\ [z]^2. \tag{1.5}$$

The analytical solution of the problem is given by Bai et al. (2006) as follows:

$$u(z,t) = 2 + 2\ Tanh\ [z-2t] \text{ and } v(z,t) = -1 + 2\ \ Sech\ [z-2t]^2. \tag{1.6}$$

2. The Definitions and Operations of Differential Transform

The basic definitions and fundamental theorems of the two- dimensional transform are defined by Ayaz (2003) as follows:

$$W(k,h) = \frac{1}{k!\ h!}\left[\frac{\partial^{k+h} w(x,y)}{\partial x^k\ \partial y^h}\right]_{(0,0)}, \tag{2.1}$$

where $W(x,y)$ is the original function and $W(k,h)$ is the transformed function. The transformation is called T-function and the lower case and upper case letters represent the original and transformed functions respectively. The differential inverse transform of $W(k,h)$ is defined as:

$$W(x,y) = \sum_{h=0}^{\infty}\sum_{k=0}^{\infty} W(k,h) x^k y^h. \tag{2.2}$$

From Equations (2.1) and (2.2) we obtain

$$W(x,y) = \sum_{h=0}^{\infty}\sum_{k=0}^{\infty} \frac{1}{k!\ h!}\left[\frac{\partial^{k+h} W(x,y)}{\partial x^k\ \partial v^h}\right]_{(0,0)} x^k y^h. \tag{2.3}$$

The fundamental theorems of the two-dimensional transform have been proved in Chen and Ho (1999) and we only mention them here in order to use them in Theorem 8.

Theorem 1:

If $w(x,y)=u(x,y)\pm v(x,y)$, then $W(k,h)=U(k,h)\pm V(k,h)$.

Theorem 2:

If $w(x,y)=\lambda u(x,y)$, then $W(k,h)=\lambda\ U(k,h)$ where λ is constant.

Theorem 3:

If $w(x,y)=\dfrac{\partial u(x,y)}{\partial x}$, then $W(k,h)=(k+1)\ U(k+1,h)$.

Theorem 4:

If $w(x,y)=\dfrac{\partial u(x,y)}{\partial y}$, then $W(k,h)=(h+1)\ U(k,h+1)$.

Theorem 5:

If $w(x,y)=\dfrac{\partial^{r+s} u(x,y)}{\partial x^r \partial y^s}$, then

$$W(k,h)=(k+1)(k+2)\cdots(k+r)(h+1)(h+2)\cdots(h+s)\ U(k+r,h+s).$$

Theorem 6:

If $w(x,y)=u(x,y)v(x,y)$, then $W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{k} U(r,h-s)\ V(k-r,s)$.

Theorem 7:

If $w(x,y)=x^n y^m$, then $W(k,h)=\delta(k-m,h-n)=\delta(k-m)\delta(h-n)$, where

$$\delta(k-m,h-n)=\begin{cases}1, & k=m \text{ and } h=n,\\ 0, & \text{Otherwise.}\end{cases}$$

Theorem 8:

If $w(x,y) = u(x,y)\dfrac{\partial u(x,y)}{\partial x}$, then

$$W(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{k}(k-r+1)U(r,h-s)\;U(k-r+1,s).$$

Proof:

The proof of Theorem 8 is a consequence of Theorem 3 and Theorem 6.

3. Analysis of the ADM method

We consider Equations (1.3) and (1.4) in the operator form

$$L_t u = -L_z v - N(u), \tag{3.1}$$

$$L_t v = -M(u,v) - K(u,v) - L_z u - L_{zzz} u, \tag{3.2}$$

where $L_t = \dfrac{\partial}{\partial t}$, $L_z = \dfrac{\partial}{\partial z}$ and $L_{zzz} = \dfrac{\partial^3}{\partial z^3}$ symbolize the linear differential operators and the notations $N(u) = uu_z, M(u,v) = uv_z$ and $K(u,v) = vu_z$ symbolize the nonlinear operators.

Applying integration inverse operator $L_t^{-1} = \int_0^t (\square)dt$ to the system (3.1) and (3.2) and using the specified initial conditions yields

$$u(z,t) = u(z,0) - L_t^{-1}L_z v - L_t^{-1}N(u), \tag{3.3}$$

and

$$v(z,t) = v(z,0) - L_t^{-1}M(u) - L_t^{-1}K(u) - L_t^{-1}L_z u - L_t^{-1}L_{zzz}u. \tag{3.4}$$

The Adomain decomposition method assumes an infinite series solution for unknown function $u(z,t)$ and $v(z,t)$ given by

$$u(z,t)=\sum_{n=0}^{\infty}u_n(z,t), \tag{3.5}$$

and

$$v(z,t)=\sum_{n=0}^{\infty}v_n(z,t). \tag{3.6}$$

The nonlinear operators $N(u)=uu_z, M(u,v)=uv_z$ and $K(u,v)=vu_z$ which is defined by the infinite series of Adomain polynomials given by

$$N(u)=\sum_{n=0}^{\infty}D_n(u_0,u_1,\cdots,u_n), \quad M(u,v)=\sum_{n=0}^{\infty}M_n(u_0,u_1,\cdots,u_n,v_0,v_1,\cdots,v_n),$$

and

$$K(u,v)=\sum_{n=0}^{\infty}W_n(u_0,u_1,\cdots,u_n,v_0,v_1,\cdots,v_n),$$

where D_n, M_n and W_n are the appropriate Adomian's polynomials which are generated according to algorithm determined in Zhou et al. (2005).

For nonlinear operator $N(u)$ these polynomials can be defined as

$$D_n(u_0,u_1,\cdots u_n)=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^k u_k\left(\sum_{n=0}^{\infty}\lambda^k u_k\right)_z\right)\right]_{\lambda=0}, \quad n\geq 0. \tag{3.7}$$

Similarly, for nonlinear operator $M(u,v)$, we have

$$M_n(u_0,u_1,\cdots u_n,v_0,v_1,\cdots,v_n)=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^k u_k\left(\sum_{n=0}^{\infty}\lambda^k v_k\right)_z\right)\right]_{\lambda=0}, \quad n\geq 0, \tag{3.8}$$

$$K_n(u_0,u_1,\cdots u_n,v_0,v_1,\cdots,v_n)=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^k v_k\left(\sum_{n=0}^{\infty}\lambda^k u_k\right)_z\right)\right]_{\lambda=0}, \quad n\geq 0. \tag{3.9}$$

These formulae are easy to set computer code to get as many polynomials as we need in calculation of the numerical as well as explicit solutions. For the sake of convenience of the readers, we can give the first few Adomian polynomials for

$$N(u)=uu_z, M(u,v)=uv_z \text{ and } K(u,v)=vu_z$$

of the nonlinearity as

$$\begin{aligned}
D_0 &= u_0(u_0)_z \\
D_1 &= u_1(u_0)_z + u_0(u_1)_z \\
D_2 &= u_2(u_0)_z + u_1(u_1)_z + u_0(u_2)_z \\
D_3 &= u_3(u_0)_z + u_2(u_1)_z + u_1(u_2)_z + u_0(u_3)_z \\
D_4 &= u_4(u_0)_z + u_3(u_1)_z + u_2(u_2)_z + u_1(u_3)_z + u_1(u_4)_z \\
&\cdots
\end{aligned}$$

and

$$\begin{aligned}
M_0 &= u_0(v_0)_z \\
M_1 &= u_1(v_0)_z + u_0(v_1)_z \\
M_2 &= u_2(v_0)_z + u_1(v_1)_z + u_0(v_2)_z \\
M_3 &= u_3(v_0)_z + u_2(v_1)_z + u_1(v_2)_z + u_0(v_3)_z \\
M_4 &= u_4(v_0)_z + u_3(v_1)_z + u_2(v_2)_z + u_1(v_3)_z + u_1(v_4)_z \\
&\cdots
\end{aligned}$$

and

$$\begin{aligned}
W_0 &= v_0(u_0)_z \\
W_1 &= v_1(u_0)_z + v_0(u_1)_z \\
W_2 &= v_2(u_0)_z + v_1(u_1)_z + v_0(u_2)_z \\
W_3 &= v_3(u_0)_z + v_2(u_1)_z + v_1(u_2)_z + v_0(u_3)_z \\
W_4 &= v_4(u_0)_z + v_3(u_1)_z + v_2(u_2)_z + v_1(u_3)_z + v_1(u_4)_z \\
&\cdots
\end{aligned}$$

and so on. The rest of the polynomials can be constructed in a similar manner. Substituting the initial conditions into (3.3) and (3.4) identifying the zeroth components u_0 and v_0, then we obtain the subsequent components by the following recursive equations by using the standard ADM

$$u_{n+1} = -L_t^{-1} L_z v_n - L_t^{-1} A_n, \tag{3.10}$$

and

$$v_{n+1} = -L_t^{-1} M_n - L_t^{-1} W_n - L_t^{-1} L_z u_n - L_t^{-1} L_{zzz} u_n. \tag{3.11}$$

the one can formulate the recursive algorithm for u_0 and v_0 and general term u_{n+1} and v_{n+1} in a form of the modified recursive scheme as follows:

$$\begin{aligned} &u_0 = u(z,0), \\ &u_1 = -L_t^{-1} L_z v_0 - L_t^{-1} A_0, \\ &u_{n+1} = -L_t^{-1} L_z v_n - L_t^{-1} A_n, \; n \geq 1, \end{aligned} \tag{3.12}$$

and

$$\begin{aligned} &v_0 = v(z,0), \\ &v_1 = -L_t^{-1} M_0 - L_t^{-1} W_0 - L_t^{-1} L_z u_0 - L_t^{-1} L_{zzz} u_0, \\ &v_{n+1} = -L_t^{-1} M_n - L_t^{-1} W_n - L_t^{-1} L_z u_n - L_t^{-1} L_{zzz} u_n, \qquad n \geq 1. \end{aligned} \tag{3.13}$$

This type of modification is giving more flexibility to the ADM in order to solve complicate nonlinear differential equations. In many cases the modified decomposition scheme avoids the unnecessary computation especially in calculation of the Adomian polynomials. The computation of these polynomials will be reduced very considerably by using the ADM.

It is worth noting that the zeroth components u_0 and v_0 are defined then the remaining components u_n and v_n, $n \geq 1$ can be completely determined. As a result, the components u_0, u_1, ..., and v_0, v_1, ..., are identified and the series solutions thus entirely determined. However, in many cases the exact solution in a closed form may be obtained.

4. Numerical Illustrations

4.1. Differential transformation method

Taking the differential transformation of (1.3) and (1.4), can be obtained

$$(h+1)U[k,h+1] = -(k+1)V[k+1,h] - \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} (k-r+1)U[r,h-s]U[k-r+1,s], \tag{4.1.1}$$

$$(h+1)V[k,h+1] = -\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}(k-r+1)U[r,h-s]V[k-r+1,s]$$
$$-\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}(k-r+1)V[r,h-s]U[k-r+1,s]-(k+1)U[k+1,h]$$
$$-(k+1)(k+2)(k+3)U[k+3,h]. \tag{4.1.2}$$

From the initial conditions (1.5), we can write

$$U[0,0]=2;\quad U[1,0]=2;\quad U[2,0]=0;\quad U[3,0]=-\frac{2}{3};\quad U[4,0]=0;$$
$$U[5,0]=\frac{4}{15};\quad U[6,0]=0;\quad U[7,0]=-\frac{34}{315};\quad U[8,0]=0;\quad U[9,0]=\frac{124}{2835};$$
$$U[10,0]=0;\quad U[11,0]=-\frac{2764}{155925};\quad U[12,0]=0. \tag{4.1.3}$$

$$V[0,0]=1;\quad V[1,0]=0;\quad V[2,0]=-2;\quad V[3,0]=0;\quad V[4,0]=\frac{4}{3};$$
$$V[5,0]=0;\quad V[6,0]=-\frac{34}{45};\quad V[7,0]=0;\quad V[8,0]=\frac{124}{315};\quad V[9,0]=0;$$
$$V[10,0]=-\frac{2764}{14175};\quad V[11,0]=0;\quad V[12,0]=\frac{43688}{467775}. \tag{4.1.4}$$

Substituting from (4.1.3) and (4.1.4) into (4.1.1) and (4.1.2) and by recursive method we have:

$$U[0,1]=-4;\quad U[1,1]=0;\quad U[2,1]=4;\quad U[3,1]=0;\quad U[0,2]=0;$$
$$U[4,1]=-\frac{8}{3};\quad U[5,1]=0;\quad U[6,1]=\frac{68}{45};\quad U[7,1]=0;\quad U[1,2]=-8;$$
$$U[2,2]=0;\quad U[3,2]=\frac{32}{3};\quad U[4,2]=0;\quad U[5,2]=-\frac{136}{15};\quad U[8,1]=-\frac{248}{315};$$

$$U[6,2]=0;\quad U[9,1]=0;\quad U[3,3]=0;\quad U[4,3]=\frac{272}{9};\quad U[7,2]=\frac{1984}{315};$$
$$U[2,3]=-\frac{64}{3};\quad U[0,3]=\frac{16}{3};\quad U[1,3]=0;\quad U[5,3]=0;\quad U[8,2]=0;$$
$$U[4,4]=0;\quad U[11,1]=0;\quad U[7,3]=0;\quad U[0,4]=0;\quad U[1,4]=\frac{64}{3};$$
$$U[2,4]=0;\quad U[3,4]=-\frac{544}{9};\quad U[10,1]=\frac{5528}{14175};\quad U[6,3]=-\frac{3968}{135}. \tag{4.1.5}$$

and

$$V[0,1]=0;\quad V[1,1]=8;\quad U[2,1]=0;\quad U[3,1]=-\frac{32}{3};\quad V[0,2]=-8;$$
$$V[4,1]=0;\quad V[5,1]=\frac{136}{15};\quad V[6,1]=0;\quad V[7,1]=-\frac{1984}{315};\quad V[1,2]=0;$$
$$V[2,2]=32;\quad V[3,2]=0;\quad V[4,2]=-\frac{136}{3};\quad V[5,2]=0;\quad V[8,1]=0;$$
$$V[6,2]=\frac{1984}{45};\quad V[9,1]=\frac{11056}{2835};\quad V[3,3]=\frac{1088}{9};\quad V[4,3]=0;\quad V[7,2]=0;$$
$$V[2,3]=0;\quad V[0,3]=0;\quad V[1,3]=-\frac{128}{3};\quad V[5,3]=-\frac{7936}{45};$$
$$V[8,2]=-\frac{11056}{2835};\quad V[4,4]=\frac{3968}{9};\quad V[7,2]=0;\quad V[0,4]=\frac{64}{3};$$
$$V[1,4]=0;\quad V[2,4]=-\frac{544}{3};\quad V[3,4]=0. \tag{4.1.6}$$

Substituting all $U(k, h)$ and $V(k, h)$ into (2.2) we have :

$$u(z,t)=2-4t+\frac{16t^3}{3}+2z-8t^2z+\frac{64t^4z}{3}+4tz^2-\frac{64t^3z^2}{3}-\frac{2z^3}{3} +\frac{32t^2z^3}{3}-\frac{544t^4z^3}{9}-\frac{8tz^4}{3}+\frac{272t^3z^4}{9}+\cdots, \tag{4.1.7}$$

and

$$v(z,t)=1-8t^2+\frac{64t^4}{3}+8tz-\frac{128t^3z}{3}-2z^2+32t^2z^2-\frac{544t^4z^2}{3}-\frac{32tz^3}{3}+\frac{1088t^3z^3}{9}-\frac{136t^2z^4}{3}+\frac{4z^4}{3}+\frac{3968t^4z^4}{9}+\cdots. \tag{4.1.8}$$

4.2. Implementation of the ADM Method

Using (3.10)-(3.13) with (3.7)-(3.9) for Equations (1.3) and (1.4) with the initial conditions (1.5) gives:

$$u_0=2+2Tanh[z]$$
$$u_1=-4tSech[z]^2$$
$$u_2=-8t^2Sech[z]^2Tanh[z]$$
$$u_3=-\frac{16}{3}t^3\left(-2+Cosh[2z]\right)Sech[z]^4$$
$$u_4=-\frac{8}{3}t^4Sech[z]^5\left(-11Sinh[z]+Sinh[3z]\right)$$
$$u_5=-\frac{16}{15}t^5\left(33-26Cosh[2z]+Cosh[4z]\right)Sech[z]^6,\cdots$$

and

$$v_0=-1+2Sech[z]^2$$
$$v_1=8tSech[z]^2Tanh[z]$$
$$v_2=8t^2\left(-2+Cosh[2z]\right)Sech[z]^4$$
$$v_3=\frac{16}{3}t^3\left(-11Sinh[z]+Sinh[3z]\right)Sech[z]^5$$
$$v_4=\frac{8}{3}t^4Sech[z]^6\left(33-26Cosh[2z]+Cosh[4z]\right)$$
$$v_5=\frac{16}{15}t^5\left(302Sinh[z]-57Sinh[3z]+Sinh[5z]\right)Sech[z]^7,\cdots,$$

and so on. In this manner the other components of the decomposition series can be easily obtained of which u(z, t) and v(z, t) were evaluated in a series form

$$u(z,t) = 2 - 4tSech[z]^2 - \frac{16}{3}t^3\left(-2 + Cosh[2z]\right)Sech[z]^4 - \frac{16}{15}t^5\left(33 - 26Cosh[2z] + Cosh[4z]\right)Sech[z]^6 - \frac{8}{3}t^4 Sech[z]^5\left(-11Sinh[z] + Sinh[3z]\right) + \cdots \tag{4.2.1}$$

$$v = -1 + 2Sech[z]^2 + 8tSech[z]^2 Tanh[z] + 8t^2\left(-2 + Cosh[2z]\right)Sech[z]^4 + \frac{16}{3}t^3\left(-11Sinh[z] + Sinh[3z]\right)Sech[z]^5 + \frac{8}{3}t^4 Sech[z]^6 (33 - 26Cosh[2z] + Cosh[4z]) + \frac{16}{15}t^5\left(302Sinh[z] - 57Sinh[3z] + Sinh[5z]\right)Sech[z]^7 + \cdots. \tag{4.2.2}$$

Table 1. Comparison of exact solution, the (DTM) [formula 4.1.7] and error of $u(x, t)$, ($x = -0.01$).

t	U exact	U numerical	Error
0.01	1.9400179935223598	1.9400179944009066	8.7855×10^{-10}
0.02	1.9000832500842400	1.9000832773454222	2.72612×10^{-8}
0.03	1.8602282193671420	1.8602284259237865	2.06557×10^{-7}
0.04	1.8204844305056798	1.8204852993740800	8.68868×10^{-7}
0.05	1.7808830595711410	1.7808857057488887	2.64618×10^{-6}
0.06	1.7414548327878834	1.7414614019153070	6.56913×10^{-6}
0.07	1.7022299327533639	1.7022440935549332	1.41608×10^{-5}
0.08	1.6632379082583706	1.6632654351638756	2.75269×10^{-5}
0.09	1.6245075882634292	1.6245570300527468	4.94418×10^{-5}
0.10	1.5860670005410948	1.5861504303466670	8.34298×10^{-5}

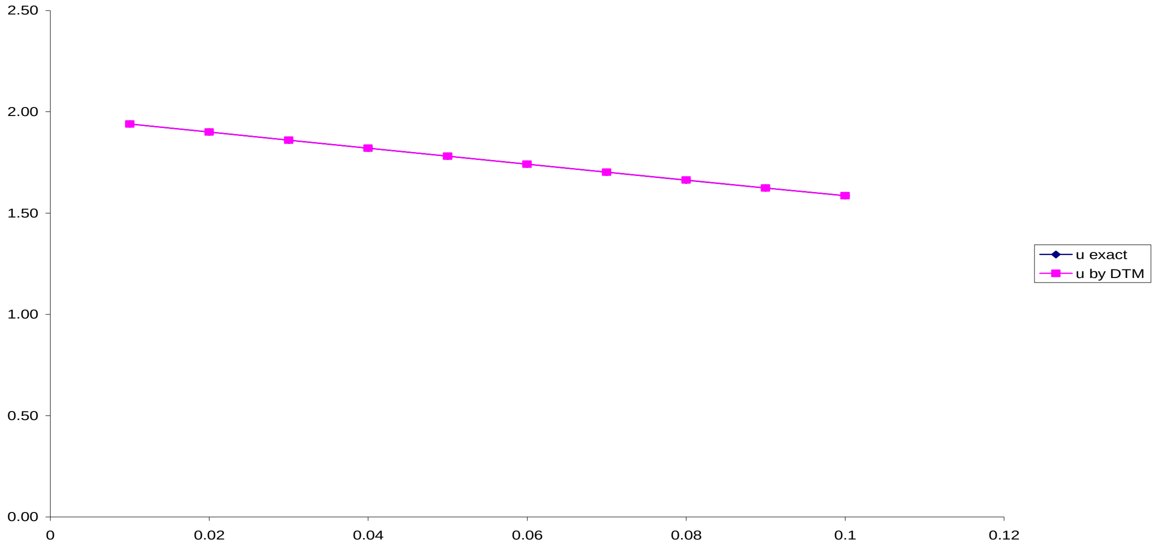

Figure 1. Plot of exact solution and numerical solution by the (DTM) [formula 4.1.7] of $u(x,t)$

Table 2. Comparison of exact solution, the (DTM) [formula 4.1.8] and error of $v(x, t)$, (x = -0.01).

t	V exact	V numerical	Error
0.01	0.9982010794494582	0.9982010796524885	2.0303×10^{-10}
0.02	0.9950083215431356	0.9950083292842610	7.74113×10^{-9}
0.03	0.9902319246693598	0.9902319949769045	7.03075×10^{-8}
0.04	0.9838870801545647	0.9838874251277311	3.44973×10^{-7}
0.05	0.9759938832085482	0.9759950837831111	1.20057×10^{-6}
0.06	0.9665771982556295	0.9665805506384725	3.35238×10^{-6}
0.07	0.9556664935259673	0.9556745210383017	8.02751×10^{-6}
0.08	0.9432956467829017	0.9433128059761436	1.71592×10^{-5}
0.09	0.9295027243641272	0.9295363320946007	3.36077×10^{-5}
0.10	0.9143297359794771	0.9143911416853333	6.14057×10^{-5}

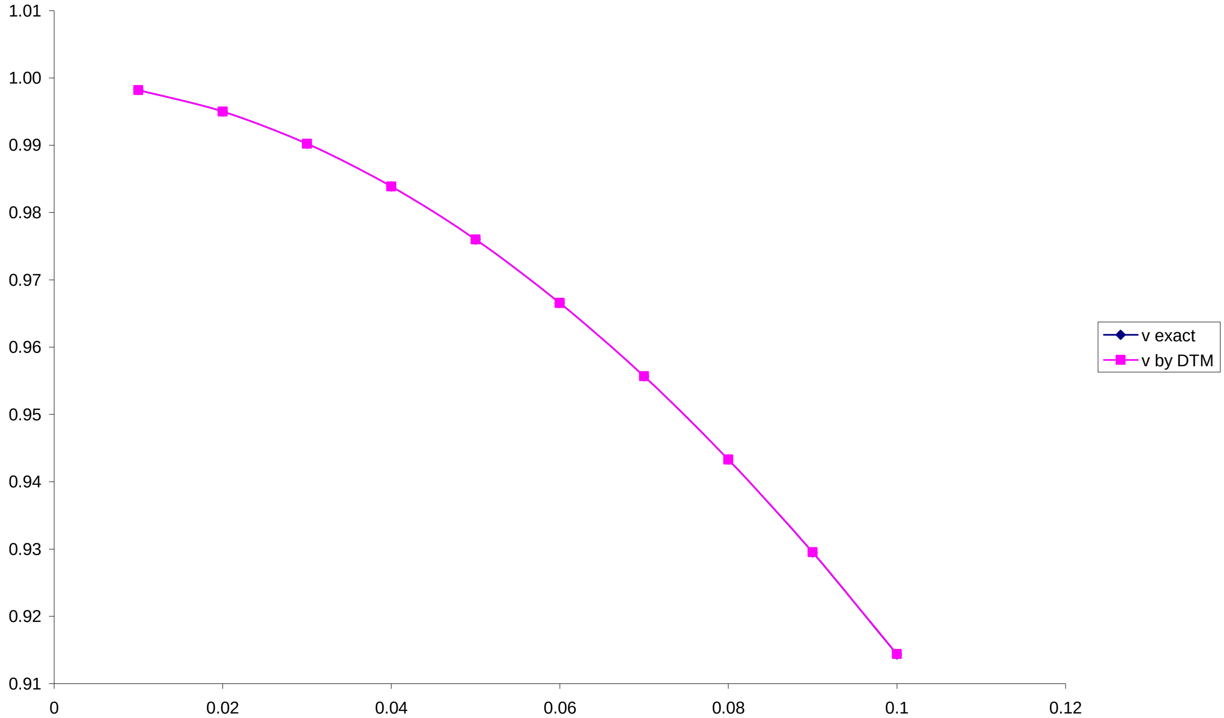

Figure 2. Plot of exact solution and numerical solution by the (DTM) [formula 4.1.8] of $v(x,t)$

Table 3. Comparison of exact solution, the (ADM) [formula 4.2.1] and error of $u(x, t)$, (x = -0.01).

t	U exact	U numerical	Error
0.01	1.9400179935223598	1.9400179935217388	6.19949×10^{-13}
0.02	1.9000832500842400	1.9000832500356863	4.856×10^{-11}
0.03	1.8602282193671420	1.8602282187141876	6.5296×10^{-10}
0.04	1.8204844305056798	1.8204844262781286	4.22755×10^{-9}
0.05	1.7808830595711410	1.7808830413239556	1.82472×10^{-8}
0.06	1.7414548327878834	1.7414547720106959	6.07772×10^{-8}
0.07	1.7022299327533639	1.7022297637469743	1.69006×10^{-7}
0.08	1.6632379082583706	1.663237496878035	4.1138×10^{-7}
0.09	1.6245075882634292	1.6245066843727567	9.03891×10^{-7}
0.10	1.5860670005410948	1.5860651695106742	1.83103×10^{-6}

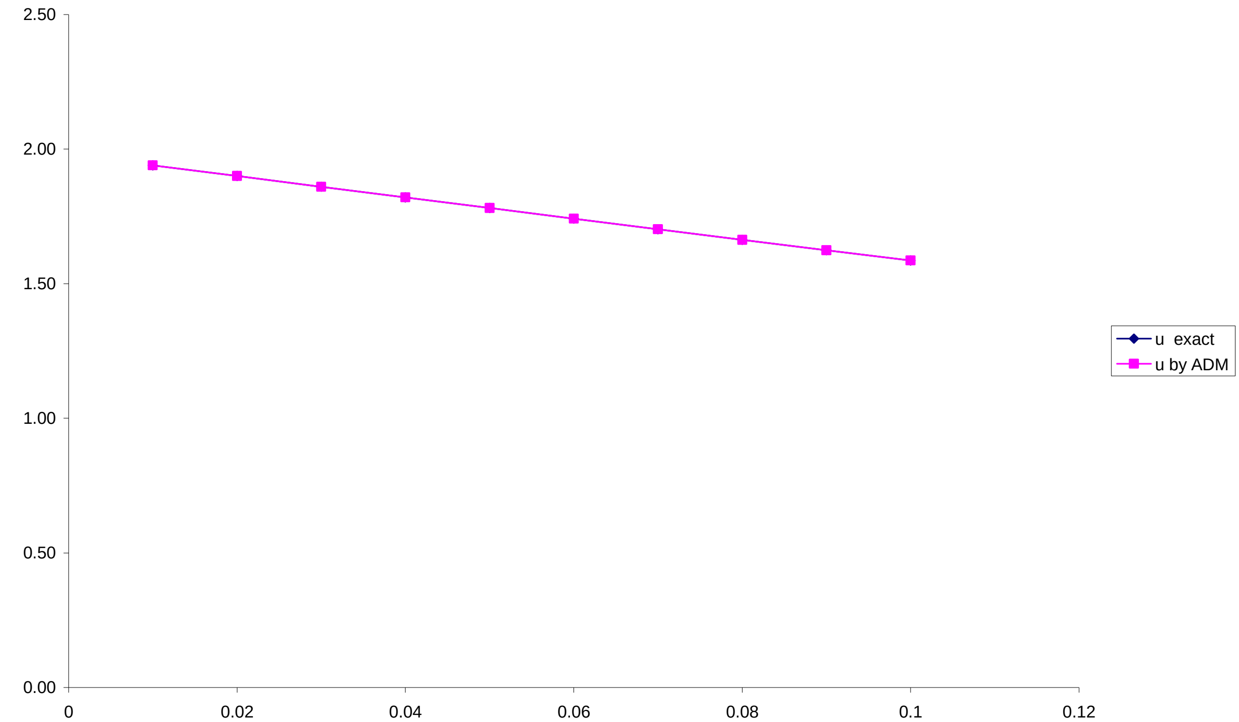

Figure 3. Plot of exact solution and numerical solution by the (ADM) [formula 4.2.1] of $u(x, t)$

Table 4. Comparison of exact solution, the (ADM) [formula 4.2.2] and error of $v(x, t)$, (x = -0.01).

t	V exact	V numerical	Error
0.01	0.9982010794494582	0.9982010794976929	4.8234×10^{-11}
0.02	0.9950083215431356	0.9950083246256554	3.08252×10^{-9}
0.03	0.9902319246693598	0.990231959715404	3.5046×10^{-8}
0.04	0.9838870801545647	0.9838872766158613	1.96461×10^{-7}
0.05	0.9759938832085482	0.9759946306263977	7.47418×10^{-7}
0.06	0.9665771982556295	0.9665794230972938	2.22484×10^{-6}
0.07	0.9556664935259673	0.9556720840302048	5.5905×10^{-6}
0.08	0.9432956467829017	0.9433080546786226	1.24079×10^{-5}
0.09	0.9295027243641272	0.9295277701483391	2.50458×10^{-5}
0.10	0.9143297359794771	0.9143766419979097	4.6906×10^{-5}

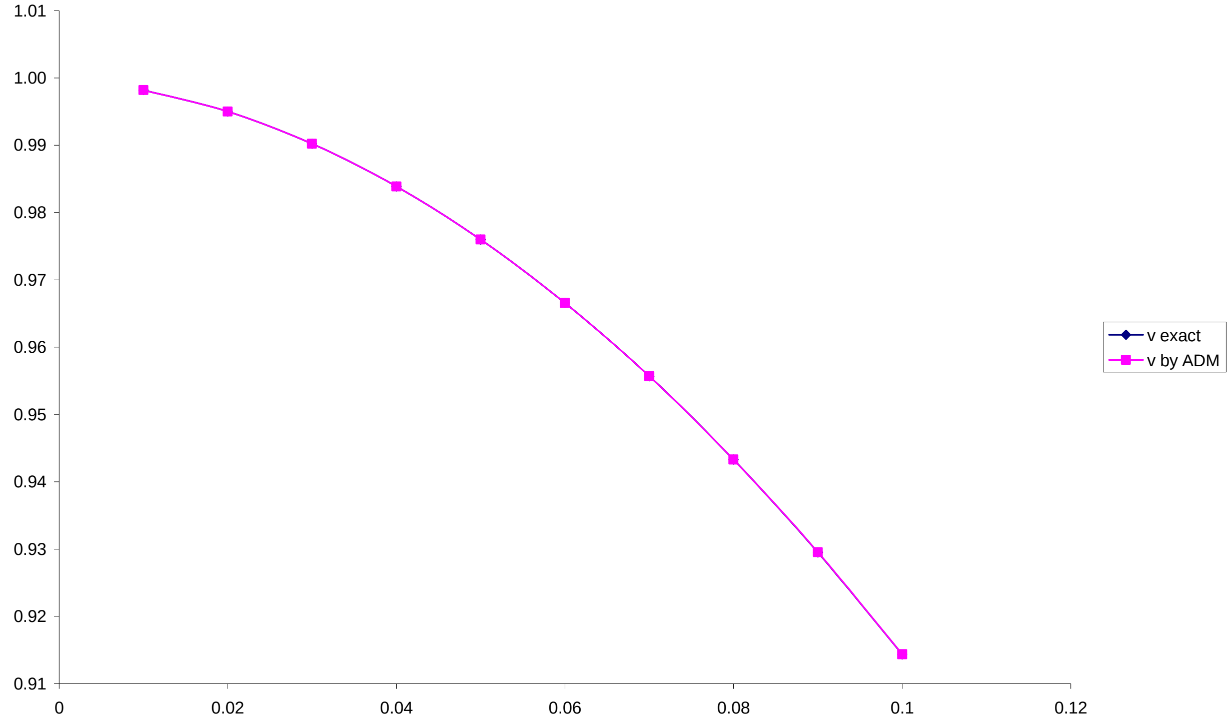

Figure 4. Plot of exact solution and numerical solution by the (ADM) [formula 4.2.2] of $v(x, t)$

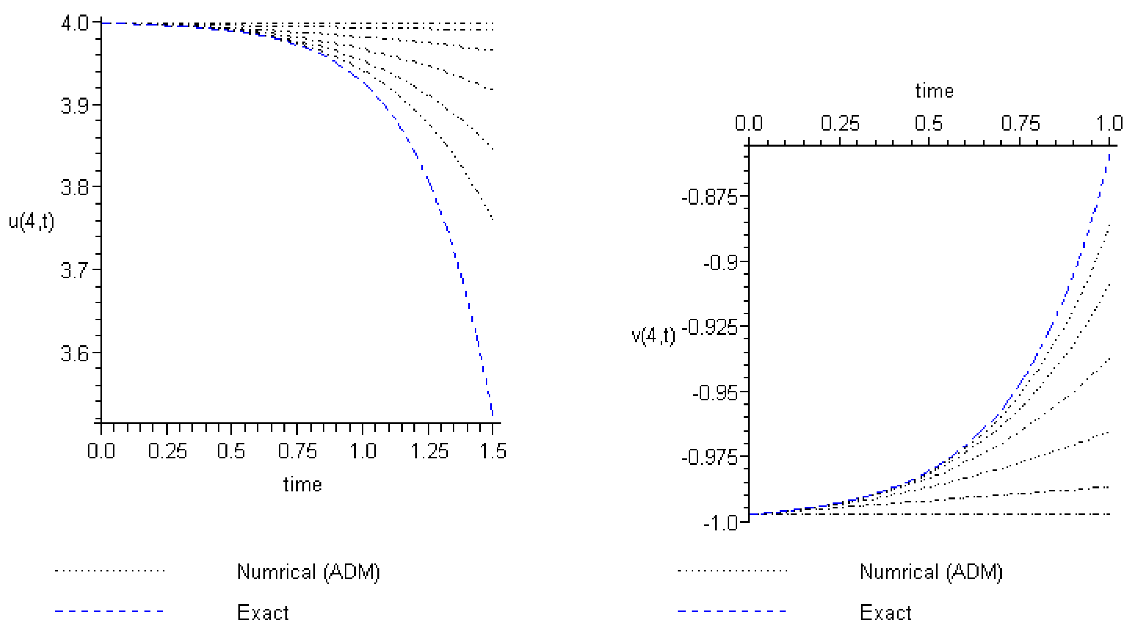

Figure 5. Comparison between the exact solution and the behavior of the solution obtained by ADM [formula 4.2.1, 4.2.2 respectively] method, $z = 4$

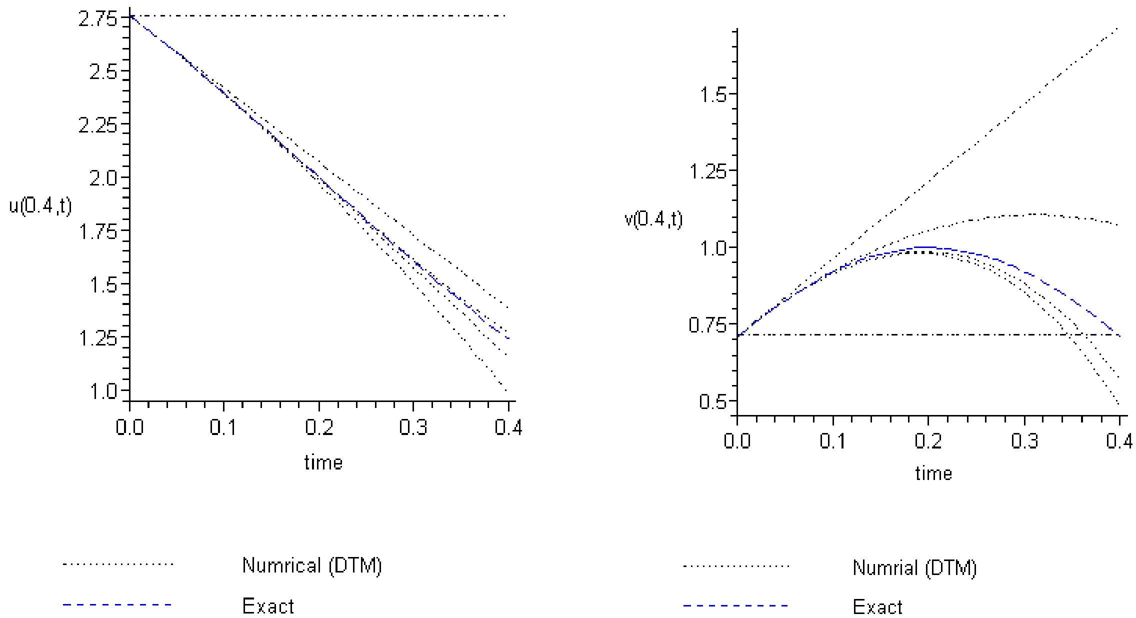

Figure 6. Comparison between the exact solution and the behavior of the solution obtained by DTM [formula 4.1.7, 4.1.8 respectively] method, $z = 0.4$

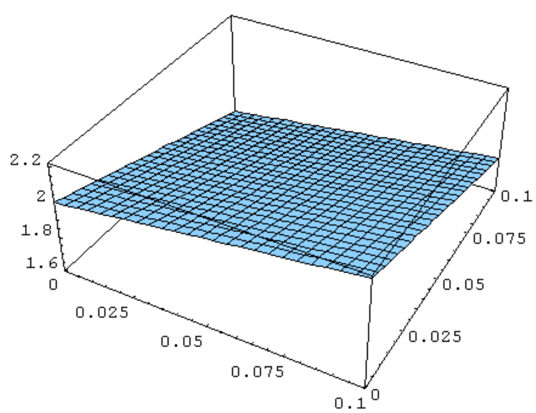

Figure 7a. The Exact solution of $u(x, t)$

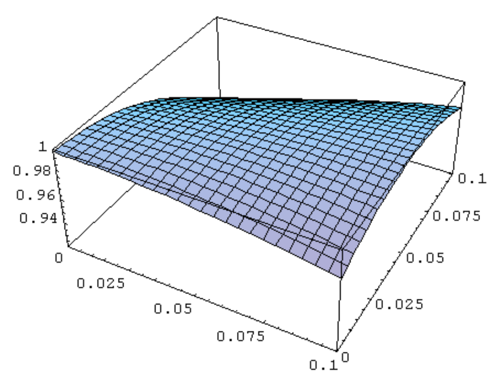

Figure 7b. The Exact solution of $v(x, t)$

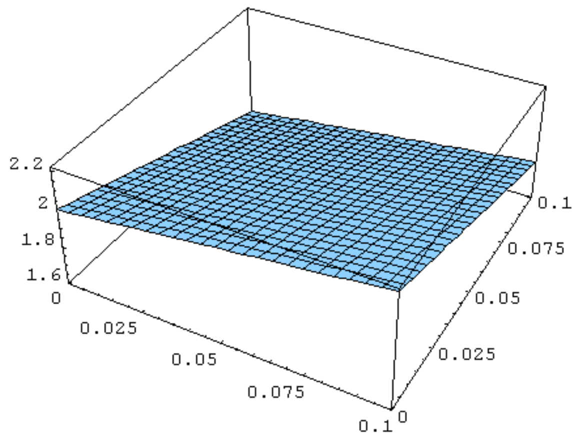

Figure 8b. The solution of $v(x, t)$ by (DTM)

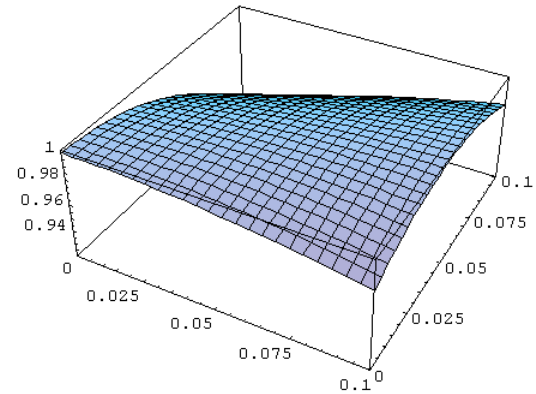

Figure 8a. The solution of $u(x, t)$ by (DTM)

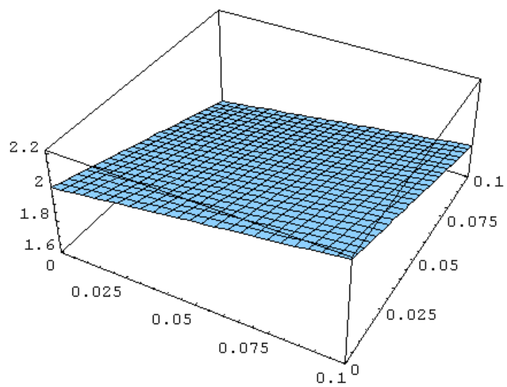

Figure 9a. The solution of $u(x, t)$ by (ADM)

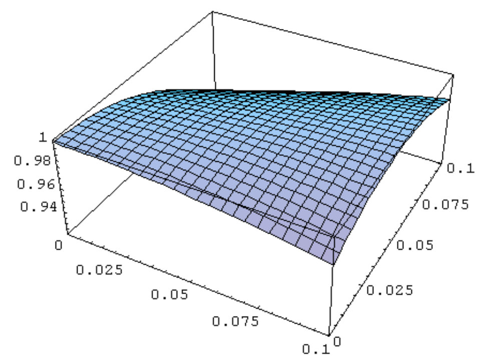

Figure 9b. The solution of $v(x, t)$ by (ADM)

5. Conclusion

This paper applied the differential transformation technique and Adomian decomposition method to solve initial value problem. Throughout the result of our example which are found by using the two methods are compared with the analytic solutions, we show that the convergence are quite close. Too the results of our example tell us the two successfully methods can be alternative way for the solution of the linear and nonlinear higher-order initial value problems. In fact, DTM and ADM are very efficient methods to find the numerical and analytic solutions of differential-difference equations, delay differential equations as well as integral equations.

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REFERENCES

Adomian, G. (1984). Convergent series solution of nonlinear equation, Comput. Appl. Math., 11, 113-117.

Arikoglu, A., Ozkol, I. (2006). Solution of differential-difference equations by using differential transform method. Appl. Math. Comput., 181, 153-162.

Ayaz, F. (2003). On the two-dimensional differential transform method, Appl. Math. Comput.,143, 361-374.

Ayaz, F. (2004). Solutions of the system of differential equations by differential transform method, Appl. Math. Comput., 147, 547–567.

Ayaz, F. (2004). Applications of differential transform method to differential-algebraic equations, Appl. Math. Comput., 152 649–657.

Bai, C. L., Zhao, H., Ji, Han G. (2006). Application of an extended homogeneous balance method to new exact solutions of nonlinear evolution equations, Czechoslovak Journal of Physics,, Vol. 56, No.3.

Chen, C.K., Ho, S. H. (1999). Solving partial differential equations by two dimensional differential transform, Appl. Math. Comput., 106, 171–179.

El-Wakil, S. A., Abdou, M. A. (2007). New applications of Adomian decomposition method, Chaos, Solitons and Fractals, 33, 513–522.

Jin, C., Liu, M. (2005). A new modification of Adomian decomposition method for solving a kind of evolution equation, Applied Mathematics and Computation, 169, 953–962.

Kangalgil, F., Ayaz, F. (2009). Solitary wave solutions for the KdV and mKdV equations by differential transform method, Chaos, Solitons and Fractals, 41, 464-472.

Karakoc, F., Bereketoglu, H. (2009). Solutions of delay differential equations by using differential transform method. Int. J. Comput. Math., 86, 914-923.

Khalifa, A. K., Raslan, K. R., Alzubaidi, H. M. (2007). Numerical study using ADM for the modified regularized long wave equation, Applied Mathematical Modelling.

Rahman, A. A., Fatt, F. M. (2009). Adomian decomposition method for two-dimensional nonlinear Volterra integral equations of the second kind. Far East J. Appl. Math., 34, 167-179.

Zhou, JK. (1986). Differential transformation and its application for electrical circuits, Wuhan, China: Huarjung University Press; [in Chinese].

Zhu, Y., Chang, Q., Wu, S. (2005). A new algorithm for calculating Adomian polynomials. Applied Mathematics and Computation, 169, 402–416.