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A Resource Based Stage-Structured Fishery Model With Selective Harvesting of Mature Species

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Abstract

In this paper we have considered a model in which revenue is generated from fishing and the growth of the fish depends upon the plankton which in turn follows a logistic law of growth. Here the fish population has two stages, a juvenile stage and a mature stage and we consider the harvesting of the mature fish species. Stability and permanence of the system are discussed. Maximum sustainable yield, maximum economic yield and optimal sustainable yield are obtained and different tax policies are discussed to achieve the reference points.

Key Words: Plankton, global stability, permanence, maximum sustainable yield, optimal sustainable yield.

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1. Introduction

Nowadays, exploitation of biological resources has been increased by people's multifarious material needs, which attract a global concern to protect the limited biological resources. Therefore, regulation of exploitation of biological resources has become a problem of major concern in view of dwindling resource stocks and the deteriorating environment. It is necessary to establish a constructive management of commercial exploitation of the biological resources.

A sustainable management policy is to be implemented by taking some specific objectives as (i) to set the goals to be achieved in order to ensure the exploitation of the resources, (ii) to identify possible indicators of sustainability for each goal, (iii) to establish the reference values for each indicators, (iv) to identify the measures to be implemented in order to achieve objectives determined on the basis of the specific conditions of the systems.

On logical consideration, random fishing of all fishes is not advisable for the persistence of the fishery. Generally, speaking, the exploitation of a population should be the mature population, which is more appropriate to the economic and biological views of renewable resources management Matsuda and Nishimori (2002) and Song and Chen (2001). Though harvesting models have been studied by many authors Kar and Chaudhuri (2000, 2003), Ragozin and Brown (1985), Mesterton-Gibbons (1988), and Leung (1995), the stage structure of the species has received very little attention. Some of the stage-structured models are studied by Arino et al. (2001), Gambell (1985), Cao et al. (1992), Bosch and Gabriel (1997) and Kar (2003) and the references therein.

To facilitate the interpretation of our mathematical findings we assume that the plankton, density of which denoted by X, can be modeled by a logistic equation when the consumer (fish) is absent. We assume that the fish is divided into two stage groups: juveniles and adults and their densities are denoted by Y and Z respectively. Here we also assume that only adult fish are capable of preying on the prey species and that the juvenile predators live on their parents. Another key and somewhat novel feature of our model is to account for the universally prevalent intra-specific competition in the consumer growth dynamic Kuang et al. (2003). This intra-specific competition is assumed to induce additional instantaneous deaths only to the adult population and the increased death rate is proportional to the square of the adult population. These terms describes either a self limitation of consumers or the influence of predation. Self limitation can occur if there is some other factor (other than food) which becomes limiting at high population densities.

With these assumptions, we have the following plausible two stage prey-predator interaction model:

$$\frac{dX}{dt} = r_1 X\left(1-\frac{X}{k}\right) - \alpha X Z,$$
$$\frac{dY}{dt} = \beta Z - r_2 Y, \tag{1.1}$$
$$\frac{dZ}{dt} = -r_3 Z + m\alpha XZ + \gamma Y - \delta Z^2.$$

Model is assumed to be closed in which plankton species are growing logistically with a growth rate r_1 and has the carrying capacity k. α is the predation parameter; m is the conversion factor; r_3 is the death rate of mature predator species; γ is the proportionality constant of transformation of immature to mature predators; $r_2 = \mu + \gamma$, where μ is the death rate; β is the birth rate of the immature populations.

To reduce the number of parameters and to determine which combinations of parameters control the behavior of the system, we nondimensionalize the system (1.1). We choose

$$X = \frac{k r_2}{r_1} x_1,\ Y = \frac{\beta x_2}{m\alpha},\ Z = \frac{r_2 x_3}{m\alpha},\ t = \frac{\tau}{r_2}.$$

Then the system takes the form

$$\frac{dx_1}{d\tau} = a x_1 - x_1^2 - b x_1 x_3,$$
$$\frac{dx_2}{d\tau} = x_3 - x_2, \tag{1.2}$$
$$\frac{dx_3}{d\tau} = -c x_3 + d x_1 x_3 + e x_2 - f x_3^2,$$

where $a = \frac{r_1}{r_2},\ b = \frac{1}{m},\ c = \frac{r_3}{r_2},\ d = \frac{m\alpha k}{r_1},\ e = \frac{\gamma\beta}{r_2^2},\ f = \frac{\delta}{m\alpha}.$

In order to study the effect of harvesting on the system, let us consider the following system

$$\frac{dx_1}{d\tau} = a x_1 - x_1^2 - b x_1 x_3,$$
$$\frac{dx_2}{d\tau} = x_3 - x_2, \tag{1.3}$$
$$\frac{dx_3}{d\tau} = -c x_3 + d x_1 x_3 + e x_2 - f x_3^2 - qE x_3.$$

Here, qEx_3 is based on the catch-per-unit-effort hypothesis Clark (1990), where q is the catch ability co-efficient and E is effort applied for fishing.

Total sustainable revenue (TR) is equal to $pqEx_3$, where 'p' is the price per unit harvested biomass. c_1E is the total cost (TC), where c_1 is the cost per unit effort. Sustainable economic rent is the difference of TR and TC, i.e., sustainable economic rent is $TR-TC$.

In Section 2, we discuss the boundedness, equilibria and their stability of system (1.3). A reasonable harvesting policy is indisputably one of the major and interesting problems from ecological and economical point of view. Maximum Sustainable Yield, Maximum Economic Yield and Optimum Sustainable Yield are studied in section 3.

2. Boundedness, Equilibria and Stability Analysis

Boundedness of a model guarantees its validity. The following theorem establishes the uniform boundedness of the system (1.3).

Theorem 2.1:

All the solutions of the system (1.3) which start in R_3^+ are uniformly bounded.

***Proof*:**

We define the function

$$w = \frac{x_1}{b} + x_2 + \frac{x_3}{d}.$$

Now, for each $v > 0$, we have

$$\frac{dw}{dt} + vw \le \frac{1}{4b}(a+k)^2 + \frac{d}{4f}\left(1 - \frac{c}{d} - \frac{qE}{d} + \frac{k}{d}\right)^2 < \mu,$$

for some bounded $\mu > 0$.

Applying the theory of differential inequalities Birkoff and Rota (1982), we obtain

$$0 < w(x_1, x_2, x_3) < \frac{\mu}{v}(1 - e^{-vt}) + w\big(x_1(0), x_2(0), x_3(0)\big)e^{-vt},$$

which upon letting $t \to \infty$, yields $0 < w < \dfrac{\mu}{v}$.

Hence, all solutions of system (1.3) that start in R_3^+ are confined to the region B, where $B = \left\{ (x_1, x_2, x_3) \in R_3^+ \, : \, w = \frac{\mu}{\nu} + \varepsilon, \text{ for any } \varepsilon > 0 \right\}$. Hence, the theorem.

System (1.3) has to be analyzed with the following initial conditions: $x_1(0) > 0$, $x_2(0) > 0$ and $x_3(0) > 0$. We observe that the right-hand side of the system (1.3) is smooth function of the variables (x_1, x_2, x_3) and the parameters, as long as these quantities are non-negative, so local existence and uniqueness properties hold in the positive octant. The state space for system (1.3) is in the positive octant, $\{(x_1, x_2, x_3): x_1 > 0, x_2 > 0 \text{ and } x_3 > 0\}$, which is clearly an invariant set, since the vector field on the boundary does not point to the exterior. Our next result concerns the existence of equilibrium points.

We observe that the possible non-negative equilibria of system (1.3) are $P_0(0, 0, 0)$, $P_1(a, 0, 0)$ and $P_2(x_1^*, x_2^*, x_3^*)$, where

$$x_1^* = \frac{af + b(c+qE) - be}{db + f} \quad \text{and} \quad x_2^* = x_3^* = \frac{e + da - (c+qE)}{db + f}.$$

We like to point out here that $e > (c+qE)$ implies the existence of another equilibrium in the absence of prey. But, it is not possible and so we assume that $(c+qE) \geq e$ throughout the paper. Therefore, P_2 is feasible if $c+qE<e+ad$ hold.

Particularly we are interested in the interior equilibrium point $P_2(x_1^*, x_2^*, x_3^*)$ for its usual importance.

In order to investigate the stability of system (1.3) near P_0, P_1 and P_2, we compute the variational matrix given by

$$M(x_1, x_2, x_3) = \begin{bmatrix} a - 2x_1 - bx_3 & 0 & -bx_1 \\ 0 & -1 & 1 \\ dx_3 & e & -c - qE + dx_1 - 2fx_3 \end{bmatrix}.$$

It is easy to check that $P_0(0, 0, 0)$ is unstable and $P_1(a, 0, 0)$ is asymptotically stable for $c + qE > e + ad$.

According to Routh-Hurwitz criteria, $P_2(x_1^*, x_2^*, x_3^*)$ is locally asymptotically stable if $c + qE < e + ad$ hold. Here we observe that the existence of P_2 implies P_1 is unstable.

Now we shall discuss the condition of global stability, permanence and extinction of system (1.3).

Theorem 2.2:

(i) If $c+qE \geq e + ad$, then the equilibrium $P_1(a, 0, 0)$ is globally asymptotically stable in R_3^+.

(ii) If $c+qE < e + ad$, then the only interior equilibrium point $P_2(x_1^*, x_2^*, x_3^*)$ is globally asymptotically stable in $\text{Int } R_3^+$.

***Proof*:**

(i) We construct the following Lyapunov function

$$V_1 = \alpha_1 \left(x_1 - a - a \ \ln \frac{x_1}{a} \right) + \alpha_2 \, x_2 + \alpha_3 \, x_3,$$

where α_i, $i =$ 1, 2, 3, are positive constants to be determined in the subsequent steps. Calculating the derivative of V_1 along each solution of (1.3), we have

$$\frac{dV_1}{d\tau} = -\alpha_1 (x_1 - a)^2 - \alpha_1 b (x_1 - a) x_3 + \left(\alpha_2 \, x_3 - (c + qE) \alpha_3 \, x_3 \right)$$
$$- \alpha_2 \, x_2 + \alpha_3 \, e \, x_2 + \alpha_3 \, d \, x_1 \, x_3 - \alpha_3 \, f \, x_3^2.$$

Let $\alpha_1 = d/b$, $\alpha_2 = e$ and $\alpha_3 = 1$. Then, $\frac{dV_1}{d\tau} = -\frac{d}{b}(x_1 - a)^2 - (c - qE - e - a\,d)\, x_3 - f\, x_3^2 < 0$ in Int. R_3^+, for $c+qE \geq$ e + ad. This establishes the global asymptotic stability.

(ii) Let us take the Lyapunov function

$$V_2(x_1, x_2, x_3) = \sum \alpha_i \left(x_i - x_i^* - x_i^* \ \ln \frac{x_i}{x_i^*} \right),$$

where α_i, i = 1, 2, 3 are positive constants to be determined in the subsequent steps. Calculating the derivative along each solution of (1.3), we have

$$\frac{dV_2}{d\tau} = -\alpha_1 (x_1 - x_1^*)^2 - b\,\alpha_1 (x_1 - x_1^*)(x_3 - x_3^*) + \alpha_2 (x_2 - x_3^*) \left(\frac{x_3}{x_2} - \frac{x_3^*}{x_2^*} \right)$$
$$+ \alpha_3 \, d \, (x_3 - x_3^*)(x_1 - x_1^*) + e\,\alpha_3 (x_3 - x_3^*) \left(\frac{x_2}{x_3} - \frac{x_2^*}{x_3^*} \right) - f\,\alpha_3 \, (x_3 - x_3^*)^2.$$

Let $\alpha_1 = d/b$, $\alpha_2 = e$ and $\alpha_3 = 1$. Therefore,

$$\frac{dV_2}{d\tau} = -\frac{d}{b}(x_1 - x_1^*)^2 - f\ (x_3 - x_3^*)^2 - e\,x_2^* \left[\sqrt{\frac{x_3}{x_2}}\ (x_2 - x_2^*)^2 - \sqrt{\frac{x_2}{x_3}}\ (x_3 - x_3^*) \right]^2 < 0.$$

This establishes $P_2\ (x_1^*,\ x_2^*,\ x_3^*)$ is globally asymptotically stable if $c+qE < e + ad$ hold.

Definition 2.1:

System (1.3) is said to be permanent if there are positive constants m and M such that each positive solution $x(t, x_0)$ of (1.3) with initial condition $x_0 \in \text{Int}\, R_3^+$ satisfies

$$m \le \liminf_{t\to\infty} x_i\,(t,\ x_0) \le \limsup_{t\to\infty} x_i\,(t,\ x_0) \le M,\ i = 1,2,3.$$

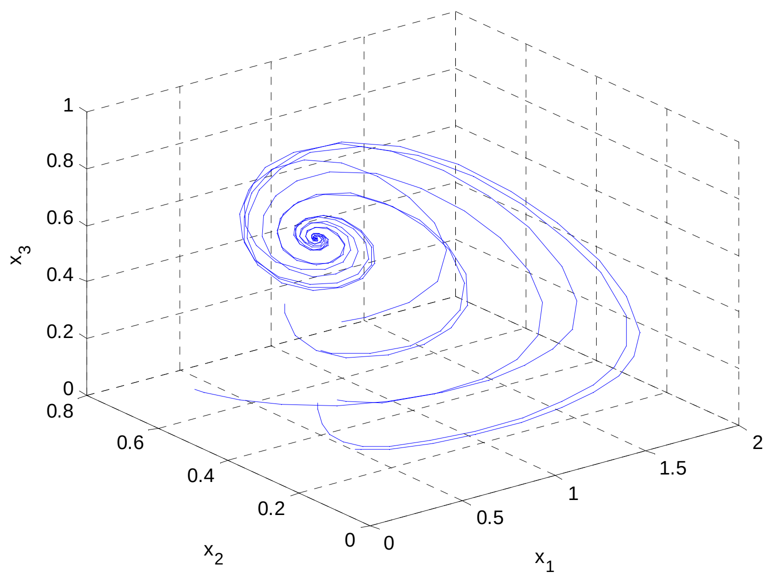

Figure 1. Phase space trajectories of system (1.3) beginning with different initial levels. It is seen that P_2 (0.77, 0.56, 0.56) is a global attractor, where a=3.0, f=0.04, b=4, c=2, e=0.2, d=3.0, q=0.05, E=10.

We have the following theorem:

Theorem 2.3:

(i) The fish species of system (1.3) is extinctive and the plankton species is not extinctive if and only if $c+qE \ge e + ad$ hold.

(ii) System (1.3) is permanent if and only if $c + qE < e + ad$ hold.

***Proof*:**

By Definition 2.1 and the Theorem 2.2, we can easily prove the Theorem 2.3.

3. Fisheries Economics

At any point of time, harvest is a function of fishing effort and size of the fish stock. For a given population size increasing the effort will give more harvest and on the other hand for a given effort, larger the stock will give the larger harvest. Since the harvest varies with the level of effort a different equilibrium population will result at each level of effort.

From an economic point of view, maximum sustainable yield does not imply the effect harvesting of resources. To attain efficiency in the economic sense, we need to take into account the costs of fishing and revenues from selling the harvest fish.

In this model the relationship between cost and effort is assumed to be linear. If c_1 is the unit cost of fishing effort E, the total cost in the fishery is defined as: $TC(E)=c_1E$.

In order to calculate the value of the fishery, the total revenue function is calculated using the following formula:

$$TR(E)=pH(E),$$

where p is the unit price and H is the total harvest. Therefore, the economic rent of the fishery is, then

$$\pi(E) = \text{TR(E)-TC(E)}.$$

3.1. Open Access Equilibrium (OAE)

If the fishery follows basic economic laws, fishers would continue enter the fishery until their average revenue equals with their marginal cost of effort. Assuming fishing homogeneous fleet and all input factors have the same opportunity costs, the situation of open access may be defined as follows in equilibrium:

$$\frac{TR(E)}{E} = TC^{'}(E),$$

which gives

$$E_{OAE} = \frac{1}{q}\left[(e+ad-c) - \frac{c_1(bd+f)}{pq}\right],$$

$$H_{OAE} = qE_{OAE}\frac{(e+ad-c-qE_{OAE})}{db+f}.$$

3.2. Maximum Sustainable Yield (MSY)

The maximum sustainable yield (MSY) of a biological resource population is the maximum rate at which it can be harvested even after maintaining the population at a constant level.

Theorem 3.1:

The maximum sustainable yield $MSY = \frac{(e-c+da)^2}{4(db+f)}$ occurs at the effort level $E_{\text{MSY}} = \frac{e-c+da}{2q}$.

***Proof*:**

Corresponding to a given effort E, the sustainable yield is given by

$$h(E) = qE\bar{x}_3 = qE\frac{[e-c+da-qE]}{db+f}.$$

Then, $\frac{dh}{dE} = 0$ when $E = \bar{E} = \frac{e-c+da}{2q}$, and $\frac{d^2h}{dE^2} = -\frac{2q^2}{db+f} < 0$ always. Therefore, $h(E)$ is maximum when $E = \bar{E}$. Hence,

$$MSY = q\bar{E}\bar{x}_3 = \frac{(e-c+da)^2}{4(db+f)}.$$

Thus, the MSY occurs at the effort level $E_{\text{MSY}} = \bar{E}$ and for any value of $E > E_{\text{MSY}}$, the yield $h(E)$ monotonically decreases with E towards zero(see Figure 2). Biologists call it a case of biological over exploitation whenever the effort level exceeds its MSY level. It is observed that at E_{MSY}, P_2 is globally asymptotically stable.

For simulation, let us take a=3.0, f=0.7, b=4.0, c= 0.62, d= 3.0, e=0.25, q=0.5. For these values, we get E_{MSY}=8.61 and MSY=1.47.

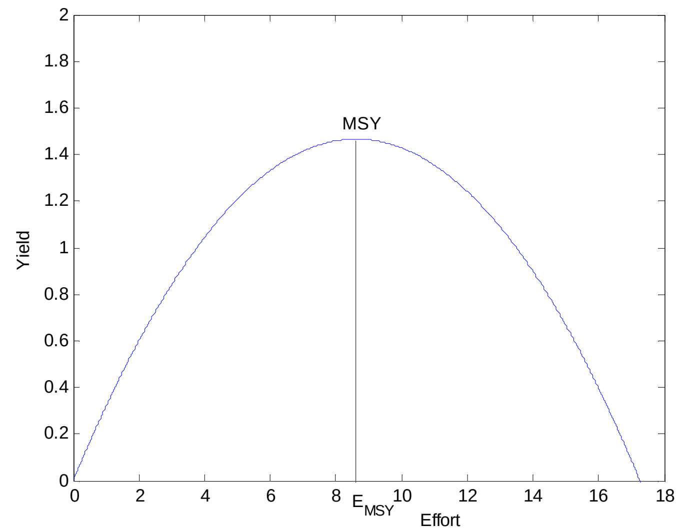

Figure 2. Yield-effort curve. The curve shows that when $E > E_{MSY}$, yield monotonically decreases with effort E towards zero.

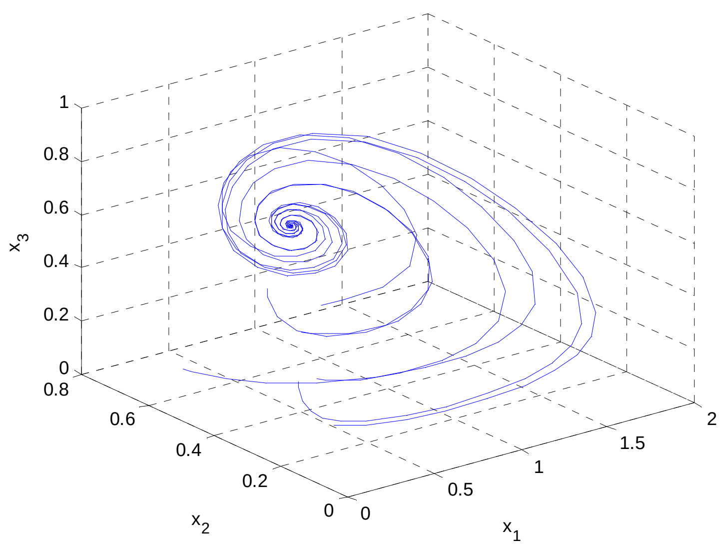

Figure 3. Phase space trajectories of system (1.3) for $E= E_{MSY}$. It is seen that corresponding equilibrium point (0.76, 0.56, 0.58) is a global attractor

3.3. Maximum Economic Yield (MEY)

Maximum economic yield (MEY) is defined as the level of landings that would maximize profits to the harvesting sector. The long term economic optimum can be found where the marginal sustainable yield is equal in value to the cost of an additional unit of effort. Let us assume *MR* (*E*) is the marginal revenue of effort, which is to be the change in total revenue when production of effort changes by an additional unit and *MC* (*E*) be the marginal cost of effort, which is to be the change in total cost when the level of fishing effort changes by an additional unit. Thus the maximum economic yield (MEY) can be obtained from the fishery when the difference between total revenue and total cost is at a maximum. Therefore at a point where *MR* (*E*) =*MC* (*E*) we get maximum economic yield, which implies

$$\frac{d\big(TR(E)\big)}{dE} = \frac{d\big(TC(E)\big)}{dE}.$$

This gives us

$$E_{MEY} = \frac{1}{2q}\left[(e + ad - c) - \frac{c_1(bd + f)}{pq}\right],$$

and

$$H_{MEY} = qE_{MEY}\,\frac{(e + ad - c - qE_{MEY})}{db + f}.$$

3.4. Optimum Sustainable Yield

Confronted with the inadequacy of the MSY, people tried to replace it by the "optimal sustainable yield", which is based on the standard cost benefit criterion used to maximize the revenues. Optimum sustainable yield is the yield which would maximize the present value of the flow of resource rent from the fishery in all future.

The objective is therefore to solve the following optimization problem

$$\max \int_0^\infty e^{-\theta\tau}\,(pqx_3 - c_1)\,E(\tau)\,d\tau, \tag{3.1}$$

subject to the state equations of (1.3) and the control constraint $0 \le E(\tau) \le E_{\max}$, where θ is the instantaneous annual discount rate.

To solve this optimization problem, we employ the Pontryagin's Maximal Principle, Pontryagin et al. (1962). The maximum principle is most conveniently formulated in terms of the following expression, called the Hamiltonian:

$$H = e^{-\theta\tau}(pqx_3 - c_1)E + \lambda_1[ax_1 - x_1^2 - bx_1x_3] + \lambda_2[x_3 - x_2] + \lambda_3[-cx_3 + dx_1x_3 + ex_2 - fx_3^2 - qEx_3],$$

where λ_1, λ_2 and λ_3 are adjoint variables and

$$\phi(\tau) = e^{-\theta\tau}(pqx_3 - c_1) - \lambda_3 q\,x_3$$

is called the switching function.

Since H is linear in the control variable, E, the optimal control will be a combination of extreme control and the singular control. The optimal control $E(\tau)$ which maximizes H must satisfy the following conditions:

$$E = E_{max}, \text{ when } \phi(\tau) > 0 \text{ i.e., when } \lambda_3(\tau)e^{\theta\tau} < p - \frac{c_1}{q\,x_3},$$

and

$$E = 0, \text{ when } \phi(\tau) < 0 \text{ i.e., when } \lambda_3(\tau)e^{\theta\tau} > p - \frac{c_1}{q\,x_3}.$$

$\lambda_1(\tau)\, e^{\theta\tau}$ is the usual shadow price and $p - c_1/qx_3$ is the net economic revenue on a unit harvest. This shows that $E = E_{\max}$ or zero according to the shadow price is less than or greater than the net economic revenue on a unit harvest. Economically, the first condition implies that if the profit after paying all the expenses is positive, then it is beneficial to harvest up to the limit of available effort. Second condition implies that when the shadow price exceeds the fisherman's net economic revenue on a unit harvest, then the fisherman will not exert any effort.

When $\phi(\tau) = 0$, i.e. when the shadow price equals the net economic revenue on a unit harvest, then the Hamiltonian H becomes independent of the control variable $E(\tau)$, *i.e.*, $\partial H / \partial E = 0$. This is the necessary and sufficient condition for the singular control $E^*(\tau)$ to be optimal over the control set $0 < E^* < E_{\max}$.

Thus, the optimal harvest policy is

$$E(\tau) = \begin{cases} E_{\max}, & \phi(\tau) > 0 \\ 0, & \phi(\tau) < 0 \\ E^* & \phi(\tau) = 0. \end{cases} \tag{3.2}$$

When $\phi(\tau) = 0$, it follows that

$$\lambda_3 = e^{-\theta\tau} \frac{(pqx_3 - c_1)}{qx_3}. \tag{3.3}$$

The adjoint equations are

$$\frac{d\lambda_1}{d\tau} = -\frac{\partial H}{\partial x_1} = -[\lambda_1 (a - 2x_1 - bx_3) + \lambda_3 dx_3], \tag{3.4}$$

$$\frac{d\lambda_2}{d\tau} = -\frac{\partial H}{\partial x_2} = -[-\lambda_2 + \lambda_3 e], \tag{3.5}$$

$$\frac{d\lambda_3}{d\tau} = -\frac{\partial H}{\partial x_3} = -\left[e^{-\theta\tau}\ pqE\,\lambda_1(-bx_1) + \lambda_2 + \lambda_3(-c + d._1 - 2\,fx_3 - qE)\right]. \tag{3.6}$$

We seek to find optimal equilibrium solution of the problem so that x_1, x_2, x_3 and E can be treated as constants.

By solving (3.4)-(3.6) we get the singular path

$$\frac{(pqx_3 - c_1)}{q\,x_3}\left\{\theta + c_1 - dx_1 + 2\,fx_3 + qE + \frac{bdx_1x_3}{\theta - a + 2x_1 + bx_3} - \frac{e}{\theta + 1}\right\} = pqE. \tag{3.7}$$

Equation (3.7) gives the optimal equilibrium population (x_1^*, x_2^*, x_3^*) and corresponding optimal harvesting effort E^*.

For simulation, let us take a=3.0, f=0.7, b=4.0, c= 0.62, d= 3.0, e=0.25, q=0.5, p=10, c=1.5, $\theta = 0.1$. For these values we get E_{OSY}=4.17 and OSY=1.074.

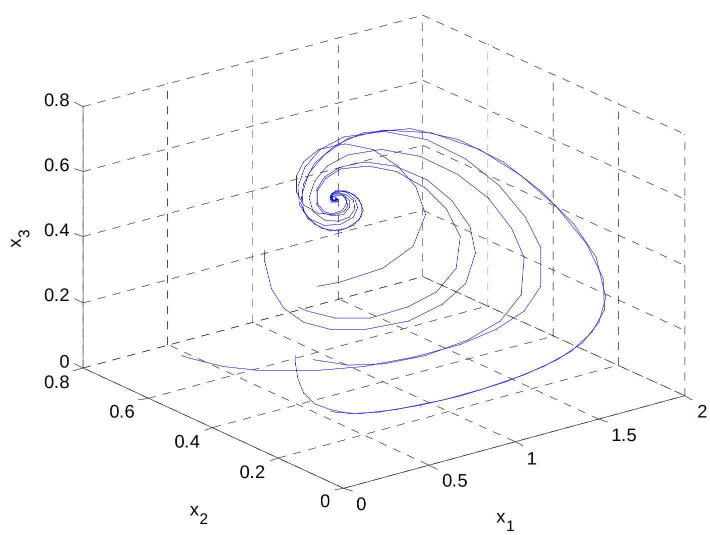

Figure 4. Phase space trajectories of system (3.1) for E= E_{OSY}. It is seen that corresponding equilibrium point (0.94, 0.52, 0.52) is a global attractor

3.5. Tax Policies to Achieve Reference Points

The idea of Reference Points (RPs) is strictly related to the management objectives involving economic, social, environmental and biological issues. The position of the sustainability indicator associated with reference values will describe the current state of the system and provide us with the relevant input to evaluate the situation and make management oriented decisions.

Landing Tax:

Assume that the fishery is in the open access situation. Let τ is the landing tax that needs to be created in order to achieve OSY (or MEY), then τ is defined by an equation:

$$(p-\tau)\, OSY = c_1 E_{OSY}.$$

Hence, $\tau = (p - c_1 E_{OSY})/OSY$. So the landing tax that needs to be created to achieve OSY is 4.18.

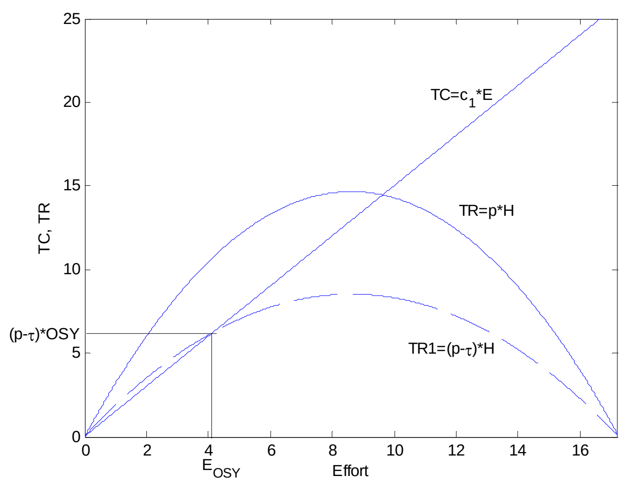

Figure 5. Revenue and cost curves using the landing tax

Effort tax:

Assume that the fishery is in open access situation. Let τ is the effort tax that needs to be created in order to achieve OSY (or MEY), then τ is defined by an equation:

$$p^*OSY = (c_1 + \tau) E_{OSY}.$$

Hence, $\tau = p^*OSY/(E_{OSY} - c_1)$. So the landing tax that needs to be created to achieve OSY is 1.08.

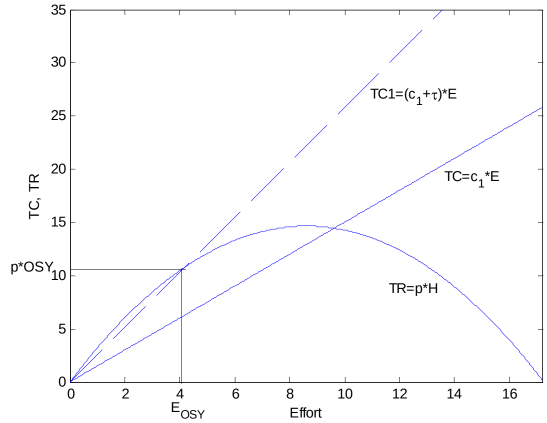

Figure 6. Revenue and cost curves using the effort tax

Entry tax:

Assume that the fishery is in open access situation. Let τ is the entry tax that needs to be created in order to achieve OSY (or MEY), then τ is defined by an equation:

$$p*OSY=c_1E_{OSY}+\tau\ .$$

Hence , $\tau=p*OSY-c_1E_{OSY}$. So the landing tax that needs to be created to achieve OSY is 4.49.

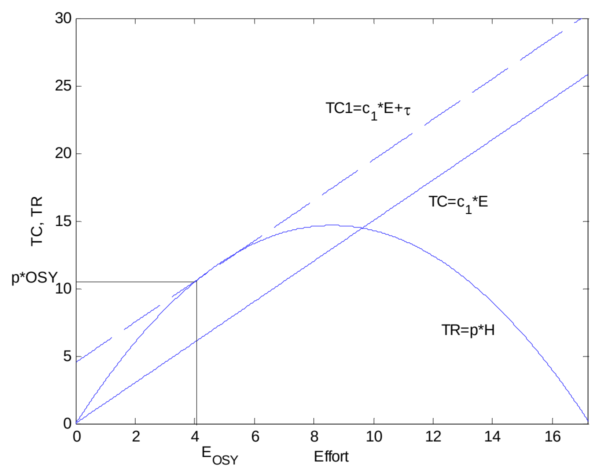

Figure 7. Revenue and cost curves using the entry tax

Thus, by imposing a tax on landing, effort or entry the management authority can force the competitive fishery into optimal mode.

4. Concluding Remarks

In this paper, we have considered a resource based fishery model with stage-structure and harvesting of mature species. We have first discussed the existence of possible steady states and then local as well as global stability. An important and one of the interesting questions in mathematical ecology is permanence, which ensures the survival of biological species and exclude extinction of species for all positive initial conditions. The question of permanence of biological species is of particular interest to fishery, forestry and wildlife managers. If it is known that a system exhibits such a permanent behavior, then ecological planning based on a fixed eventual population can be carried out. Realizing the problem we have obtained the conditions for permanence of the solutions of our system. Next the MSY, MEY & OSY are obtained. Tax policies to achieve the reference points are also discussed.

The dynamics exhibited by the system show good consistence with the observation in biological reality. If the unharvested system is permanent, then a sufficiently small harvesting rate will not change drastically the qualitative behavior of the system, but the region of coexistence shrinks as the harvesting rate increases. The result provides a theoretical support for safe harvesting in biological resource management.

For the Simulation purposes we have used the software MATLAB. As the real world data are not available to us, we have used some hypothetical data with the sole purpose of illustrating the results that we have established analytically.

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