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Maximum Contraflow Evacuation Planning Problems On Multi-network

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Abstract

Contraflow approach for the evacuation planning problem increases outbound capacity of the evacuation routes by the reversal of anti-parallel arcs, if such arcs exist. The existing literature focuses on network contraflow problems that allow only anti-parallel arcs with equal transit time. However, the problems modeled on multi-network, allowing parallel as well as anti-parallel arcs with not necessarily equal transit time, seem more realistic. In this paper, we study the maximum dynamic contraflow problem for multi-network and propose efficient solution techniques to them with discrete as well as continuous time settings. We also extend the results to solve earliest version of the problem for two terminal series parallel (TTSP) multi-network.

Keywords: Contraflow; Multi-network; TTSP network; Evacuation planning problem

MSC 2010 No.: 90B06, 90B20

1. Introduction

Evacuation planning problems attempt to find an optimal evacuation plan with a realistic flow model where each evacuee is supposed to be evacuated from risk site (source) to safe site (sink). The plan minimizes the loss of human lives and their property during natural and human-created disasters and also mitigates the rush hour traffic in the crowded urban areas. Contraflow approach, which is lane reversal strategy, could be an efficient idea for increasing the outbound capacity of an evacuation route by reversing the direction of anti-parallel arcs, i.e., two directed arcs joining the same pair of nodes with opposite directions, if such arcs exist, so that the flow of evacuees could be increased over given time horizon or given number of evacuees could be evacuated in a minimum time horizon. Despite about a two-decade long history on study of network contraflow evacuation problems, there is limited implementation in real emergency evacuations (e.g., Urbina and Wolshon (2003), Hamza-Lup et al. (2004), Kim and Shekhar (2005), Kim et al. (2008), Tuydes and Ziliaskopoulos (2004, 2006)) due to difficulty in using commonly employed methods to duplicate traffic conditions of real contraflow lane during an emergency (Wei et al. (2019)).

The evacuation scenario in which as much evacuees as possible are to be shifted from the source to the sink within a given time horizon can be modeled as a maximum dynamic flow (MDF) problem (Ford and Fulkerson (1958; 1962)). The problem and its applications have been extensively studied in the literature (e.g., Borradaile et al. (2017), Borrmann et al. (2012), Göttlich et al. (2011), Hamacher et al. (2011)). Evacuation problems with non-conservation flow constraints for intermediate nodes have also been studied (e.g., Khadka and Bhandari (2019), Bhandari et al. (2020), Bhandari and Khadka (2020b, 2020c). The problem with contraflow approach, also known as a maximum dynamic contraflow (MDCF) problem, has been analytically studied by Rebennack et al. (2010). The problem has been modeled in a dynamic network with a single source and a single sink. It has been solved in polynomial time where the arc reversal ability has been adapted only once at the beginning of the time horizon. The results are based on the reduction of given network into the network on which ordinary maximum dynamic flow problem can be solved in polynomial time. The problem in which time parameter varies continuously has been studied and a polynomial time solution procedure for the problem has been proposed by Khadka and Bhandari (2017) (see also Pyakurel and Dhamala (2017)).

Evacuation scenarios in which the number of evacuees is known in advance and all the evacuees are supposed to be evacuated from the source to the sink in a minimum possible time can be modeled as a quickest flow (QF) problem, (Burkard (1993), Hoppe and Tardos (2000), Lin and Jaillet (2015)). An efficient solution procedure has been proposed for the quickest contraflow (QCF) problem by Rebennack et al. (2010). The earliest arrival flow (EAF) problem which maximizes the flow at each time step within the given time horizon (Gale (1959), Minieka (1973), Wilkinson (1971), Baumann (2007), Steiner (2009), Ruzika et al. (2011)). The earliest arrival contraflow (EACF) problem in TTSP network, allowing arc reversibility only once at the beginning of time horizon, has been solved in polynomial time by Dhamala and Pyakurel (2013). Contraflow approach has been incorporated in network flow model by Dhungana and Dhamala (2019) to study facility location problem, and the notion of abstract flow has been applied to network contraflow problems by

Dhungana et al. (2018). The partial contraflow approach over the abstract network setting has been introduced by Pyakurel et al. (2019). Bhandari and Khadka (2020a) studied contraflow problems for network with not necessarily equal transit time on anti-parallel arcs.

A limitation of existing contraflow models is that they do not capture real world situation where multiple parallel lanes of different transit time do exist. In this paper, we consider multi-network to cope the situation with anti-parallel lanes with unequal to and fro transit time as well as parallel lanes with unequal transit time. We study maximum and earliest versions of evacuation planning problems, and propose efficient solutions algorithms for them with discrete as well as continuous time settings. The paper is organized as follows. Mathematical model of the problem is described in Section 2. Solutions to maximum dynamic contraflow problem and earliest arrival contraflow problem are proposed in Subsections 3.1 and 3.2, respectively. Section 4 concludes the paper.

2. Model Description

We consider an evacuation multi-network over a time horizon T as $N = (V, E, c_e, \tau_e, s, d, T)$ where V is set of nodes v, the crossings of routes from dangerous place, the source s, to safer place, the sink d; E is set of route-segments, the arc $e = (v, w)$ joining any two different nodes $v, w \in V$. We assume the number of nodes and arcs on N to be finite, i.e., we assume $n := |V|$ and $m := |E|$. Further, $c : E \rightarrow Z^{\geq 0}$ is a capacity function denoting the upper bound for flow units to travel the arc at a time slot, and $\tau : E \rightarrow Z^{>0}$ is the transit time parameter denoting time required for a flow unit to travel the arc.

The number of evacuees which are sent along the routes from the dangerous place to the specified safe place while sending from one road crossing to another during the specified transit time is represented by the flow function $f : E \times [0, T) \rightarrow R^{\geq 0}$ in the network N.

The number of flow units entering arc e at time $\theta \in [0, T)$ is assumed to be bounded by the capacity of an arc, i.e.

$$0 \leq f(e, \theta) \leq c_e, \;\; \forall \;\; e \in E \;\; \text{and} \;\; \forall \;\; \theta \in [0, T). \tag{1}$$

Flow units that enter into node v for all $v \in V \backslash \{s, d\}$ must exit from it within given time horizon T, i.e.,

$$\sum_{e \in \delta^-(v)} \int_0^{T-\tau_e} f(e, \theta) d\theta = \sum_{e \in \delta^+(v)} \int_0^{T} f(e, \theta) d\theta \, ; \forall \;\; v \in V \backslash \{s, d\}, \tag{2}$$

where $\delta^-(v)$ and $\delta^+(v)$ denote for the set of arcs entering into the node v and leaving from it, respectively. An $s - d$ flow of value f, from the source s to the sink d, is said to be a feasible flow if it satisfies capacity constraints (1). We assume, without loss of generality, $f(e, \theta) = 0$ for $\theta \notin [0, T)$ and all flow units leave the network before time T. Thus, a feasible continuous dynamic

$s-d$ flow of value $\mathbf{f}_c$ is an optimal flow on N for time horizon T, if $\mathbf{f}_c$ given by

$$\begin{aligned} \mathbf{f}_c &:= \sum_{e\in\delta^+(s)} \int_0^T f(e,\theta)d\theta - \sum_{e\in\delta^-(s)} \int_0^T f(e,\theta)d\theta \\ &= \sum_{e\in\delta^-(d)} \int_0^T f(e,\theta)d\theta - \sum_{e\in\delta^+(d)} \int_0^T f(e,\theta)d\theta, \end{aligned} \tag{3}$$

is maximum.

If we treat time in a discrete manner, i.e., if we discretize time horizon T into time steps $\{0,1,\ldots,T\}$, the flow function f defined as $f : E \times \{0,1,\ldots,T\} \to Z^{\geq 0}$ satisfies the capacity constraints and flow conservation constraints in the following forms:

$$0 \leq f(e,\theta) \leq c_e, \ \ \forall \ e\in E \ \text{ and } \forall \ \theta \in \{0,1,\ldots,T\}, \tag{4}$$

and

$$\sum_{e\in\delta^-(v)} \sum_{\theta=0}^{T-\tau_e} f(e,\theta) = \sum_{e\in\delta^+(v)} \sum_{\theta=0}^{T} f(e,\theta)\,;\forall \ v\in V\backslash\{s,d\}, \tag{5}$$

respectively.

A feasible discrete dynamic $s-d$ flow of value $\mathbf{f}_d$ is an optimal flow on N for time horizon T, if $\mathbf{f}_d$ given by

$$\begin{aligned} \mathbf{f}_d &:= \sum_{e\in\delta^+(s)} \sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in\delta^-(s)} \sum_{\theta=0}^{T} f(e,\theta) \\ &= \sum_{e\in\delta^-(d)} \sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in\delta^+(d)} \sum_{\theta=0}^{T} f(e,\theta), \end{aligned} \tag{6}$$

is maximum.

Contraflow Problems on Multi-network. For the multi-network $N = (V,E,c_e,\tau_e,s,d,T)$, an arc $e=(v,w)\in E$ in which the flow travels from node v to node w is replaced by the arc (w,v), for contraflow purpose. The important feature of the network N, here, is that the capacities and the transit time on anti-parallel arcs could be unequal, that is, $c_{(v,w)}$ is not necessarily equal to $c_{(w,v)}$ and $\tau_{(v,w)}$ is not necessarily equal to $\tau_{(w,v)}$ for $(v,w),(w,v)\in E$. Also, the network could have parallel arcs of different transit time; however, we do not allow any loop on N. To this end, the objective of the continuous time maximum dynamic contraflow (CT-MDCF) problem on N is to maximize the net feasible continuous dynamic flow $\mathbf{f}_c$ from the source s to the sink d given by Equation (3), if the direction of the arcs on N are allowed to reverse. A continuous time earliest arrival contraflow (CT-EACF) problem asks to maximize the net $s-d$ flow on network N by every time $\theta\in[0,T)$, if the direction of the arcs on N are allowed to reverse. The problems with discrete time setting can also be defined in similar ways.

3. Solution Discussion

The existing literature in contraflow models considers equal transit time but not necessarily the same arc capacity at both anti-parallel arcs. In contraflow approach, the two anti-parallel arcs are replaced by a single undirected arc with capacity equal to the sum of capacities on both the arcs. In the case of the problem on multi-network, the anti-parallel arcs are replaced by a single undirected arc with the same procedure as described by Rebennack et al. (2010) if all the arcs have the same transit time and all the arcs are kept parallel and undirected with the existed arc capacity and transit time if different transit time exist on them. This generates an undirected multi-network. The parallel arcs will be labeled to avoid obstruction on the multi-network while applying algorithms so that desired arc could be chosen to yield optimal solution.

3.1. Maximum Dynamic Contraflow Problem

The maximum dynamic flow problem can be solved efficiently using the minimum cost flow computation on the given network. Ford and Fulkerson (1958) showed that an optimal solution to this minimum cost flow problem can be turned into a maximal dynamic flow. Let Γ be the set of all $s-d$ chains γ_r for some $r \in Z^{\geq 0}; r \leq m$ with flow values $\mathbf{v}(\gamma_r)$ determined by the decomposition of the optimal minimum cost flow obtained by considering the transit times on the arcs as cost over time horizon T. If τ_{γ_r} denotes the transit time along the chain γ_r, the maximum dynamic flow with discrete time setting denoted by $\mathbf{f}_d$, in terms of temporally repeated flows, on any network is given by

$$\mathbf{f}_d := \sum_{k=1}^{r} v(\gamma_k).(T - \tau_{\gamma_k} + 1). \tag{7}$$

The dynamic flow of value $\mathbf{f}_d$ given by Equation (7) is optimal on N for time horizon T (Ford and Fulkerson (1962)).

Although use of the notion of time expanded network introduced by Ford and Fulkerson (1958) to find a maximum dynamic flow leads a pseudo polynomial time complexity algorithm, it is important for proving the optimality of the solution executed by algorithms proposed in this paper. We have an important relation, due to Ford and Fulkerson (1958), which states that the maximum flow for two terminal case of the maximum dynamic problem on N does not exceed the optimal flow for the corresponding time expanded network N^T. The dynamic network N is transformed into the time expanded (static) network $N^T = (V^T, E^T)$ where

$$V^T = \{v(\theta) : v \in V \text{ and } \theta \in \{0, 1, \ldots, T\}\},$$

and

$$E^T = \{(v(\theta), w(\theta + \tau(v, w))) : v \neq w,\ v, w \in V \text{ and } \theta \in \{0, 1, \ldots, T - \tau_e\}\}.$$

An exact solution procedure for solving the discrete time maximum dynamic contraflow (DT-MDCF) problem modeled on multi-network N, if the arc reversibility is permitted only once at time zero, has been presented in Algorithm 1. During the procedure, the arc $(w, v) \in E$ is reversed, if the flow along arc (v, w) exceeds $c_{(v,w)}$ for $\tau_{(v,w)} \leq \tau_{(w,v)}$; or if, disregard of flow value on (v, w), $\tau_{(w,v)} < \tau_{(v,w)}$. This can be viewed, alternatively, as follows. For $(v, w), (w, v) \in E$ such that $\tau_{(v,w)} = \tau_{(w,v)}$, the flow value at arc $\tilde{e} = (v, w) \in \tilde{E}$ greater than the capacity $c_{(v,w)}$ of the corresponding arc $(v, w) \in E$ means there is flipping of the direction of arc $(w, v) \in E$. Similarly, in the case with unequal transit times, we can see the sense of flipping the direction of arc $e = (w, v) \in E$, if there is some positive flow on the corresponding arc $\tilde{e} = (v, w)$. The minimum cost flow (MCF) algorithm applied to generate a dynamic temporally repeated flow in step 4 ensures that there is flow along the arc (v, w) with less or equal transit time in comparison to the transit time of corresponding anti-parallel arc (w, v), regardless of the arc (w, v) is saturated. The parallel arcs $(v, w) \in \tilde{N}$ have been labeled as $(v, w)_i$ such that $\tau_{(v,w)_i} < \tau_{(v,w)_{i+1}}$, for $i = 1, 2, \ldots, q; q \leq m$, to avoid obstruction on the multi-network while applying MCF algorithm.

Algorithm 1: Discrete Time Maximum Dynamic Contraflow (**DT-MDCF)**

(1) Given a multi-network $N = (V, E, c_e, \tau_e, s, d, T)$ with single source s, single sink d and integer inputs.
(2) Transform N into undirected multi-network $\tilde{N} = (V, \tilde{E}, c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ where
$\tilde{e} = (v, w) \in \tilde{E}$, if $(v, w), (w, v) \in E$ such that $\tau_{(v,w)} = \tau_{(w,v)}$, with $c_{\tilde{e}} = c_{(v,w)} + c_{(w,v)}$ and $\tau_{\tilde{e}} = \tau_{(v,w)}$; and
$\tilde{e} = (v, w) \in \tilde{E}$, if $(v, w) \in E$ and $(w, v) \notin E$ such that $\tau_{(v,w)} = \tau_{(w,v)}$, with $c_{\tilde{e}} = c_{(v,w)}$ and $\tau_{\tilde{e}} = \tau_{(v,w)}$.
(3) Label parallel arcs $(v, w) \in \tilde{N}$ as $(v, w)_i$ such that $\tau_{(v,w)_i} < \tau_{(v,w)_{i+1}}$, for $i = 1, 2, \ldots, q; q \leq m$.
(4) Generate a dynamic, temporally repeated flow on network $\tilde{N}$.
(5) Perform flow decomposition into path and cycle flows of the flow resulting from step 4. Remove the cycle flows.
(6) Arc $(w, v) \in E$ is reversed, if and only if the flow along arc (v, w) is greater than $c_{(v,w)}$, or if there is a non-negative flow along arc $(v, w) \in E$.
(7) Get a discrete time maximum dynamic contraflow on N.

If we consider the evacuation scenario on a static network $N = (V, E, c_e, s, d)$ (that is, without transit times on the arcs) then the flow problem considered in here coincides with the maximum static contraflow (MSCF) problem. The maximum static contraflow problem in a static network N is equivalent to the maximum flow problem in the corresponding transformed network $\tilde{N}$ (Rebennack et al. (2010)).

Following theorems (Theorem 3.1 and 3.2) show that the Algorithm 1 solves maximum dynamic contraflow problem on multi-network N optimally in strongly polynomial time.

Theorem 3.1.

Given a multi-network $N = (V, E, c_e, \tau_e, s, d, T)$ with integer inputs. Then a discrete time maximum dynamic flow on $\tilde{N}$ is equivalent to a discrete time maximum dynamic contraflow on N.

Proof:

The auxiliary network $\tilde{N}$ of the original network N obtained in step 2 is an undirected multi-network. Now, the discrete time maximum dynamic contraflow problem on N can be viewed as a discrete time maximum dynamic flow problem on $\tilde{N}$. While solving the latter problem on $\tilde{N}$, the network is to be further transformed by replacing each undirected arc by two oppositely directed arcs with capacities and transit times of both arcs equal to that of original arc. This allows us to send flow on either direction of the arc. However, the flow direction, once chosen, remains fixed throughout the procedure. That is, there is only a flow on one direction of any arc, and never in both directions at the same time as well as at different time periods. However, there could be a flow along arc (v, w) and (w, v) such that $\tau(v, w) \neq \tau(w, v)$ for $(v, w), (w, v) \in E$ at the same time or at different time periods. The latter situation does not make the flow on $\tilde{\mathcal{N}}$ an infeasible since, in fact, arcs (v, w) and (w, v) are physically different arcs for $\tau(v, w) \neq \tau(w, v)$, due to the labeling of arcs in step 3. Thus, the flow constructed by Algorithm 1 is feasible.

Since every feasible flow of the maximum dynamic flow problem on the transformed network $\tilde{N}$ is feasible to the maximum dynamic contraflow problem on network N, the maximum dynamic flow on $\tilde{N}$ is not greater than the maximum dynamic contraflow on N. On the other hand, since maximum dynamic flow on network N does not exceed maximum flow for the corresponding time expanded network N^T (Ford and Fulkerson (1958)), the maximum dynamic contraflow on N is not greater than the maximum static contraflow in time expanded network N^T. This static contraflow is equivalent to the optimal static flow in $\tilde{N}^T$ due to the fact that any maximum static contraflow on network N has equivalent maximum flow in the corresponding transformed network $\tilde{N}$, (Rebennack et al. (2010)). Again, since there exists a temporally repeated flow which is maximal over the time horizon T (Ford and Fulkerson (1962)), the optimal static flow in $\tilde{N}^T$ is equivalent to the temporally repeated chain flow on $\tilde{N}$. Thus, the optimal dynamic contraflow on N is not greater than the optimal dynamic flow on $\tilde{N}$. ■

Theorem 3.2.

For multi-network $N = (V, E, c_e, \tau_e, s, d, T)$ with integer inputs, Algorithm 1 runs in strongly polynomial time.

Proof:

Construction of auxiliary network in step 2 and labeling parallel arcs in step 3 require only linear time on m. Computing a maximum dynamic flow in step 3 dominates the running time of Algorithm 1. It is computed with the help of temporally repeated flow on $\tilde{N}$. Finding a temporally repeated flow is equivalent to solving a minimum cost flow problem. The minimum mean cycle-canceling algorithm of Goldberg and Tarjan (1989), for instance, requires $O(n^2m^3logn)$ time for solving this problem. Next effort is to decompose the maximum static flow which requires $O(mn)$ time (Ahuja et al. (1993)). Thus, Algorithm 1 runs in a strongly polynomial time. ■

It is equivalent to replace T by $T-1$ in the definition of time expanded network N^T of N, if the flow problem is modeled with continuous time setting. For the $s - d$ chain flows as described above, the maximum dynamic flow with continuous time setting denoted by $\mathbf{f}_c$, in terms of temporally repeated flows, on N is given by

$$\mathbf{f}_c := \sum_{k=1}^{r} v(\gamma_k).(T - \tau_{\gamma_k}). \tag{8}$$

The temporally repeated flow given by Equation (8) is maximal over the time horizon T (Anderson and Philpott (1994)).

Now, we give a solution procedure (Algorithm 2) that combines the approach of natural transformation suggested by Fleischer and Tardos (1998) together with Algorithm 1 to solve the continuous time maximum dynamic contraflow problem on multi-network, when arc reversibility is allowed only once at time zero. The approach states that feasible discrete dynamic flow, of value $f_d(e, \theta)$, entering an arc $e \in E$ at time $\theta \in \{0, 1, \ldots, T - \tau_e\}$ can be interpreted as a continuous dynamic flow, of rate $f_c(e, \theta)$, into arc e during the whole time interval $[\theta, \theta + 1)$. That is, for any arc $e \in E$, $f_d(e, \theta) := f_c(e, [\theta, \theta + 1)) \ \forall \ \theta \in \{0, 1, \ldots, T - 1 - \tau_e\}$. This transformation is a bidirectional, if the time horizon T and all transit times on network N are integral (Baumann (2007)).

Algotithm 2: Continuous Time Maximum Dynamic Contraflow (**CT-MDCF)**

(1) Given a network $N = (V, E, c_e, \tau_e, s, d, T)$ with single source s, single sink d and integer inputs.
(2) Transform N into undirected multi-network $\tilde{N} = (V, \tilde{E}, c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ as in Algorithm 1.
(3) Label parallel arcs $(v, w) \in \tilde{N}$ as $(v, w)_i$ such that $\tau_{(v,w)_i} < \tau_{(v,w)_{i+1}}$, for $i = 1, 2, \ldots, q; q \leq m$.
(4) Generate a dynamic, temporally repeated flow on network $\tilde{N}$ for time horizon $T - 1$.
(5) Perform flow decomposition into path and cycle flows of the flow resulting from Step 4. Remove the cycle flows.
(6) Transform the discrete dynamic flow into continuous dynamic flow using the natural transformation as $f_d(e, \theta) := f_c(e, [\theta, \theta + 1)) \ \forall \ \theta \in \{0, 1, \ldots, T - 1 - \tau_e\}$.
(7) Arc $(w, v) \in E$ is reversed, if and only if the flow along arc (v, w) is greater than $c_{(v,w)}$, or if there is a non-negative flow along arc $(v, w) \in E$.
(8) Get a continuous time maximum dynamic contraflow on N.

Optimality of the solution of maximum dynamic contraflow problem with continuous time setting can be shown as in the case of discrete time setting. Only the additional effort in Algorithm 2 is to apply the notion of natural transformation that always yields a feasible flow (Fleischer and Tardos (1998)). Also, the time complexity of finding a temporally repeated continuous flow is equal to the time complexity of finding a temporally repeated discrete flow. Therefore, since temporally repeated flow with continuous time setting is maximal over the time horizon (Anderson and Philpott (1994)), the following theorem holds true.

Theorem 3.3.

For multi-network $N = (V, E, c_e, \tau_e, s, d, T)$ with integer inputs, Algorithm 2 solves the continuous time maximum dynamic contraflow problem optimally in strongly polynomial time.

3.2. Earliest Arrival Contraflow Problem on TTSP Network

Let us consider a special class of network known as two terminal series-parallel (TTSP) multi-network N. A two terminal series-parallel network $N = (V, E)$ is a directed network with a single source s and a single sink d which has a single arc (s, d) or is obtained from two series parallel networks N_1 and N_2 by one of the two operations: Parallel Composition and Series Composition. The first suggests to merge source nodes s_1 of N_1 and s_2 of N_2 to form the source node s of N and merge sink nodes d_1 of N_1 and d_2 of N_2 to form the sink node d of N. The second suggests to merge the sink node d_1 of N_1 with the source node s_2 of N_2 to form the network N with source node s_1 and sink node d_2.

Ruzika et al. (2011) modified the minimum cost flow (MCF) algorithm of Bein et al. (1985) to incorporate time horizon T in the flow model and proposed maximum dynamic flow algorithm for TTSP network. They showed that this maximum dynamic flow has the earliest arrival property. The application of this modified MCF algorithm in Algorithm 1 to generate a temporally repeated flow on a TTSP network yields a maximum dynamic flow which has earliest arrival property. This idea is also applicable for the case of multi-network. An exact solution procedure that solves the discrete time earliest arrival contraflow (DT-EACF) problem on a TTSP multi-network N, if the direction of arcs are allowed to reverse only once at time zero, has been presented in Algorithm 3.

Algorithm 3: Discrete Time Earliest Arrival Contraflow (**DT-EACF)**

(1) Given a TTSP multi-network $N = (V, E, c_e, \tau_e, s, d, T)$ with single source s, single sink d and integer inputs.
(2) Transform N into undirected multi-network $\tilde{N} = (V, \tilde{E}, c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ as in Algorithm 1.
(3) Label parallel arcs $(v, w) \in \tilde{N}$ as $(v, w)_i$ such that $\tau_{(v,w)_i} < \tau_{(v,w)_{i+1}}$, for $i = 1, 2, \ldots, q; q \leq m$.
(4) Generate a dynamic, temporally repeated flow on network $\tilde{N}$ applying Minimum Cost Circulation Algorithm of Ruzika et al. (2011).
(5) Perform flow decomposition into path and cycle flows of the flow resulting from Step 4. Remove the cycle flows.
(6) Arc $(w, v) \in E$ is reversed, if and only if the flow along arc (v, w) is greater than $c_{(v,w)}$, or if there is a non-negative flow along arc $(v, w) \in E$.
(7) Get a discrete time earliest arrival contraflow on N.

Theorem 3.4.

Algorithm 3 solves the earliest arrival contraflow problem with discrete time setting on a TTSP multi-network N for time horizon T optimally in strongly polynomial time.

Proof:

Construction of auxiliary network $\tilde{N}$ of network N is well defined due the same arguments as mentioned in the proof of Theorem 3.1. Also, two terminal series parallel multi-network N, after transforming into its auxiliary network $\tilde{N}$, remains two terminal series parallel. The dynamic flow obtained from Step 4 is optimal for time horizon T and N being a TTSP network the obtained flow has earliest arrival property (Ruzika et al. (2011)). Construction of auxiliary network in Step 2 and labeling parallel arcs in Step 3 require only linear time on m. We apply the minimum cost flow algorithm of Bein et al. (1985) to find $s-d$ paths which requires time of order $O(nm + mlogm)$. Flow decomposition takes $O(nm)$ times (Ahuja et al. (1993)). Thus, Algorithm 3 solves the earliest arrival contraflow problem with discrete time setting on N for time horizon T optimally in strongly polynomial time. ∎

Algorithm 3, together with the notion of natural transformation discussed in Subsection 3.1, solves the continuous time earliest arrival contraflow problem on a TTSP multi-network N, when the arc reversal capability is allowed only once at time zero. And, of course, the solution to the problem is optimal and can be found in strongly polynomial time as in the case of discrete time setting.

4. Conclusion

Contraflow approach seems to be a crucial tool in evacuating people at risk during disasters. Existing network contraflow model fails to capture the situation with multiple lanes connecting two places with unequal transit time on them. In this paper, we modeled the situation as a multi-network contraflow problem where anti-parallel lanes with not necessarily equal to-and-fro transit time as well as parallel lanes with unequal transit time do exist. We studied maximum dynamic contraflow problem and earliest arrival contraflow problem modeled on two terminal general multi-network and two terminal series parallel (TTSP) multi-network, respectively. We also proposed exact solutions to both problems, if the arc reversibility is permitted only once at time zero, that run with polynomial time complexity. By repeatedly solving the maximum dynamic contraflow problem using Algorithm 1, the quickest contraflow (QCF) problem on multi-network can also be solved efficiently. Searching of solutions to the problems at which the arc reversibility is permitted at any time point within the specified time horizon would be further research in the field considered here.

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