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Spherically Symmetric Charged Anisotropic Solution In Higher Dimensional Bimetric General Relativity

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Abstract

In this paper we have obtained a solution of field equations of Rosen's bimetric general relativity (BGR) for the static spherically symmetric space-time with charged anisotropic fluid distribution in $(n + 2)$-dimensions. An exact solution is obtained and a special case is considered. This work is an extension of our previous work where four-dimensional case was discussed.

Keywords: Bimetric general relativity; Static spherically symmetric space-time; Charged anisotropic fluid; Exact solution in BGR; Higher dimensional BGR

MSC 2010 No.: 83D05, 83E15

1. Introduction

General relativity is one of the most successful theory giving explanation of gravitation in accordance with observations. The only criticism associated with this theory is singularities occurring in the solutions. Many alternative theories are continuously proposed to circumvent singularities. In line with this Rosen (1940a), Rosen (1940b), Rosen (1963), Rosen (1973) proposed a modification to general relativity known as bimetric theory of gravitation. Several researchers studied various aspects of this theory, the interest is continuing and in last decade number of research papers were published on this theory. Numerous cosmological models based on distinct space-times have been investigated in recent years by Khadekar and Tade (2007), Sahoo (2008), Tripathy et al. (2010), Mahurpawar and Ronghe (2011), Sahoo et al. (2011), Jain et al. (2012), Sahoo and Mishra (2013a), Sahoo and Mishra (2013b) , Sahoo and Mishra (2013c), Sahoo and Mishra (2014b), Sahoo and Mishra (2014a), Sahu et al. (2015), Borkar and Gayakwad (2017), etc., in the context of this theory. But in most of the cases only vacuum solutions exist so we can conclude that this bimetric theory of gravitation does not help in any way to describe the early era of the universe.

2. Bimetric General Relativity

A modified version of the previous bimetric theory, called the bimetric general relativity (BGR), was proposed by Rosen (1978), Rosen (1980a), and Rosen (1980b). In this theory, gravity is attributed to a curved space-time described by the metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{1}$$

and a second metric tensor in the background space is described by

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu. \tag{2}$$

The field equations in BGR are written in the form of Einstein's field equations, but with an additional term on the right hand side,

$$G^\mu_\nu = -8\pi T^\mu_\nu + S^\mu_\nu, \tag{3}$$

where G^μ_ν is the Einstein tensor, T^μ_ν energy-momentum tensor of matter distribution and

$$S^\mu_\nu = \frac{3}{a^2}(\gamma_{\nu\alpha} g^{\alpha\mu} - \frac{1}{2}\delta^\mu_\nu g^{\alpha\beta}\gamma_{\alpha\beta}), \tag{4}$$

where a is a constant scale parameter. The order of this scale parameter is related to the size of the universe.

In view of observed anisotropy and presence of free charges it is worth investigating the space-times filled with anisotropic charged fluid. This paper is intended to study an exact solution of the field equations for charged anisotropic fluid in $(n + 2)$-dimensional BGR proposed by Rosen (1980a). We have followed the method developed by Khadekar and Kandalkar (2004) by introducing the *generating function* $G(r)$ which determines the relevant physical variables as well as the metrical coefficients and a function $w(r)$ measuring the degree of anisotropy. This function is called *anisotropic function*. The general relativity analogue of the charged anisotropic fluid in 4-dimensions was considered by Singh et al. (1995) and results obtained here match with those of obtained there. Also in absence of charge, results obtained in this paper match with the one obtained by Kandalkar and Gawande (2008) for the case of higher dimensional general theory of relativity. Moreover for $n = 2$, results in this paper match with the results obtained by Hasmani and Pandya (2017) for 4-dimensional anisotropic charged matter in BGR.

3. Metrics and Field Equations

The general static spherically symmetric line element may be expressed as

$$ds^2 = -exp[\lambda(r)]dr^2 - r^2 d\Omega^2 + exp[\nu(r)]dt^2, \tag{5}$$

where

$$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + \cdots + \left[\prod_{i=1}^{n-1} \sin^2\theta_i\right] d\theta_n^2. \tag{6}$$

Consider the background flat metric $\gamma_{\mu\nu}$ in $(n+2)$-dimensional analogue of static de-Sitter form as

$$d\sigma^2 = -\left(1 - \frac{r^2}{a^2}\right)^{-1} dr^2 - r^2 d\Omega^2 + \left(1 - \frac{r^2}{a^2}\right) dt^2. \tag{7}$$

For a region very small compared to a, i.e., for $r \ll a$, this line element has Minkowski form

$$d\sigma^2 = -dr^2 - r^2 d\Omega^2 + dt^2. \tag{8}$$

The convention used here for coordinates is

$$x^1 = r, x^2 = \theta_1, x^3 = \theta_2, \cdots, x^{n+1} = \theta_n, x^{n+2} = t.$$

The energy momentum tensor for charged anisotropic fluid is of the form

$$T_{\mu\nu} = (\rho + p_\perp)U_\mu U_\nu - p_\perp g_{\mu\nu} + (p_r - p_\perp)\chi_\mu \chi_\nu + \frac{1}{4\pi}\left(g^{\lambda\alpha}F_{\mu\lambda}F_{\nu\alpha} - \frac{1}{4}g_{\mu\nu}F_{\lambda\alpha}F^{\lambda\alpha}\right), \tag{9}$$

with matter density ρ, p_r being the radial pressure in the direction of χ_μ, $p_\perp$ being the tangential pressure orthogonal to χ_μ, the $(n+2)$-velocity vector of the fluid U_μ and χ_μ being the unit space-like vector orthogonal to U_μ.

The skew symmetric Maxwell Tensor $F_{\mu\nu}$ satisfies the Maxwell's equations in the form

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0, \tag{10}$$

$$F^{\mu\nu}{}_{;\nu} = -4\pi J^\mu, \tag{11}$$

where $J^\mu = \sigma U^\mu$ is the $(n+2)$-current of the charge distribution with proper charge density σ within the n-sphere. It is known that due to spherical symmetry the only non-vanishing components of $F_{\mu\nu}$ are $F_{1(n+2)}$ and $F_{(n+2)1}$.

Choosing the comoving system we write

$$U^\mu = \left(\underbrace{0, 0, 0, \cdots, 0}_{(n+1)times}, \exp\left[-\frac{\nu}{2}\right]\right), \tag{12}$$

$$\chi^\mu = \left(\exp\left[-\frac{\lambda}{2}\right], \underbrace{0, 0, \cdots, 0}_{(n+1)times}\right), \tag{13}$$

where $U_\mu U^\mu = -\chi_\mu \chi^\mu = 1$.

In the region $r \ll a$ and neglecting the terms which are small throughout this region, the non-vanishing components of S^{μ}_{ν} can be written in accordance with Falik and Rosen (1980) as,

$$-S^1_1 = -S^2_2 = -S^3_3 = \cdots = -S^{n+1}_{n+1} = S^{n+2}_{n+2} = \frac{3}{2a^2}\text{exp}[-\nu]. \tag{14}$$

Using the procedure given by Rosen (1980a), the final form of field equations (3) using Einstein tensor G^{μ}_{ν} for metric (5), energy-momentum tensor (9), background metric (8) and values from equation (14) are written as,

$$\text{exp}[-\lambda]\left(\frac{n\nu'}{2r} + \frac{n(n-1)}{2r^2}\right) - \frac{n(n-1)}{2r^2} = 8\pi p_r - E^2 - \frac{3}{2a^2}\text{exp}[-\nu], \tag{15}$$

$$\begin{aligned}\text{exp}[-\lambda]\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{(n-1)(\lambda'-\nu')}{2r} + \frac{(n-1)(n-2)}{2r^2}\right)& \\ - \frac{(n-1)(n-2)}{2r^2} = 8\pi p_{\perp} + E^2 - \frac{3}{2a^2}\text{exp}[-\nu],&\end{aligned} \tag{16}$$

$$\text{exp}[-\lambda]\left(\frac{n\lambda'}{2r} - \frac{n(n-1)}{2r^2}\right) + \frac{n(n-1)}{2r^2} = 8\pi\rho + E^2 - \frac{3}{2a^2}\text{exp}[-\nu], \tag{17}$$

$$(r^n E)' = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})} r^n \sigma(r) exp[\frac{\lambda}{2}], \tag{18}$$

where E is the electric field strength and a prime for λ and ν denotes a differentiation with respect to r.

The energy-momentum conservation equations $T^{\mu}_{\nu;\mu} = 0$ gives,

$$(\rho + p_r)\frac{\nu'}{2} + p_r' = \frac{n}{r}(p_{\perp} - p_r) + \frac{1}{8\pi r^4}\frac{dQ^2}{dr} + \frac{(n-2)E^2}{4\pi r}, \tag{19}$$

where charge Q is related with the electric field strength E, through the integral form of the Maxwell's equation (18), which can be written as

$$Q(r) = Er^n = 4\pi \int_0^r r^n exp[\lambda/2]\sigma(r)dr. \tag{20}$$

Following Harpaz and Rosen (1985), we can define the *effective* density ρ_e, *effective* radial pressure p_{r_e} and *effective* tangential pressure $p_{\perp_e}$ as,

$$\begin{aligned}\rho_e &= \rho - \frac{3}{16\pi a^2}\text{exp}[-\nu],\\ p_{r_e} &= p_r - \frac{3}{16\pi a^2}\text{exp}[-\nu],\\ p_{\perp_e} &= p_{\perp} - \frac{3}{16\pi a^2}\text{exp}[-\nu].\end{aligned} \tag{21}$$

So the field equations (15)-(17) take the form,

$$\text{exp}[-\lambda]\left(\frac{n\nu'}{2r} + \frac{n(n-1)}{2r^2}\right) - \frac{n(n-1)}{2r^2} = 8\pi p_{r_e} - \frac{Q^2}{r^{2n}}, \tag{22}$$

$$\exp[-\lambda]\left(\frac{\nu''}{2}-\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}-\frac{(n-1)(\lambda'-\nu')}{2r}+\frac{(n-1)(n-2)}{2r^2}\right) \\ -\frac{(n-1)(n-2)}{2r^2}=8\pi p_{\perp_e}+\frac{Q^2}{r^{2n}}, \tag{23}$$

$$\exp[-\lambda]\left(\frac{n\lambda'}{2r}-\frac{n(n-1)}{2r^2}\right)+\frac{n(n-1)}{2r^2}=8\pi\rho_e+\frac{Q^2}{r^{2n}}. \tag{24}$$

Equation (19) can be rewritten as,

$$(\rho_e+p_{r_e})\frac{\nu'}{2}+p'_{r_e}=\frac{n}{r}(p_{\perp_e}-p_{r_e})+\frac{1}{8\pi r^4}\frac{dQ^2}{dr}+\frac{(n-2)Q^2}{4\pi r^{2n+1}}. \tag{25}$$

4. Solutions of the Field Equations

Now from Equation (24),

$$\exp[-\lambda]=1-\frac{2m_e(r)}{r}+\frac{2}{n(n-1)}\frac{Q^2}{r^{2n-2}}, \tag{26}$$

where $m_e(r)$ is the *effective* mass function defined as,

$$m_e(r)=\frac{1}{nr^{n-2}}\int_0^r\left(8\pi\rho_e r^n+\frac{2QQ'}{(n-1)r^{n-1}}\right)dr. \tag{27}$$

Now from equation (25),

$$\nu'=-\frac{2p'_{r_e}}{(\rho_e+p_{r_e})}+\frac{2n(p_{\perp_e}-p_{r_e})}{r(\rho_e+p_{r_e})}+\frac{QQ'}{2\pi r^4(\rho_e+p_{r_e})}+\frac{(n-2)Q^2}{2\pi r^{2n+1}(\rho_e+p_{r_e})}. \tag{28}$$

Using Equations (26) and (28) in (22), one can get

$$\left[1-\frac{2m_e}{r}+\frac{2}{n(n-1)}\frac{Q^2}{r^{2n-2}}\right]\left[-\frac{nrp'_{r_e}}{(\rho_e+p_{r_e})}+\frac{n^2(p_{\perp_e}-p_{r_e})}{(\rho_e+p_{r_e})}+\frac{nQQ'}{4\pi r^3(\rho_e+p_{r_e})} \\ +\frac{n(n-2)Q^2}{4\pi r^{2n}(\rho_e+p_{r_e})}+\frac{n(n-1)}{2}\right]=8\pi p_{r_e}r^2+\frac{n(n-1)}{2}-\frac{Q^2}{r^{2n-2}}. \tag{29}$$

Defining a *generating function* $G(r)$ as,

$$G(r)=\frac{1-\frac{2m_e}{r}+\frac{2}{n(n-1)}\frac{Q^2}{r^{2n-2}}}{8\pi p_{r_e}r^2+\frac{n(n-1)}{2}-\frac{Q^2}{r^{2n-2}}}, \tag{30}$$

and introducing an *anisotropic function* $w(r)$ as,

$$w(r)=\frac{n^2(p_{r_e}-p_{\perp_e})}{(\rho_e+p_{r_e})}G(r). \tag{31}$$

Using Equations (30) and (31) in Equation (29), we get

$$8\pi(\rho_e+p_{r_e})=\frac{-8\pi nrp'_{r_e}G+\frac{2nQQ'G}{r^3}+\frac{2n(n-2)Q^2G}{r^{2n}}}{(1-\frac{n(n-1)}{2}G+w)}. \tag{32}$$

Differentiation of Equation (26) gives,

$$\exp[-\lambda]\lambda' = \frac{2m_e'}{r} - \frac{2m_e}{r^2} - \frac{4QQ'}{n(n-1)r^{2n-2}} + \frac{4Q^2}{nr^{2n-1}}. \tag{33}$$

Adding $8\pi p_{r_e}$ on both sides of Equation (24) and then using Equations (33), (26) and (30), we get

$$\begin{aligned} 8\pi(\rho_e + p_{r_e}) = &\frac{nm_e'}{r^2} - \frac{nm_e}{r^3} - \frac{2QQ'}{(n-1)r^{2n-1}} + \frac{2Q^2}{r^{2n}} \\ &+ \left(\frac{n(n-1)}{2r^2} + 8\pi p_{r_e} - \frac{Q^2}{r^{2n}}\right)\left(1 - \frac{n(n-1)}{2}G\right). \end{aligned} \tag{34}$$

Differentiation of Equation (30) gives,

$$\begin{aligned} \frac{nm_e'}{r^2} - \frac{nm_e}{r^3} - \frac{2QQ'}{(n-1)r^{2n-1}} + \frac{2Q^2}{r^{2n}} = &-\frac{nG'}{2r}\left(8\pi p_{r_e} r^2 + \frac{n(n-1)}{2} - \frac{Q^2}{r^{2n-2}}\right) \\ &- 4\pi nrp_{r_e}'G - 8\pi np_{r_e}G + \frac{nQQ'G}{r^{2n-1}} - \frac{n(n-1)Q^2G}{r^{2n}}. \end{aligned} \tag{35}$$

Using Equations (35) and (32) into Equation (34), we get

$$\begin{aligned} &8\pi p_{r_e}' + \frac{(2 - n(n+1)G - nrG')(1 - \frac{n(n-1)}{2}G + w)}{nrG(1 + \frac{n(n-1)}{2}G - w)} 8\pi p_{r_e} \\ &+ \frac{(n-1)(2 - n(n-1)G - nrG')(1 - \frac{n(n-1)}{2}G + w)}{2r^3G(1 + \frac{n(n-1)}{2}G - w)} \\ &- \frac{(2 + n(n-1)G - nrG')(1 - \frac{n(n-1)}{2}G + w)}{nrG(1 + \frac{n(n-1)}{2}G - w)} \frac{Q^2}{r^{2n}} \\ &+ \frac{(1 - \frac{n(n-1)}{2}G + w)}{(1 + \frac{n(n-1)}{2}G - w)} \frac{2QQ'}{r^{2n}} \\ &- \frac{4QQ'}{r^4(1 + \frac{n(n-1)}{2}G - w)} - \frac{4(n-2)}{(1 + \frac{n(n-1)}{2}G - w)} \frac{Q^2}{r^{2n+1}} = 0, \end{aligned} \tag{36}$$

which is a linear differential equation in p_{r_e}. Hence,

$$8\pi p_{r_e} = exp\left[-\int B(r)dr\right]\left[p_0 + \int C(r)exp\left[\int B(r)dr\right]dr\right], \tag{37}$$

where p_0 is a constant of integration and we have taken

$$B(r) = \frac{(2 - n(n+1)G - nrG')(1 - \frac{n(n-1)}{2}G + w)}{nrG(1 + \frac{n(n-1)}{2}G - w)},$$

$$\begin{aligned} C(r) = &- \frac{(n-1)(2 - n(n-1)G - nrG')(1 - \frac{n(n-1)}{2}G + w)}{2r^3 G(1 + \frac{n(n-1)}{2}G - w)} \\ &+ \frac{(2 + n(n-1)G - nrG')(1 - \frac{n(n-1)}{2}G + w)}{nrG(1 + \frac{n(n-1)}{2}G - w)} \frac{Q^2}{r^{2n}} \\ &- \frac{(1 - \frac{n(n-1)}{2}G + w)}{(1 + \frac{n(n-1)}{2}G - w)} \frac{2QQ'}{r^{2n}} \\ &+ \frac{4QQ'}{r^4(1 + \frac{n(n-1)}{2}G - w)} + \frac{4(n-2)}{(1 + \frac{n(n-1)}{2}G - w)} \frac{Q^2}{r^{2n+1}}. \end{aligned} \tag{38}$$

From Equation (27),

$$\frac{m_e'}{r} = \frac{8\pi\rho_e r}{n} + \frac{2QQ'}{n(n-1)r^{2n-2}}. \tag{39}$$

Using Equation (39) in (35) and then using Equation (30), we get

$$\begin{aligned} 8\pi\rho_e = &\left(1 - \frac{n(n-1)}{2}G\right)\frac{n}{2r^2} - \left(\frac{2(2n-3)}{n(n-1)} + (2n-3)G\right)\frac{nQ^2}{2r^{2n}} \\ &-4n\pi G\left(3p_{r_e} + rp_{r_e}' - \frac{QQ'}{4\pi r^{2n-1}}\right) - \frac{nrG'}{2}\left(8\pi p_{r_e} + \frac{n(n-1)}{2r^2} - \frac{Q^2}{r^{2n}}\right), \end{aligned} \tag{40}$$

which is the expression for effective density ρ_e. Equation (31) gives

$$p_{\perp_e} = p_{r_e} - \frac{w}{n^2 G}(\rho_e + p_{r_e}). \tag{41}$$

Equations (26) and (30) yields,

$$\exp[-\lambda] = G\left(8\pi p_{r_e} r^2 + \frac{n(n-1)}{2} - \frac{Q^2}{r^{2n-2}}\right). \tag{42}$$

Using this in Equation (22), we obtain

$$\frac{d\nu}{dr} = \frac{2}{nrG} - \frac{(n-1)}{r}, \tag{43}$$

which after integration gives,

$$\exp[\nu] = \frac{A^2}{r^{(n-1)}}\exp\left[\frac{2}{n}\int (1/rG)dr\right], \tag{44}$$

where A is a constant of integration.

Using values from Equations (44) and (26), the space-time (5) becomes

$$\begin{aligned} ds^2 = &-\left[1 - \frac{2m_e(r)}{r} + \frac{2}{n(n-1)}\frac{Q^2}{r^{2n-2}}\right]^{-1} dr^2 - r^2 d\Omega^2 \\ &+ \frac{A^2}{r^{(n-1)}} exp\left[\frac{2}{n}\int (1/rG)dr\right] dt^2. \end{aligned} \tag{45}$$

5. Special case

The local flatness at the origin is required for a physically meaningful solution. Thus we assume the non divergent effective pressure at origin, as $r \to 0$, $\frac{m_e(r)}{r} \to 0$ and $\frac{Q^2}{r^{2n-2}} \to 0$ which results into $G(r) \to \frac{2}{n(n-1)}$. If $G(r) = \frac{2}{n(n-1)}, w(r) = 0 (i.e. p_r = p_\perp)$ and $Q(r) = 0$, one obtains $\lambda = 0$.

By considering the charge density σ to be constant, we can get $Q(r) \sim r^3$ from Equation (20). The appropriate junction condition at the surface $r = r_0$ yields

$$Q(r) = e(r/r_0)^3. \tag{46}$$

If we denote $e/r_0^3 = K$ then we can write,

$$Q(r) = Kr^3. \tag{47}$$

Further we define *generating function* and *anisotropic function* from Equations (30) and (31), respectively, as

$$G(r) = \frac{2}{n(n-1)}(1 - \alpha r^2), \tag{48}$$

$$w(r) = -\alpha r^2, \tag{49}$$

where α is a constant, this choice is physically appropriate because function $G(r) \sim \frac{2}{n(n-1)}$ as $r \sim 0$.

From Equation (38), $B(r) = 0, C(r) = 6K^2 r + 2(n-2)K^2 r^{5-2n}$. So from Equation (37),

$$p_{r_e} = \frac{p_0}{8\pi} + \frac{3K^2 r^2}{8\pi} + \frac{(n-2)}{(3-n)}\frac{K^2}{8\pi} r^{6-2n}. \tag{50}$$

If constant $p_0 = 0$, then

$$p_{r_e} = \frac{3K^2 r^2}{8\pi} + \frac{(n-2)}{(3-n)}\frac{K^2}{8\pi} r^{6-2n}. \tag{51}$$

Hence, from Equation (21) the radial pressure is given by,

$$p_r = \frac{3}{16\pi a^2}\left(\frac{(1-\alpha r^2)^{\frac{n-1}{2}}}{A^2}\right) + \frac{3K^2 r^2}{8\pi} + \frac{(n-2)}{(3-n)}\frac{K^2}{8\pi} r^{6-2n}. \tag{52}$$

Also, from Equation (40) the effective density is obtained as,

$$\rho_e = \frac{3n\alpha}{16\pi} + \frac{21\alpha K^2 r^4}{8\pi(n-1)} - \frac{15K^2 r^2}{8\pi(n-1)} - \frac{(4n^2 - 32n + 55)}{(n-1)(3-n)}\frac{\alpha K^2}{8\pi} r^{8-2n} + \frac{(6n^2 - 37n + 54)}{(n-1)(3-n)}\frac{K^2}{8\pi} r^{6-2n}, \tag{53}$$

which gives the energy density using Equation (21) as,

$$\rho = \frac{3}{16\pi a^2}\left(\frac{(1-\alpha r^2)^{\frac{n-1}{2}}}{A^2}\right) + \frac{3n\alpha}{16\pi} + \frac{21\alpha K^2 r^4}{8\pi(n-1)} - \frac{15K^2 r^2}{8\pi(n-1)} - \frac{(4n^2 - 32n + 55)}{(n-1)(3-n)}\frac{\alpha K^2}{8\pi} r^{8-2n} + \frac{(6n^2 - 37n + 54)}{(n-1)(3-n)}\frac{K^2}{8\pi} r^{6-2n}. \tag{54}$$

Using values from Equations (48), (49), (51) and (53) into Equation (41), one can obtain

$$\begin{aligned} p_{\perp_e} = & \frac{3K^2r^2}{8\pi} + \frac{3(n-1)\alpha^2 r^2}{32\pi(1-\alpha r^2)} + \frac{3(n-6)\alpha K^2 r^4}{16\pi n(1-\alpha r^2)} \\ & + \frac{21\alpha^2 K^2 r^6}{16\pi n(1-\alpha r^2)} - \frac{(4n^2-32n+55)\alpha^2 K^2 r^{10-2n}}{16\pi n(3-n)(1-\alpha r^2)} \\ & + \frac{K^2 r^{6-2n}}{8\pi(3-n)}\left\{\frac{(7n^2-40n+56)\alpha r^2}{2n(1-\alpha r^2)} + (n-2)\right\}. \end{aligned} \tag{55}$$

Further, from Equation (21) the tangential pressure is given by,

$$\begin{aligned} p_{\perp} = & \frac{3}{16\pi a^2}\left(\frac{(1-\alpha r^2)^{\frac{n-1}{2}}}{A^2}\right) + \frac{3K^2r^2}{8\pi} + \frac{3(n-1)\alpha^2 r^2}{32\pi(1-\alpha r^2)} + \frac{3(n-6)\alpha K^2 r^4}{16\pi n(1-\alpha r^2)} \\ & + \frac{21\alpha^2 K^2 r^6}{16\pi n(1-\alpha r^2)} - \frac{(4n^2-32n+55)\alpha^2 K^2 r^{10-2n}}{16\pi n(3-n)(1-\alpha r^2)} \\ & + \frac{K^2 r^{6-2n}}{8\pi(3-n)}\left\{\frac{(7n^2-40n+56)\alpha r^2}{2n(1-\alpha r^2)} + (n-2)\right\}. \end{aligned} \tag{56}$$

Using Equations (48) and (51), Equation (42) can be written as

$$\exp[-\lambda] = \frac{2}{n(n-1)}(1-\alpha r^2)\left(\frac{n(n-1)}{2} + 3K^2r^4 + \frac{(2n-5)}{(3-n)}K^2 r^{8-2n}\right). \tag{57}$$

Using Equation (48) into Equation (44), we get

$$\exp[\nu] = \frac{A^2}{(1-\alpha r^2)^{\frac{n-1}{2}}}. \tag{58}$$

These expressions of $\exp[-\lambda]$ and $\exp[\nu]$ obtained above give metric (5) in the form

$$\begin{aligned} ds^2 = & -\left[\frac{2}{n(n-1)}(1-\alpha r^2)\left(\frac{n(n-1)}{2} + 3K^2r^4 + \frac{(2n-5)}{(3-n)}K^2 r^{8-2n}\right)\right]^{-1} dr^2 \\ & - r^2 d\Omega^2 + \frac{A^2}{(1-\alpha r^2)^{\frac{n-1}{2}}} dt^2. \end{aligned} \tag{59}$$

This metric describes geometry of the cosmological model proposed by us.

6. Conclusion

In this paper we have presented exact analytical solution of field equations of bimetric general relativity for the case of static spherically symmetric anisotropic distribution of charged matter in $(n+2)$-dimensions by introducing the *generating function* and *anisotropic function*.

From the equation of effective mass function $m_e(r)$, we note that along with material density the electromagnetic anisotropy also contributes to the mass.

It can also be noted that for $Q(r) = 0$, the solution obtained here match with the solution of Kandalkar and Gawande for a neutral matter in higher dimensional general relativity. Thus, our solution is more general.

From the expressions of ρ, p_r and $p_\perp$ it is apparent that the last terms contribute negatively to these quantities. For large n, these terms tend to zero as they are in reciprocal powers of r.

Moreover, the present results reduce to the Einstein's general relativity for a physical system which is small compared to the size of the universe because in such case the term $\frac{3}{2a^2}exp[-\nu]$ in the field equations is negligible. This conclusion is derived by matching our results with the one obtained by Singh et al. for the 4-dimensional general relativity.

Moreover for $n = 2$, results in this paper match with our results obtained earlier for 4-dimensional anisotropic charged matter in BGR.

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