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Explicit and Exact Solutions with Multiple Arbitrary Analytic Functions of Jimbo–Miwa Equation

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Abstract

In this paper, a generalized F -expansion method is used to construct exact solutions of the (3+1)-dimensional Jimbo–Miwa equation. As a result, many new and more general exact solutions are obtained including single and combined non-degenerate Jacobi elliptic function solutions, hyperbolic function solutions and trigonometric function solutions, each of which contains six arbitrary analytic functions. It is shown that with the aid of symbolic computation the generalized F -expansion method may provide a straightforward and effective mathematical tool for solving nonlinear partial differential equations.

Keywords: Nonlinear partial differential equations; F -expansion method; Jacobi elliptic function solutions; Hyperbolic function solutions; Trigonometric function solutions

MSC (2000) No.: 35Q53, 35Q99

1. Introduction

Nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs) which are involved in many fields from physics to biology, chemistry, mechanics, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help one to understand these phenomena better. With the development of soliton theory, many effective methods for obtaining exact solutions of NLPDEs have been presented such as Hirota's bilinear method [Hirota (1971)], Bäcklund transformation [Miurs (1978)], Painlevé expansion [Weiss, Tabor and Carnevale (1983)], sine-cosine method [Yan (1996)], homogeneous balance method [Wang (1996)], homotopy perturbation method

[El-Shahed (2005), He (2005), Mohyud-Din and Noor (2008)], variational iteration method [He (1999), Noor and Mohyud-Din (2008a,b)], tanh-function method [Zayed et al. (2004), Abudsalam (2005), Zhang (2007a), Zhang and Xia (2006c)], algebraic method [Hu (2005), Yomba (2006), Zhang and Xia (2006b)], auxiliary equation method [Sirendaoreji (2003), Zhang and Xia (2007a,b)], and Exp-function method [Boz and Bekir (2008), He and Xu (2006), Yusufoglu (2008), Zhang (2008), Zhu (2007)].

Recently, a useful method called F -expansion method [Wang, et al. (2003), Wang and Zhou (2003), Zhou et al. (2003)] was proposed to construct periodic wave solutions of NLPDEs, which can be thought of as an over-all generalization of Jacobi elliptic function expansion method [Liu et al. (2001), Fu, et al. (2001), Parkes et al. (2002)]. The F -expansion method was later extended in different manners [Chen et al. (2005), Liu and Yang (2004), Ren and Zhang (2006), Wang and Li (2005), Zhang (2007c)]. More recently, we proposed a generalized F -expansion method [Zhang (2006, 2007b), Zhang and Xia (2006a)] to construct exact solutions which contain not only the results obtained by the methods of Chen, et al. (2005), Wang and Li (2005), Zhang (2007c) but also a series of new and more general exact solutions, in which the restriction on ξ as merely a linear function of t, x_1, x_2, \dots, x_m and the restriction on the coefficients being constants are all removed.

In the present paper, we further improve and develop our work made in [Zhang (2006), (2007b), Zhang and Xia (2006a)] for obtaining new and more general exact solutions of the (3+1)-dimensional Jimbo–Miwa (JM) equation [Senthilvelan (2001)]:

$$u_{xxxy} + 3(uu_y)_x + 3u_{xx}\partial_x^{-1}u_y + 3u_xu_y + 3u_{yt} - 3u_{zz} = 0, \tag{1}$$

the Painlevé property, Bäcklund transformation, soliton solutions and doubly periodic solutions were investigated by Lou (1996), Hong and Oh (2000), Zhang and Wu (2002).

The rest of this paper is organized as follows. In Section 2, we give the description of the generalized F -expansion method. In Section 3, we use this method to obtain more general exact solutions of the (3+1)-dimensional JM equation. In Section 4, some conclusions are given.

2. Description of the Generalized F -expansion Method

For a given NLPDE with independent variables $x = (t, x_1, x_2, \dots, x_m)$ and dependent variable u

$$F(u, u_t, u_{x_1}, u_{x_2}, \dots, u_{x_m}, u_{x_1t}, u_{x_2t}, \dots, u_{x_mt}, u_{tt}, u_{x_1x_1}, u_{x_2x_2}, \dots, u_{x_mx_m}, \dots) = 0, \tag{2}$$

we seek its solutions in the more general form

$$u = a_0 + \sum_{i=1}^n \{a_i F^i(\xi) + b_i F^{-i}(\xi) + c_i F^{i-1}(\xi) F'(\xi) + d_i F^{-i}(\xi) F'(\xi)\}, \tag{3}$$

where $a_0 = a_0(x)$, $a_i = a_i(x)$, $b_i = b_i(x)$, $c_i = c_i(x)$, $d_i = d_i(x)$ ($i = 1, 2, \dots, n$) and $\xi = \xi(x)$ are undetermined differentiable functions, $F(\xi)$ in (3) satisfies

$$F'^2(\xi) = PF^4(\xi) + QF^2(\xi) + R, \quad (4)$$

and, hence, holds for $F(\xi)$ and $F'(\xi)$

$$\begin{cases} F''(\xi) = 2PF^3(\xi) + QF(\xi) \\ F'''(\xi) = (6PF^2(\xi) + Q)F'(\xi) \\ F^{(4)}(\xi) = 24P^2F^5(\xi) + 20PQF^3(\xi) + (Q^2 + 12PR)F(\xi) \\ \vdots \end{cases} \quad (5)$$

where P , Q and R are all parameters, the prime denotes $d/d\xi$. Given different values of P , Q and R , the different Jacobi elliptic function solutions $F(\xi)$ can be obtained from (4).

Some special solutions of (4) are listed in Table 1 including several new ones which weren't reported by Zhang (2006, 2007b), Zhang and Xia (2006a). To determine u explicitly, we take the following four steps:

- Step 1.** Determine the integer n by balancing the highest order nonlinear term(s) and the highest order partial derivative of u in (2).
- Step 2.** Substitute (3) along with (4) and (5) into (2) and collect all the coefficients of $F^i(\xi)F^j(\xi)$ ($i = 0, 1; j = 0, \pm 1, \pm 2, \dots$), then set each coefficient to zero to derive a set of over-determined partial differential equations for a_0, a_i, b_i, c_i, d_i ($i = 1, 2, \dots, n$) and ξ .
- Step 3.** Solve the system of over-determined partial differential equations obtained in Step 2 for a_0, a_i, b_i, c_i, d_i and ξ by use of *Mathematica*.
- Step 4.** Select the appropriate P, Q, R and $F(\xi)$ from Table 1 and substitute them along with a_0, a_i, b_i, c_i, d_i and ξ into (3) to obtain non-degenerate Jacobi elliptic function solutions of (2) (see Table 2 for $F'(\xi)$), from which hyperbolic function solutions and trigonometric function solutions can be obtained in the limit cases when $m \rightarrow 1$ and $m \rightarrow 0$ (see Tables 3 and 4).

Table 1. Relations between values of P, Q, R and corresponding $F(\xi)$ of (4)

P	Q	R	$F(\xi)$
m^2	$-(1+m)^2$	1	$\text{sn}\xi, \text{cd}\xi = \frac{\text{cn}\xi}{\text{dn}\xi}$
$-m^2$	$2m^2 - 1$	$1 - m^2$	$\text{cn}\xi$
-1	$2 - m^2$	$m^2 - 1$	$\text{dn}\xi$
1	$-(1+m)^2$	m^2	$\text{ns}\xi = \text{sn}^{-1}\xi, \text{dc}\xi = \frac{\text{dn}\xi}{\text{cn}\xi}$
$1 - m^2$	$2m^2 - 1$	$-m^2$	$\text{nc}\xi = \text{cn}^{-1}\xi$
$m^2 - 1$	$2 - m^2$	-1	$\text{nd}\xi = \text{dn}^{-1}\xi$

$1 - m^2$	$2 - m^2$	1	$sc\xi = \frac{sn\xi}{cn\xi}$
$-m^2(1 - m^2)$	$2m^2 - 1$	1	$sd\xi = \frac{sn\xi}{dn\xi}$
1	$2 - m^2$	$1 - m^2$	$cs\xi = \frac{cn\xi}{sn\xi}$
1	$2m^2 - 1$	$-m^2(1 - m^2)$	$ds\xi = \frac{dn\xi}{sn\xi}$
$\frac{1}{4}$	$\frac{1 - 2m^2}{2}$	$\frac{1}{4}$	$ns\xi \pm cs\xi, \frac{cn\xi}{\sqrt{1 - m^2}sn\xi \pm dn\xi}$
$\frac{1 - m^2}{4}$	$\frac{1 + m^2}{2}$	$\frac{1 - m^2}{4}$	$nc\xi \pm sc\xi$
$\frac{1}{4}$	$\frac{m^2 - 2}{2}$	$\frac{m^2}{4}$	$ns\xi \pm ds\xi$
$\frac{m^2}{4}$	$\frac{m^2 - 2}{2}$	$\frac{m^2}{4}$	$sn\xi \pm icn\xi, \frac{dn\xi}{\sqrt{1 - m^2}sn\xi \pm cn\xi}$
$-\frac{1}{4}$	$\frac{1 + m^2}{2}$	$-\frac{(1 - m^2)^2}{4}$	$mcn\xi \pm dn\xi$
$\frac{1}{4}$	$\frac{1 - 2m^2}{2}$	$\frac{1}{4}$	$msn\xi \pm idn\xi, \frac{sn\xi}{1 \pm cn\xi}$
$\frac{1}{4m^2}$	$\frac{1 - 2m^2}{2}$	$\frac{m^2}{4}$	$\frac{dn\xi}{\sqrt{\frac{1 - m^2}{m^2} \pm cn\xi}}$
$\frac{m^2}{4}$	$\frac{m^2 - 2}{2}$	$\frac{1}{4}$	$\frac{sn\xi}{1 \pm dn\xi}$
$\frac{1 - m^2}{4}$	$\frac{1 + m^2}{2}$	$\frac{m^2 - 1}{4}$	$\frac{dn\xi}{1 \pm msn\xi}$
$\frac{1 - m^2}{4}$	$\frac{1 + m^2}{2}$	$\frac{1 - m^2}{4}$	$\frac{cn\xi}{1 \pm sn\xi}$
$\frac{(1 - m^2)^2}{4}$	$\frac{1 + m^2}{2}$	$\frac{1}{4}$	$\frac{sn\xi}{cn\xi \pm dn\xi}$
$\frac{m^4}{4}$	$\frac{2 - m^2}{2}$	$\frac{1}{4}$	$\frac{cn\xi}{\sqrt{1 - m^2} \pm dn\xi}$

Table 2. Derivatives of Jacobi Elliptic Functions

$sn'\xi = cn\xi dn\xi$	$cd'\xi = -(1 - m^2)sd\xi nd\xi$	$cn'\xi = -sn\xi dn\xi$
$dn'\xi = -m^2sn\xi cn\xi$	$ns'\xi = -cs\xi ds\xi$	$dc'\xi = (1 - m^2)nc\xi sc\xi$
$nc'\xi = sc\xi dc\xi$	$nd'\xi = m^2cd\xi sd\xi$	$sc'\xi = dc\xi nc\xi$
$cs'\xi = -ns\xi ds\xi$	$ds'\xi = -cs\xi ns\xi$	$sd'\xi = nd\xi cd\xi$

Table 3. Jacobi Elliptic Functions Degenerate into Hyperbolic Functions when $m \rightarrow 1$

$\operatorname{sn}\xi \rightarrow \tanh\xi$	$\operatorname{cn}\xi \rightarrow \operatorname{sech}\xi$	$\operatorname{dn}\xi \rightarrow \operatorname{sech}\xi$
$\operatorname{sc}\xi \rightarrow \sinh\xi$	$\operatorname{sd}\xi \rightarrow \sinh\xi$	$\operatorname{cd}\xi \rightarrow 1$
$\operatorname{ns}\xi \rightarrow \operatorname{coth}\xi$	$\operatorname{nc}\xi \rightarrow \cosh\xi$	$\operatorname{nd}\xi \rightarrow \cosh\xi$
$\operatorname{cs}\xi \rightarrow \operatorname{csch}\xi$	$\operatorname{ds}\xi \rightarrow \operatorname{csch}\xi$	$\operatorname{dc}\xi \rightarrow 1$

Table 4. Jacobi Elliptic Functions Degenerate into Trigonometric Functions when $m \rightarrow 0$

$\operatorname{sn}\xi \rightarrow \sin\xi$	$\operatorname{cn}\xi \rightarrow \cos\xi$	$\operatorname{dn}\xi \rightarrow 1$
$\operatorname{sc}\xi \rightarrow \tan\xi$	$\operatorname{sd}\xi \rightarrow \sin\xi$	$\operatorname{cd}\xi \rightarrow \cos\xi$
$\operatorname{ns}\xi \rightarrow \operatorname{csc}\xi$	$\operatorname{nc}\xi \rightarrow \sec\xi$	$\operatorname{nd}\xi \rightarrow 1$
$\operatorname{cs}\xi \rightarrow \cot\xi$	$\operatorname{ds}\xi \rightarrow \operatorname{csc}\xi$	$\operatorname{dc}\xi \rightarrow \sec\xi$

Remark 1. The solution hypothesis (3), based on the most existing work, is a trial solution of NLPDEs. It can be easily found that (3) is more general than those introduced in literature. To be more precise, if $b_i = c_i = d_i = 0$, a_0 and a_i are constants, and ξ is merely a linear function of x and t , then (3) becomes those constructed by Wang, et al. (2003), Wang and Zhou (2003), Zhou et al. (2003). If $c_i = d_i = 0$, a_0 , a_i and b_i are constants, and ξ is merely a linear function of x and t , then (3) reduces to that used by Wang and Li (2005). If $c_i = d_i = 0$, then (3) changes into the one employed by Chen et al. (2005). If $d_i = 0$, then (3) gives the general one proposed by Zhang (2007c).

3. Exact Solutions of the JM Equation

In order to obtain exact solutions of (1), we suppose $u_y = v_x$ and set the integral constant as zero, then (1) becomes

$$u_{xxx} + 6u_x u_y + 3uv_{xx} + 3u_{xx} v + 3u_{yt} - 3u_{zz} = 0, \quad (6)$$

$$u_y - v_x = 0. \quad (7)$$

According to Step 1, we get $n = 2$ for u and v . To search for explicit and exact solutions, we assume that (6) and (7) have the following formal solutions:

$$u = a_0 + a_1 F(\xi) + a_2 F^2(\xi) + b_1 F^{-1}(\xi) + b_2 F^{-2}(\xi) + c_1 F'(\xi) + c_2 F'(\xi)F(\xi) + d_1 F'(\xi)F^{-1}(\xi) + d_2 F'(\xi)F^{-2}(\xi), \quad (8)$$

$$v = A_0 + A_1 F(\xi) + A_2 F^2(\xi) + B_1 F^{-1}(\xi) + B_2 F^{-2}(\xi) + C_1 F'(\xi) + C_2 F'(\xi)F(\xi) + D_1 F'(\xi)F^{-1}(\xi) + D_2 F'(\xi)F^{-2}(\xi), \quad (9)$$

where $a_0 = a_0(y, z, t)$, $a_i = a_i(y, z, t)$, $b_i = b_i(y, z, t)$, $c_i = c_i(y, z, t)$, $d_i = d_i(y, z, t)$,
 $A_0 = A_0(y, z, t)$, $A_i = A_i(y, z, t)$, $B_i = B_i(y, z, t)$, $C_i = C_i(y, z, t)$, $D_i = D_i(y, z, t)$ ($i = 1, 2$),
 $\eta = \eta(y, z, t)$, $\xi = kx + \eta$, k is a nonzero constant.

With the aid of *Mathematica*, substituting (8) and (9) along with (4) and (5) into (6) and (7), the left-hand sides of (6) and (7) are converted into two polynomials of $F^i(\xi)F^j(\xi)$ ($i = 0, 1; j = 0, \pm 1, \pm 2, \dots$), then setting each coefficient to zero, we get a set of over-determined partial differential equations for $a_0, a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2, A_0, A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2$ and η . Solving the set of over-determined partial differential equations by use of *Mathematica*, we get the following results:

Case 1.

$$\begin{aligned} a_0 &= zg_1(t) + g_2(t), \quad a_1 = 0, \quad a_2 = -k^2P, \quad b_1 = 0, \quad b_2 = -k^2R, \\ c_1 &= \pm k^2\sqrt{P}, \quad c_2 = 0, \quad d_1 = 0, \quad d_2 = \pm k^2\sqrt{R}, \\ A_0 &= \frac{3\eta_z^2 + k^3(\pm 6\sqrt{PR} - Q)\eta_y - 3k(zg_1(t) + g_2(t))\eta_y - 3\eta_y\eta_t}{3k^2}, \quad A_1 = 0, \\ A_2 &= -kP\eta_y, \quad B_1 = 0, \quad B_2 = -kR\eta_y, \quad C_1 = \pm k\sqrt{P}\eta_y, \quad C_2 = 0, \quad D_1 = 0, \\ D_2 &= \pm k\sqrt{R}\eta_y, \quad \eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t), \end{aligned}$$

where $f_1(y), f_2(t), f_3(y), f_4(t), g_1(t)$ and $g_2(t)$ are arbitrary analytic functions of the indicated variables, l is an arbitrary constant.

Case 2.

$$\begin{aligned} a_0 &= zg_1(t) + g_2(t), \quad a_1 = 0, \quad a_2 = -k^2P, \quad b_1 = 0, \quad b_2 = 0, \quad c_1 = \pm k^2\sqrt{P}, \\ c_2 &= 0, \quad d_1 = 0, \quad d_2 = 0, \quad A_0 = \frac{3\eta_z^2 - k^3Q\eta_y - 3k(zg_1(t) + g_2(t))\eta_y - 3\eta_y\eta_t}{3k^2}, \\ A_1 &= 0, \quad A_2 = -kP\eta_y, \quad B_1 = 0, \quad B_2 = 0, \quad C_1 = \pm k\sqrt{P}\eta_y, \quad C_2 = 0, \quad D_1 = 0, \\ D_2 &= 0, \quad \eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t), \end{aligned}$$

where $f_1(y), f_2(t), f_3(y), f_4(t), g_1(t)$ and $g_2(t)$ are arbitrary analytic functions of the indicated variables, l is an arbitrary constant.

Case 3.

$$\begin{aligned} a_0 &= zg_1(t) + g_2(t), \quad a_1 = 0, \quad a_2 = -2k^2P, \quad b_1 = 0, \quad b_2 = -2k^2R, \quad c_1 = 0, \\ c_2 &= 0, \quad d_1 = 0, \quad d_2 = 0, \quad A_0 = \frac{3\eta_z^2 - 4k^3Q\eta_y - 3k(zg_1(t) + g_2(t))\eta_y - 3\eta_y\eta_t}{3k^2}, \\ A_1 &= 0, \quad A_2 = -kP\eta_y, \quad B_1 = 0, \quad B_2 = -kR\eta_y, \quad C_1 = 0, \quad C_2 = 0, \quad D_1 = 0, \\ D_2 &= 0, \quad \eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t), \end{aligned}$$

where $f_1(y), f_2(t), f_3(y), f_4(t), g_1(t)$ and $g_2(t)$ are arbitrary analytic functions of the indicated variables, l is an arbitrary constant.

Case 4.

$$\begin{aligned}
 a_0 &= zg_1(t) + g_2(t), \quad a_1 = 0, \quad a_2 = -2k^2P, \quad b_1 = 0, \quad b_2 = 0, \quad c_1 = 0, \\
 c_2 &= 0, \quad d_1 = 0, \quad d_2 = 0, \quad A_0 = \frac{3\eta_z^2 - 4k^3Q\eta_y - 3k(zg_1(t) + g_2(t))\eta_y - 3\eta_y\eta_t}{3k^2}, \\
 A_1 &= 0, \quad A_2 = -kP\eta_y, \quad B_1 = 0, \quad B_2 = 0, \quad C_1 = 0, \quad C_2 = 0, \quad D_1 = 0, \\
 D_2 &= 0, \quad \eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t),
 \end{aligned}$$

where $f_1(y), f_2(t), f_3(y), f_4(t), g_1(t)$ and $g_2(t)$ are arbitrary analytic functions of the indicated variables, l is an arbitrary constant.

Selecting $F(\xi) = ns\xi, P = 1, Q = -(1 + m^2), R = m^2$ from Table 1, using Case 1 and Table 2, we obtain combined non-degenerate Jacobi elliptic function solutions:

$$u = zg_1(t) + g_2(t) - k^2ns^2\xi - k^2m^2sn^2\xi \mp k^2cs\xi ds\xi \mp k^2mcn\xi dn\xi, \tag{10}$$

$$\begin{aligned}
 v &= \frac{3\eta_z^2 + k^3(\pm 6m + 1 + m^2)\eta_y - 3k(zg_1(t) + g_2(t))\eta_y - 3\eta_y\eta_t}{3k^2} - k\eta_y ns^2\xi \\
 &\quad - km^2\eta_y sn^2\xi \mp k\eta_y cs\xi ds\xi \mp km\eta_y cn\xi dn\xi,
 \end{aligned} \tag{11}$$

where $\xi = kx + \eta, \eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t)$.

Selecting again $F(\xi) = ns\xi \pm cs\xi, P = 1/4, Q = -(1 - 2m^2)/2, R = 1/4$ from Table 1, using Case 1 and Table 2, we obtain combined non-degenerate Jacobi elliptic function solutions:

$$\begin{aligned}
 u &= zg_1(t) + g_2(t) - \frac{k^2}{4}(ns\xi \pm cs\xi)^2 - \frac{k^2}{4} \frac{1}{(ns\xi \pm cs\xi)^2} \\
 &\quad \mp \frac{k^2}{2}(cs\xi ds\xi \pm ns\xi ds\xi) \mp \frac{k^2}{2} \frac{\pm ds\xi}{ns\xi \pm cs\xi},
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 v &= \frac{6\eta_z^2 + k^3(\pm 3 - 1 + 2m^2)\eta_y - 6k(zg_1(t) + g_2(t))\eta_y - 6\eta_y\eta_t}{6k^2} - \frac{k}{4}\eta_y(ns\xi \pm cs\xi)^2 \\
 &\quad - \frac{k}{4}\eta_y \frac{1}{(ns\xi \pm cs\xi)^2} \mp \frac{k}{2}\eta_y(cs\xi ds\xi \pm ns\xi ds\xi) \mp \frac{k}{2}\eta_y \frac{\mp ds\xi}{ns\xi \pm cs\xi},
 \end{aligned} \tag{13}$$

where $\xi = kx + \eta, \eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t)$.

In the limit case when $m \rightarrow 1$, from (12) and (13) we get hyperbolic function solutions:

$$u = zg_1(t) + g_2(t) - \frac{k^2}{4} (\coth \xi \pm \operatorname{csch} \xi)^2 - \frac{k^2}{4} \frac{1}{(\coth \xi \pm \operatorname{csch} \xi)^2} \mp \frac{k^2}{2} (\operatorname{csch}^2 \xi \pm \coth \xi \operatorname{csch} \xi) \mp \frac{k^2}{2} \frac{\mp \operatorname{csch} \xi}{\coth \xi \pm \operatorname{csch} \xi}, \tag{14}$$

$$v = \frac{6\eta_z^2 + k^3(\pm 3 + 1)\eta_y - 6k(zg_1(t) + g_2(t))\eta_y - 6\eta_y\eta_t}{6k^2} - \frac{k}{4}\eta_y(\coth \xi \pm \operatorname{csch} \xi)^2 - \frac{k}{4}\eta_y \frac{1}{(\coth \xi \pm \operatorname{csch} \xi)^2} \mp \frac{k}{2}\eta_y(\operatorname{csch}^2 \xi \pm \coth \xi \operatorname{csch} \xi) \mp \frac{k}{2}\eta_y \frac{\mp \operatorname{csch} \xi}{\coth \xi \pm \operatorname{csch} \xi}, \tag{15}$$

where $\xi = kx + \eta$, $\eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t)$.

In the limit case when $m \rightarrow 0$, from (12) and (13) we get trigonometric function solutions:

$$u = zg_1(t) + g_2(t) - \frac{k^2}{4} (\csc \xi \pm \cot \xi)^2 - \frac{k^2}{4} \frac{1}{(\csc \xi \pm \cot \xi)^2} \mp \frac{k^2}{2} (\cot \xi \csc \xi \pm \csc^2 \xi) \mp \frac{k^2}{2} \frac{\mp \csc \xi}{\csc \xi \pm \cot \xi}, \tag{16}$$

$$v = \frac{6\eta_z^2 + k^3(\pm 3 - 1)\eta_y - 6k(zg_1(t) + g_2(t))\eta_y - 6\eta_y\eta_t}{6k^2} - \frac{k}{4}\eta_y(\csc \xi \pm \cot \xi)^2 - \frac{k}{4}\eta_y \frac{1}{(\csc \xi \pm \cot \xi)^2} \mp \frac{k}{2}\eta_y(\cot \xi \csc \xi \pm \csc^2 \xi) \mp \frac{k}{2}\eta_y \frac{\mp \csc \xi}{\csc \xi \pm \cot \xi}, \tag{17}$$

where $\xi = kx + \eta$, $\eta = l(z^3 + 6yzt + z^2 + 2yt) + zf_1(y) + zf_2(t) + f_3(y) + f_4(t)$.

To the best of our knowledge, the solutions obtained above have not been reported in the literature. From Cases 1–4, we can also obtain other Jacobi elliptic function solutions, hyperbolic function solutions and trigonometric function solutions, here we omit them for simplicity.

4. Conclusions

In this paper, we have successfully constructed new and more general exact solutions with six arbitrary analytic functions of the (3+1)-dimensional JM equation including single and combined non-degenerate Jacobi elliptic function solutions, hyperbolic function solutions and trigonometric function solutions. These solutions contain six arbitrary analytic functions which can make us discuss the behaviors of solutions and also provide us with enough freedom to construct solutions that may be related to real physical problems. It may be important to explain some physical phenomena.

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