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Commutative Monoid on Symmetrical Difference Operator Over Intuitionistic Fuzzy Matrices

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Abstract

In this article, we extend the mathematical operation symmetrical difference ($\ominus$) to intuitionistic fuzzy matrix. Various properties of the difference operator '$\ominus$' are discussed over intuitionistic fuzzy matrices. Also associativity property of the above said operator is studied when each entry of an intuitionistic fuzzy matrix is either intuitionistic fuzzy tautological or co tautological since it is very critical when we prove that in usual manner. Finally, a commutative monoid algebraic structure is obtained on symmetrical difference operator over intuitionistic fuzzy matrices.

Keywords: Intuitionistic fuzzy matrix; Intuitionistic fuzzy tautological and cotautological

MSC 2010 No.: 03E72, 15B15, 94D05

1. Introduction

The concept of fuzzy set has been found to be an effective tool to deal with fuzziness. However it often falls short of the expected standard in the description of neutral state. As a result, a new concept called IFS was introduced by Atanassov in 1983, when it is possible to model hesitation and uncertainty by using an additional degree. Later, much fundamental works with new operations have done with the concept by Atanassov in 1986. Also, an intuitionistic fuzzy tautological set and co tautological set were developed by Atanassov ((1988), (1999)). Im et al. (2001), Pal (2001), and Khan et al. (2002) generalize a fuzzy matrix as IFM in with its operations and has been useful in dealing with the areas such as decision making, relational equations, clustering analysis, etc. IFM is also very useful in the discussion of Intuitionistic fuzzy relation. Khan and Pal ((2002-2003), (2005)) introduced intuitionistic fuzzy tautological matrix (IFTM) and interval value intuitionistic fuzzy matrices. Later, Murugadas and Lalitha (2015) developed intuitionistic fuzzy cotautological matrix (IFCTM). Adak et al. ((2012), (2013)) established nilpotent matrices over distributive lattice and distributive lattice over intuitionistic fuzzy matrices. Generalized intuitionistic fuzzy matrices and generalized interval valued intuitionistic fuzzy sets are studied by Bhowmik and Pal ((2008), (2012)).

A lot of research activities with several algebraic structures have been carried out during various years by different researchers on IFMs in (Shyamal and Pal (2002), Im et al. (2003), Pradhan and Pal (2013), Mondal and Pal (2014), Boobalan and Sriram (2016), Muthuraji and Sriram (2016), Atanassov (2017), Muthuraji and Sriram (2017), Silambarasan and Sriram (2019), Silambarasan and Sriram (2020)). Recently Mondal and Pal (2019) developed bipolar fuzzy matrices; also, Dogra and Pal (2020) studied picture fuzzy matrices with its applications.

Symmetrical difference over ordinary sets is defined in Kuratowski (2001) through basic operations like union, intersection and negation as $A \ominus B = (A \wedge B^c) \vee (A^c \wedge B)$. Anton Antonov (2004) defined the above said operator to IFSs with some properties and have shown that associativity was not hold on IFSs.

In this article, symmetrical difference operator is extended to IFMs. Several properties are studied and a commutative monoid structure is obtained on $'\ominus'$ over IFM. The structure of this article is as follows. In Section 2 some basic concepts which are related to this work are recalled. Symmetrical difference operator is extended to IFM and some algebraic properties are studied in Section 3. A commutative monoid structure is constructed over IFM in Section 4.

2. Preliminaries

Atanassov (1986) defined an IFS A in E (universal set) as an object of the following form $A = \{(x, \mu_A(x)), \gamma_A(x)/x \in E\}$, where the functions $\mu_A(x) : E \to [0,1]$ and $\gamma_A(x) : E \to [0,1]$ define the membership and non-membership function of the element $x \in E$, respectively, for every $x \in E, 0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

For our convenience, consider the elements of IFSs as in the form (x, x'). Let $(x, x'), (y, y') \in$ IFS. Then, the basic operations on IFS are given as

(i) $(x, x') \vee (y, y') = [\max(x, y), \min(x', y')]$,

(ii) $(x, x') \wedge (y, y') = [\min(x, y), \max(x', y')]$,

(iii) $(x, x')^c = (x', x)$.

Atanassov ((1988), (1999)) established an intuitionistic fuzzy set A is an intuitionistic fuzzy tautologicl set (or cotautological) if the membership ship value $x \geq x'(x' \geq x)$ for all $x \in X$.

Im et al. (2001) and Khan et al. (2002) introduced intuitionistic fuzzy matrix $A = [(a_{ij}, a'_{ij})]_{m\times n}$ as a matrix where a_{ij} and a'_{ij} are the membership and nonmembership value of the ij^{th} element of A satisfying the condition that $0 \leq a_{ij} + a'_{ij} \leq 1$ for all i, j as well as its operations are defined as follows.

For any two elements $A = [(a_{ij}, a'_{ij})]$, $B = [(b_{ij}, b'_{ij})] \in \mathscr{F}_{mn}$ where $i = 1, 2, \cdots, m,\quad j = 1, 2, \cdots, n$, define

(i) $A \vee B = [(a_{ij}, a'_{ij}) \vee (b_{ij}, b'_{ij})]$.

(ii) $A \wedge B = [(a_{ij}, a'_{ij}) \wedge (b_{ij}, b'_{ij})]$.

(iii) $A^c = [(a'_{ij}, a_{ij})]$.

(iv) $A^T = [(a_{ji}, a'_{ji})]$.

(v) If $A \leq B$, then $a_{ij} \leq b_{ij}$ and $a'_{ij} \geq b'_{ij}$.

(vi) If $(a_{ij}, a'_{ij}) = (1, 0)$ for all i, j, then A is said to an Universal matrix denoted by U.

(vii) If $(a_{ij}, a'_{ij}) = (0, 1)$ for all i, j, then A is said to be Zero matrix denoted by O

where $\mathscr{F}_{mn}$ denotes the set of all IFMs of order $m \times n$.

Motivation of this Work

Anton Antonov (2004) developed symmetrical difference operator between the elements of two IFSs A and B as $(x, x') \in A$ and $(y, y') \in B$ then

$$A \ominus B = max(min(x, y'), min(x', y)), min(max(x', y), max(x, y')),$$

in that the author has shown that associative not hold by taking some concrete values which were not comparable elements on IFS theory. After that, no further study have been made on this operation. For any comparable elements on IFS, the operation '$\ominus$' is associative but the mathematical way of proving some algebraic properties like associativity are very critical.

My aim of this work is to develop some results of the above said operation by proving some properties in easy manner through intuitionistic fuzzy tautological and co tautological.

3. Properties of Symmetrical Difference Operator on IFM

In this section symmetrical difference operator $\ominus$ is extended to IFM. Some properties are discussed.

Definition 3.1.

Let A and B are IFMS of same order. The symmetrical difference operator $(\ominus)$ over A and B using the basic operations $\vee$, $\wedge$ and complement (c) is defined in the following way.

Let $A = [(a_{ij}, a'_{ij})]$ and $B = [(b_{ij}, b'_{ij})]$, define

$$A \ominus B = (A \wedge B^C) \vee (A^C \wedge B),$$

$$\begin{aligned} A \ominus B &= [(a_{ij} \wedge b'_{ij}) \vee (a'_{ij} \wedge b_{ij}), (a'_{ij} \vee b_{ij}) \wedge (a_{ij} \vee b'_{ij})] \\ &= [(a_{ij} \wedge b'_{ij}), (a'_{ij} \vee b_{ij})] \vee [(a'_{ij} \wedge b_{ij}), (a_{ij} \vee b'_{ij})] \\ &= [(a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij})] \vee [(a'_{ij}, a_{ij}) \wedge (b_{ij}, b'_{ij})] \\ A \ominus B &= [(a_{ij} \wedge b'_{ij}) \vee (a'_{ij} \wedge b_{ij}), (a'_{ij} \vee b_{ij}) \wedge (a_{ij} \vee b'_{ij})], \end{aligned}$$

for all i, j.

Lemma 3.1.

For any $A, B \in \mathscr{F}_{mn}$, we have the following,
(i) $A \ominus B = B \ominus A$ ($\ominus$ is commutative).
(ii) $A \ominus U = A^C$.
(iii) $A \ominus O = A$.
(iv) $A \ominus A = A \wedge A^C$.

Proof:

(i) From Definition 3.1, we have

$$\begin{aligned} A \ominus B &= [(a_{ij} \wedge b'_{ij}) \vee (a'_{ij} \wedge b_{ij}), (a'_{ij} \vee b_{ij}) \wedge (a_{ij} \vee b'_{ij})] \text{ for all i, j} \\ &= [(b'_{ij} \wedge a_{ij}) \vee (b_{ij} \wedge a'_{ij}), (b_{ij} \vee a'_{ij}) \wedge (b'_{ij} \vee a_{ij})] \\ &= [(b_{ij} \wedge a'_{ij}) \vee (b'_{ij} \wedge a_{ij}), (b'_{ij} \vee a_{ij}) \wedge (b_{ij} \vee a'_{ij})] \text{ (since `}\wedge\text{'and `}\vee\text{'are commutative)} \\ &= B \ominus A. \end{aligned}$$

(ii) Let $A = [(a_{ij}, a'_{ij})]$ and $U = [(1, 0)]$ for all i, j, then

$$\begin{aligned} A \ominus U &= [(a_{ij}, a'_{ij}) \ominus (1, 0)] \\ &= [(a_{ij} \wedge 0) \vee (a'_{ij} \wedge 1), (a'_{ij} \vee 1) \wedge (a_{ij} \vee 0)] \\ &= [0 \vee a'_{ij}, 1 \wedge a_{ij}] \\ &= [a'_{ij}, a_{ij}] \\ &= A^C. \end{aligned}$$

(iii) Let $O = [(0,1)]$ for all i, j. Then

$$\begin{aligned} A \ominus O &= [(a_{ij}, a'_{ij}) \ominus (0,1)] \\ &= [(a_{ij} \wedge 1) \vee (a'_{ij} \wedge 0), (a'_{ij} \vee 0) \wedge (a_{ij} \vee 1)] \\ &= [a_{ij} \vee 0, a'_{ij} \wedge 1] \\ &= [a_{ij}, a'_{ij}] \\ &= A. \end{aligned}$$

(iv)

$$\begin{aligned} A \ominus A &= [(a_{ij} \wedge a'_{ij}) \vee (a'_{ij} \wedge a_{ij}), (a'_{ij} \vee a_{ij}) \wedge (a_{ij} \vee a'_{ij})] \\ &= [(a_{ij} \wedge a'_{ij}), (a'_{ij} \vee a_{ij})] \\ &= A \wedge A^C. \end{aligned}$$

■

Lemma 3.2.

For any A, B in $\mathscr{F}_{mn}$, we have

$$A^C \ominus B^C = A \ominus B.$$

Proof:

$$\begin{aligned} A^C \ominus B^C &= [(a'_{ij}, a_{ij}) \ominus (b'_{ij}, b_{ij})] \text{ for all i,j} \\ &= [(a'_{ij} \wedge b_{ij}) \vee (a_{ij} \wedge b'_{ij}), (a_{ij} \vee b'_{ij}) \wedge (a'_{ij} \vee b_{ij})] \\ &= [(a_{ij} \wedge b'_{ij}) \vee (a'_{ij} \wedge b_{ij}), (a'_{ij} \vee b_{ij}) \wedge (a_{ij} \vee b'_{ij})] \\ &= A \ominus B. \end{aligned}$$

■

Lemma 3.3.

Let A, B and C be any IFM of same order with corresponding elements are comparable whenever the matrices may or may not be comparable . Then '$\ominus$' is associative which means the following holds:

$$(A \ominus B) \ominus C = A \ominus (B \ominus C).$$

Proof:

By an illustrative example we shall prove the above property as follows.

$$A = \begin{bmatrix} (0.4, 0.3) & (0.7, 0.1) \\ (0.2, 0.6) & (0.5, 0.2) \end{bmatrix},$$

$$B = \begin{bmatrix} (0.7, 0.2) & (0.5, 0.2) \\ (0.1, 0.8) & (0.6, 0.1) \end{bmatrix},$$

$$C = \begin{bmatrix} (0.0, 1.0) & (1.0, 0.0) \\ (0.4, 0.5) & (0.9, 0.1) \end{bmatrix},$$

$$A \ominus B = \begin{bmatrix} (0.3, 0.4) & (0.2, 0.5) \\ (0.2, 0.6) & (0.2, 0.5) \end{bmatrix}, \quad (A \ominus B) \ominus C = \begin{bmatrix} (0.3, 0.4) & (0.5, 0.2) \\ (0.4, 0.5) & (0.5, 0.2) \end{bmatrix},$$

$$B \ominus C = \begin{bmatrix} (0.7, 0.2) & (0.2, 0.5) \\ (0.4, 0.5) & (0.1, 0.6) \end{bmatrix}, \quad A \ominus (B \ominus C) = \begin{bmatrix} (0.3, 0.4) & (0.5, 0.2) \\ (0.4, 0.5) & (0.5, 0.2) \end{bmatrix}.$$

Hence $(A \ominus B) \ominus C = A \ominus (B \ominus C)$. ■

4. Commutative Monoid on Symmetrical Difference Operator over IFM

In this section, we obtain the theoretical proof lemma 3.4 and a commutative monoid algebraic structure.

Let S_1 be the set of all intuitionistic fuzzy tautological element and S_2 be the set of all intuitionistic fuzzy co tautological element. Any IFM whose entries are either from set S_1 or S_2. Here after $\mathscr{F}_{mn}$ means the set of all IFMs of order $m \times n$ with corresponding entries are comparable whenever the matrices may or may not be comparable.

Lemma 4.1.

Let $(a_{ij}, a'_{ij}), (b_{ij}, b'_{ij}) \in S_2$ are any ij^{th} entry of the IFMs A, B then

$$(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij}) = \begin{cases} (a_{ij}, a'_{ij}), & \text{if } (a_{ij}, a'_{ij}) \geq (b_{ij}, b'_{ij}), \\ (b_{ij}, b'_{ij}), & \text{if } (a_{ij}, a'_{ij}) \leq (b_{ij}, b'_{ij}). \end{cases}$$

Proof:

Consider the ij^{th} element of A and B as (a_{ij}, a'_{ij}) and (b_{ij}, b'_{ij}), also $(a_{ij}, a'_{ij}), (b_{ij}, b'_{ij}) \in S_2$.

Therefore, we have $a_{ij} \leq a'_{ij}, b_{ij} \leq b'_{ij}$.

Suppose $(a_{ij}, a'_{ij}) \geq (b_{ij}, b'_{ij})$. Then $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$.

From the above two equations, we have the following

$$a_{ij} \leq a'_{ij} \leq b'_{ij} \Rightarrow a_{ij} \wedge b'_{ij} = a_{ij},$$

and

$$b_{ij} \leq a_{ij} \leq a'_{ij} \Rightarrow b_{ij} \wedge a'_{ij} = b_{ij}.$$

Now, the ij^{th} element of $A \ominus B$ is

$$\begin{aligned}(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij}) &= [(a_{ij} \wedge b'_{ij}) \vee (a'_{ij} \wedge b_{ij}), (a'_{ij} \vee b_{ij}) \wedge (a_{ij} \vee b'_{ij})] \\ &= [a_{ij} \vee b_{ij}, a'_{ij} \wedge b'_{ij}] \\ &= [(a_{ij}, a'_{ij})] \\ &= A.\end{aligned}$$

Similarly, we can prove $A \ominus B = (b_{ij}, b'_{ij})$ if $(a_{ij}, a'_{ij}) \leq (b_{ij}, b'_{ij})$. ■

Remark 4.1.

From Lemma 4.1, the symmetrical difference operator acts like join "$(\vee)$" operator when the entries are in S_2.

Lemma 4.2.

Let $(a_{ij}, a'_{ij}), (b_{ij}, b'_{ij}) \in S_1$ are any ij^{th} entry of the IFMs A, B. Then

$$(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij}) = \begin{cases} (a_{ij}, a'_{ij})^c, & \text{if } (a_{ij}, a'_{ij}) \leq (b_{ij}, b'_{ij}), \\ (b_{ij}, b'_{ij})^c, & \text{if } (b_{ij}, b'_{ij}) \leq (a_{ij}, a'_{ij}). \end{cases}$$

Proof:

Consider the ij^{th} element of

$$A \ominus B = [(a_{ij} \wedge b'_{ij}) \vee (a'_{ij} \wedge b_{ij}), (a'_{ij} \vee b_{ij}) \wedge (a_{ij} \vee b'_{ij})].$$

Since $(a_{ij}, a'_{ij}), (b_{ij}, b'_{ij}) \in S_1$, we have $a_{ij} \geq a'_{ij}$ and $b_{ij} \geq b'_{ij}$ for all i, j

Suppose $(a_{ij}, a'_{ij}) \geq (b_{ij}, b'_{ij})$. Then $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$.

From the above two equations, we have the following:

$$a'_{ij} \leq b'_{ij} \leq b_{ij} \Rightarrow a'_{ij} \wedge b_{ij} = a'_{ij}, \;\; a'_{ij} \vee b_{ij} = b_{ij},$$

and also

$$b'_{ij} \leq b_{ij} \leq a_{ij} \Rightarrow b'_{ij} \wedge a_{ij} = b'_{ij}, \;\; b'_{ij} \vee a_{ij} = a_{ij}.$$

Now, $(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})$ reduces to

$$\begin{aligned}(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij}) &= [(b'_{ij} \vee a'_{ij}), (b_{ij} \wedge a_{ij})] \\ &= [(b'_{ij}, b_{ij})] \\ &= (b_{ij}, b'_{ij})^c.\end{aligned}$$

Similarly, we can prove other case also. ■

Remark 4.2.

From Lemma 4.3, $A \ominus B = A^C$ when $A \leq B$ and from Lemma 3.3, $A \ominus B = A^C \ominus B^C$. Now, $A^C \ominus B^C = A \ominus B = A^C$ when $A \leq B$.

Lemma 4.3.

Let $(a_{ij}, a'_{ij}) \in S_1$ and $(b_{ij}, b'_{ij}) \in S_2$ are any ij^{th} entry of the IFMs A, B. Then

$$(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij}) = (a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij}) \ if \ (a_{ij}, a'_{ij}) \geq (b_{ij}, b_{ij}).$$

Proof:

Consider the ij^{th} element A and B are (a_{ij}, a'_{ij}) where $a_{ij} \geq a'_{ij}$ and (b_{ij}, b'_{ij}) with $b_{ij} \leq b'_{ij}$ since the elements S_1 and S_2 are tautological and cotautological, respectively.

Suppose $(a_{ij}, a'_{ij}) \geq (b'_{ij}, b_{ij})$ means $a_{ij} \geq b_{ij}, a'_{ij} \leq b'_{ij}$.

Now, we have $a_{ij} \geq a'_{ij} \wedge b_{ij}$ and $a_{ij} \geq a'_{ij} \vee b_{ij}$ also $b'_{ij} \geq a'_{ij} \wedge b_{ij}$ and $b'_{ij} \geq a'_{ij} \vee b_{ij}$

$$\Rightarrow a_{ij} \wedge b'_{ij} \geq a'_{ij} \wedge b_{ij} \text{ and } a_{ij} \vee b'_{ij} \geq a'_{ij} \vee b_{ij}.$$

Therefore, the ij^{th} element of $A \ominus B$ becomes

$$(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij}) = [a_{ij} \wedge b'_{ij}, a'_{ij} \vee b_{ij}] \Rightarrow (a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij}).$$ ■

Remark 4.3.

Let the ij^{th} entry of any IFMs A and B be $(a_{ij}, a'_{ij}) \leq (b_{ij}, b'_{ij})$. Then, either both the entries are in S_1 or S_2 or $(a_{ij}, a'_{ij}) \in S_2$ and $(b_{ij}, b'_{ij}) \in S_1$.

Suppose not. If $(a_{ij}, a'_{ij}) \in S_1$ and $(b_{ij}, b'_{ij}) \in S_2$, then we have the following:

$a_{ij} \geq a'_{ij} \geq b'_{ij} \geq b_{ij} \Rightarrow a_{ij} \geq b_{ij}$, which contradicts $(a_{ij}, a'_{ij}) \leq (b_{ij}, b'_{ij})$.

Theorem 4.1.

The symmetrical difference operation "$\ominus$" is associative over IFMs.

Proof:

Let A, B and C are any IFMs of same order with corresponding entries are comparable.

We prove this in three cases as follows.

Case 1:
Let us assume that for any i, j, $(a_{ij}, a'_{ij}), (b_{ij}, b'_{ij}), (c_{ij}, c'_{ij}) \in S_2$.

Also $(a_{ij}, a'_{ij}) \geq (b_{ij}, b'_{ij}) \geq (c_{ij}, c'_{ij})$. Then we have the following from Lemma 4.1,

$$[(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] = (a_{ij}, a'_{ij}),$$

also

$$[(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] \ominus (c_{ij}, c'_{ij}) = (a_{ij}, a'_{ij}) \ominus (c_{ij}, c'_{ij}) = (c_{ij}, c'_{ij}).$$

Similarly,

$$(a_{ij}, a'_{ij}) \ominus [(b_{ij}, b'_{ij})] \ominus (c_{ij}, c'_{ij}) = [(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] = [(a_{ij}, a'_{ij})].$$

In this way we can prove all the other comparability between ij^{th}entry of A, B and C.

Hence in this case "$\ominus$" is associative.

Case 2:

Let us assume that for any i, j, $(a_{ij}, a'_{ij}), (b_{ij}, b'_{ij}), (c_{ij}, c'_{ij}) \in S_1$. Also $(a_{ij}, a'_{ij}) \geq (b_{ij}, b'_{ij}) \geq (c_{ij}, c'_{ij})$. Then we have the following from Lemma 4.3,

$$[(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] = (b'_{ij}, b_{ij}).$$

Now

$$\begin{aligned} [(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] \ominus (c_{ij}, c'_{ij}) &= (b'_{ij}, b_{ij}) \ominus (c_{ij}, c'_{ij}) \\ &= [(b'_{ij} \wedge c'_{ij}) \vee (b_{ij} \wedge c_{ij}), (b_{ij} \vee c_{ij}) \wedge (b'_{ij} \vee c'_{ij})] \\ &= [b'_{ij} \vee c_{ij}, b_{ij} \wedge c'_{ij}] \\ &= (c_{ij}, c'_{ij}). \end{aligned}$$

Since $c_{ij} \geq c'_{ij} \geq b'_{ij} \Rightarrow c_{ij} \geq b'_{ij}$ and also $c'_{ij} \leq c_{ij} \leq b_{ij} \Rightarrow c'_{ij} \leq b_{ij}$, similarly

$$\begin{aligned} (a_{ij}, a'_{ij}) \ominus [(b_{ij}, b'_{ij}) \ominus (c_{ij}, c'_{ij})] &= (a_{ij}, a'_{ij}) \ominus (c'_{ij}, c_{ij}) \\ &= [(a_{ij} \wedge c_{ij}) \vee (a'_{ij} \wedge c'_{ij}), (a_{ij} \vee c_{ij}) \wedge (a'_{ij} \vee c'_{ij}] \\ &= [a'_{ij} \vee c_{ij}, a_{ij} \wedge c'_{ij}] \\ &= (c_{ij}, c'_{ij}). \end{aligned}$$

Thus, from the above equations, "$\ominus$" is associative in this case also.

Case 3:

Consider the ij^{th} entry of the IFMs A,B and C are given as $(a_{ij}, a'_{ij}) \in S_1, (b_{ij}, b'_{ij}) \in S_2, (c_{ij}, c'_{ij}) \in S_2$. Also, $(a_{ij}, a'_{ij}) \geq (b_{ij}, b'_{ij}) \geq (c_{ij}, c'_{ij})$. Then we have the following from Lemma 4.5,

$$[(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] = [(a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij})].$$

Now

$$\begin{aligned} [(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] \ominus (c_{ij}, c'_{ij}) &= [(a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij})] \ominus (c_{ij}, c'_{ij}) \\ &= [(a_{ij} \wedge b'_{ij}), (a'_{ij} \vee b_{ij})] \ominus (c_{ij}, c'_{ij}). \end{aligned}$$

Now the membership value of the above expression is $(a_{ij} \wedge b'_{ij}) \geq c_{ij}$ because $a_{ij} \geq c_{ij}$ and $b'_{ij} \geq b_{ij} \geq c_{ij}$ gives $(a_{ij} \wedge b'_{ij}) \geq c_{ij}$.

Similarly, non membership value of the equation is $(a'_{ij} \vee b_{ij}) \leq c'_{ij}$.

The value of $(a_{ij} \wedge b'_{ij}) \geq$ the value of $(a'_{ij} \vee b_{ij})$, which means this element

$$(a_{ij} \wedge b'_{ij}), (a'_{ij} \vee b_{ij}) \in S_1.$$

Therefore, from Lemma 4.5 and from the above two inequalities, we have the following,

$$\begin{aligned}[(a_{ij}, a'_{ij}) \ominus (b_{ij}, b'_{ij})] \ominus (c_{ij}, c'_{ij}) &= [(a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij})] \wedge (c'_{ij}, c_{ij}) \\ &= (a_{ij}, a'_{ij}) \wedge [(b'_{ij}, b_{ij}) \wedge (c'_{ij}, c_{ij})] \\ &= [(a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij})].\end{aligned}$$

Now consider the $ijth$ entry of $A \ominus [B \ominus C]$ as

$$\begin{aligned}(a_{ij}, a'_{ij}) \ominus [(b_{ij}, b'_{ij}) \ominus (c_{ij}, c'_{ij})] &= [(a_{ij}, a'_{ij}) \wedge (b_{ij}, b_{ij})] \\ &= [(a_{ij}, a'_{ij}) \wedge (b'_{ij}, b_{ij})],\end{aligned}$$

from the RHS of above two equations. We have “$\ominus$”is associative in this case also.

Hence the operation ‘$\ominus$’is associative over IFMs. ■

Theorem 4.2.

The algebraic structure $(\mathscr{F}_{mn}, \ominus, O)$ is a commutative monoid, where O represents a zero matrix.

Proof:

From Lemma 3.2, we have $A \ominus B = B \ominus A$, which means $\ominus$ is commutative over IFMs and $A \ominus O = A$ gives zero matrix which belongs to $\mathscr{F}_{mn}$ is an identity element.

Also from Theorem 4.7, $A \ominus (B \ominus C) = (A \ominus B) \ominus C$ gives the associative property.

Thus $(\mathscr{F}_{mn} \ominus, O)$ is a commutative monoid. ■

5. Conclusion

In this article, symmetrical difference operation $(\ominus)$ has been extended to IFMs. Some algebraic properties of “$\ominus$” are discussed over IFM. Theoretical proof to check whether difference operation is associative or not is very critical one. Here we prove the above in another direction which helps anyone to move further research on this area in the future. Finally, using the operation “$\ominus$,” an algebraic structure is constructed over IFM.

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